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A Method for Optimising the Weight and Response of Brush-Type Wound-Field Direct Current Motors

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Abstract

Weight optimised direct current (DC) motors have been used lately as actuators for micro-robotics and biomedical equipment. The evolution of these motors originated from the need for lightweight compact motors with good response in the industry. Many techniques have been used in developing permanent magnet (PM) and wound-field (WF) DC motors, such as numerical analysis, selecting adequate magnetic material, finite element analysis and optimisation models. Yet, published articles on optimising the weight and response of WF DC motors reveal the development of WF weight optimised motors with problems for example non-proportional geometry, saturated armature teeth, weak output torque and high operating specific electric loading. To overcome these problems, this paper presents a method that separately optimises wound-field DC motors operating with closed-loop proportional, integral and derivative (PID) controllers. A 900-watts DC motor and its PID controllers are optimised as an example for illustrating the proposed method.

1 Introduction

Direct Current motors are reliable, robust and widely used as actuators in mechatronic systems. Until lately, these motors were known to be bulky and imprecise, which kept them away from high-tech industries. High-tech industries, such as robotics, biomedical engineering, and aerospace engineering need compact, accurate and lightweight DC motors.

In general, both weight optimised and non-optimised DC motors operate within the same ambient conditions and utilise electric power to drive their loads and may be built from standard electric and magnetic material. To make the research outcome directly useful to the industry, in this study standard electric and magnetic motor material are adopted in developing WF weight optimised motors.

The traditional design process of DC motors relies much on trial and error. This process starts with a set of arbitrary values for motor dimensions to develop a motor design. Then, this design is verified and modified to comply with a set of constraints that bound this process. These constraints limit acceptable motor designs to the ones having reasonable production cost, unsaturated magnetic material, uniform and proportional geometry, acceptable heat dissipation and sufficient output power and torque. Usually, many acceptable designs can be related to a specific motor [15,17].

Permanent magnet weight optimised DC motors may have lighter weight and higher efficiency than WF DC motors. However, WF motors have better torque/speed characteristics and wider range of speeds, especially when encountering mechanical disturbance. In fact, the development of WF optimised motors hasn't been popular yet. Research work on optimising the weight of DC motors can be categorised into a) Optimising the weight and/or response of permanent magnet (PM) DC motors [1-13] and b) Optimising the weight and response, and c) optimising the response of wound-field DC motors [14-16].

The weight optimisation of PM motors have been widely investigated by modelling their magnetic circuit, using rare-earth magnets, various magnetic alloys, optimising the air-gap between their rotor and stator, numerical analysis, applying finite element analysis and optimisation methods. However, so far only weight optimisation of WF motors has been performed. According to the literature work this has led to the optimisation of motors with problems, such as saturated armature teeth, non-proportional dimensions and weak output torque as listed in Tables 1.2 and 1.3.
In view of these, an optimisation model was adopted and improved for optimising the weight of WF motors in this study [14]. This effort successfully results in optimising WF motors having lightweight with non-saturated armature teeth and proportional shape. In addition to that, motors optimised by this method show better response than the ones presented by the previous study. A 900-watt motor is optimised in concurrent with its PID controller as an example to illustrate the achievement. The results are compared in Tables 1.2 and 1.3 and will be detailed in later sections.

2 Parameters and Variables of Motors

In this study, the weight minimisation problem of designing optimised DC motors is mapped into an optimisation problem by developing a mathematical model describing the relations between the parameters and variables of such motors in a set of equations.

Motor parameters such as the input voltage, output rotational speed, maximum delivered power, delivered toque and the rating current at that load are considered fixed during the optimisation process and are classified as design parameters. In addition to that, constants such as the resistivity and specific weight of the motor's conductors and core material describing the behaviour and characteristics of standard electrical and magnetic material forming these motors, are also classified as design parameters in this study.

On the other hand, motor length, diameter, armature slot depths and length of armature and field winding copper conductors of motors or their operating air-gap flux densities and specific electric loading are treated as design variables. Functions of motor design parameters and variables, such as the cross-sectional areas of the copper conductors forming the armature and field windings are also proposed as design variables. Table 1.1 lists the set of design parameters and variables proposed in this study.

3 Optimisation Model

The set of equations representing the weight optimisation model of field wound DC motors is listed below [14,17]:

1. **Objective function w:** links the parameters and variables of a motor representing the weight of motors.
$$w = f(\rho_{cu}, A_{wa}, L_{wa}, A_{wf}, L_{wf}, \rho_{fe}, L, D, d_s, D_n)$$
2. **Equality equation h_1:** links the specific electric loading factor of a motor, to the set of parameters and variables corresponding to it in small motors.
$$h_1 = f(ac, P_0, L_{wa}, D, V, d_s) = 0$$
3. **Equality constraint h_2:** links the current in the field winding of a motor, to the set of parameters and variables corresponding to it in small motors.
$$h_2 = f(V, L_{wf}, b_{fc}, f_{cf}, A_{wf}) = 0$$
4. **Equality equation h_3:** links the air gap flux density of a motor, to the set of parameters and variables corresponding to it in small motors.
$$h_3 = f(B_g, P_0, D, L) = 0$$
5. **Equality equation h_4:** links the field winding mean-turn length of a motor, to the set of parameters and variables corresponding to it in small motors.
$$h_4 = f(L_{wf}, L, b_p) = 0$$
6. **Equality equation h_5:** links the product of the diameter and pole of a motor, to the product of parameters and variables corresponding to it in small motors.
$$h_5 = f(D, p, ds) = 0$$
7. **Inequality constraint g_1:** links the parameters of a motor representing the pole pitch ratio to its upper bound 380mm in small motors.
$$g_1 = \pi D / p - 0.38 \le 0$$
8. **Inequality constraint g_2:** links the parameters and variables of a motor representing the armature teeth flux density to its upper bound 1.8T in small motors.
$$g_2 = 2DB_g / (D - 2d_s) - 1.8 \le 0$$
9. **Inequality constraint g_3:** links the parameters and variables of a motor representing the armature peripheral speed to its upper bound 25m/s in small motors.
$$g_3 = \pi Dn - 25 \le 0$$
10. **Inequality constraint g_4:** links the parameters and variables of a motor representing the armature peripheral speed to its lower bound 8m/s in small motors.
$$g_4 = 8 - \pi Dn \le 0$$
11. **Inequality constraint g_5:** links the parameters and variables of a motor representing the length to pole pitch ratio to its upper bound 0.9 in small motors.
$$g_5 = Lp / \pi D - 0.9 \le 0$$
12. **Inequality constraint g_6:** links the parameters and variables of a motor representing the length to pole pitch ratio to its lower bound 0.6 in small motors.
$$g_6 = 0.6 - Lp / \pi D \le 0$$
13. **Inequality constraint g_7:** links the parameters of a motor representing the ratio of the armature slot sizing per diameter ratio to its upper bound 3.5 in small motors.
$$g_7 = 2D / (D - 2d_s) - 3.5 \le 0$$
14. **Inequality constraint g_8:** links the variables of a motor representing the ratio of the armature slot sizing per diameter ratio to its lower bound 2.5 in small motors.
$$g_8 = 2.5 - 2D / (D - 2d_s) \le 0$$

15. **Inequality constraint** g_9 **:** bounds the maximum operating flux density in the air-gap of a small motor to 0.8T.

$$g_9 = B_g - 0.8 \le 0$$

16. **Inequality constraint** g_{10} **:** bounds the minimum operating flux density in the air-gap of a small motor to 0.3T.

$$g_{10} = 0.3 - B_g \le 0$$

17. **Inequality constraint** g_{11} **:** links the parameters and variables of a motor representing the maximum allowable ratio of the armature tooth width to the armature slot depth ratio in small motors.

$$g_{11} = d_s - 2\pi(D - 2d_s)/S \le 0$$

18. **Inequality constraint** g_{12} **:** limits the maximum armature slot depth of a motor to half the armature diameter in small motors.

$$g_{12} = d_s - 0.5D \le 0$$

19. **Inequality constraint** g_{13} **:** links the output power of a motor to the set of parameters and variables representing its lower bound in small motors.

$$g_{13} = P_0 - \pi^2(\Psi)(B_g)(ac)(D^2)(L)(n) \le 0$$

20. **Inequality constraint** g_{14} **:** links the output torque of a motor to the set of parameters and variables representing its lower bound in small motors.

$$g_{14} = T_a - (\pi/2)(\Psi)(B_g)(ac)(D^2)(L) \le 0$$

21. **Inequality constraint** g_{15} **:** links the parameters and variables of a motor representing the cross-sectional area of the main field conductors with the area of the armature slots and bounds their packing limit to 80 % in small motors.

$$g_{15} = f(L, D, d_s, L_{wa}, A_{wa}, p) \le 0$$

22. **Inequality constraint** g_{17} : links motor parameters and variables representing the heat dissipation in the field winding of a motor to the maximum allowable heat dissipation bound 750 in small motors.

$$g_{17} = \frac{(I_f)^2 L_{wf}\ \rho}{2\ pAwf\ \ (Lmtf\ + b_{fc})h_f} - 750 \le 0$$

23. **Inequality constraint** g_{18} **:** links motor variable representing the specific electric loading to its maximum allowable bound in small motors.

$$g_{18} = ac - 20 \le 0$$

24. **Inequality constraint** g_{19} : links motor variable representing the specific electric loading to its minimum allowable bound in small motors.

$$g_{19} = 6 - ac \le 0$$

25. **Inequality constraint** g_{20} : links motor parameters and variables representing the commutating peripheral speed to its maximum allowable bound in small motors.

$$g_{20} = 1.9nD - 15 \le 0$$

26. **Inequality constraint** g_{21} : links motor parameters and variables representing the current density in the field winding conductors of a motor to its maximum allowable bound in small motors.

$$g\,21 = \frac{A_{tf\,1}}{b_{fc} \cdot f_{cf}\ .hf\ .(10)\wedge 6} - 2.5 \le 0$$

27. **Inequality constraint** g_{22} : links motor parameters and variables representing the current density in the field winding conductors of a motor to its minimum allowable bound in small motors.

$$g\,22 = 1.2 - \frac{A_{tf\,1}}{b_{fc} \cdot f_{cf}\ .hf\ .(10)\wedge 6} \le 0\ .$$

4 Space of Motor Design

The design variables and parameters of DC motors are inserted into the optimisation model for evaluating optimal motor designs.

The length, diameter, armature slot depth and the length of the armature and field winding conductors are continuously varied between predicted upper and lower limits. However, the operating flux densities and specific electric loading in various parts of motor designs are varied within the constraints listed below to achieve optimal designs [17]:

a. A*ir-gap flux density,* B_g

The operating air-gap flux density in small motors is usually bounded between 0.3 and 0.8 T. The limiting factor in choosing this variable is the magnetic saturation of the armature teeth of motors. Motors with saturated teeth have higher power consumption and heating losses than unsaturated ones. The magnetic density in the armature teeth is proportional to the air-gap flux density in DC motors. On the other hand, motors with low operating air-gap flux densities have igher weights. Therefore, in designing motors the selection of the operating air-gap flux density is critical.

b. **Specific e*lectric loading, ac***

It is usually bounded between 8000 to 24000 ampere-conductor/m in small DC motors. For a specific machine, increasing the value of *ac* leads to the increase in the number of armature conductors and to the decrease in the length of the diameter and weight of a motor. However, increasing *ac* beyond a certain limit in small motors may results in high temperature rise, low efficiency and expensive construction cost

c. Armature t*eeth flux density, Bt*

The value of the armature teeth flux density, *Bt*, *in a* DC motor is proportional to the air- gap flux density B_g. As this value is varied the flux density in the teeth of the armature of motors should not exceed 1.8T (the saturation limit).

d. *Armature core flux density,* B_c

The operating armature core flux density B_c is proposed at 1.5T in this paper. This value is used for evaluating the area and length of the armature core of motors.

e. *Yoke flux density,* B_y

The operating yoke flux density B_y *is* selected at 1.5T in this method. The cross sectional area of the yoke is evaluated at this value to minimise its losses accordingly.

f. *Pole flux density,* B_p

The value of the pole flux density is proposed at its upper limit, the saturation limit 1.8T. As the magnetic flux in the poles vary the area of the pole is also varied. The pole dimensions are satisfactory only if the main field wound coil is accommodated on the pole successfully with acceptable heat retardation.

The space of motor designs is searched by alternating these variables with the variables corresponding to the dimensions of motors between their bounds and by simultaneously inserting them along side with motor parameters into the optimisation model. The optimal motor design having the minimum weight is selected from the set of optimal motor designs. The parameters of this motor are then mapped into the control model for evaluating the optimal controller design.

5 Optimisation Model of Motor/Controller System

The mechanical parameters of the optimal motor design corresponding to its armature resistance ***Ra***, armature inductance *Ln**a***, motor constant *Km* and its moment of inertia *Ja* are evaluated and inserted into the transfer function G(s)=(W/V) of the motor/controller system. The response of the system is evaluated against a generated bounded space of PID controller designs. The set of space of the system responses is evaluated for the first second of its operation from steady state similar to the previous study [14].

$$G(s) = \frac{k_m(k_i + s.k_p + s^2.k_d)}{s(s.J_a + T)(s.L_{na} + R_a) + k_m{}^2(k_i + s.k_p + s^2 k_d)}$$

The absolute error ***E*** between the desired rotational speed of the optimal motor design and the output speed of the motor/controller system during the first second of their operation is used in this paper for developing the performance index ***J*** of the optimal motor and controller design [14]:

$$E(s) = (W(s) - 6.28n(s))$$

$J = abs(Int(E(t))dt$ (t= 0 to 1 seconds)

The set of performance indices of the motor controller system is evaluated. The system having the least performance index is considered optimal. This discrete optimisation procedure has led to the development of systems with only one active constraint g16. Tables 1.2 and 1.3 compare the results of this method with the results presented by Papalambros and Reyer for optimising the weight and response of a 900-Watts motor [14].

6 Results and Discussions

The success of this technique is attributed to the discrete optimisation method considered in optimising the weight of motors and the change of one design parameter into variable which resulted from re-examining all the design parameters and variables.

The active (violated) non-equality constraints violated by this method and the previous method in optimising a 900-watts motor/controller system are compared in Table 1.3.

The previous optimisation method [14] was the first study to investigate and optimise the weight of WF DC motors successfully. Yet, it has led to the design of a 900-watt motor system with saturated magnetic teeth, non-proportional length to pole pitch ratios, weak output torque and utilised a high specific electric loading value. However, this proposed method when applied to the same 900-watts system has developed a system with only one active constraint and a motor having less weight than the previous method (Tables 1.2,1.3).

7 Conclusion & Future works

A design model and an optimisation method for optimising direct current motors are presented in this paper. A 900-Watts motor is optimised as a case study. As demonstrated in the example, this technique is useful in producing optimised motors with high efficiencies and lower weights.

A coupled motor/controller system will be developed in the future for simultaneously optimising the weight and performance of wound-field DC motors. A prototype experimental motor is being developed to verify the developed optimisation methodology. The results will be published in another paper.

Table 1.1: Proposed design parameters and variables

a	Number of parallel paths-function of design parameters
ac	Specific electric loading-design variable
A_{Tf1}	Total ampere turns needed to pass flux in the magnetic material of motors-function of design variables
A_{wa}	Cross-sectional area of armature conductors -function of design parameters
A_{wf}	Cross sectional area of main field conductors -function of variables and design parameters
b_{fc}	Depth of main field winding coil-design variables
b_p	Width of main field pole-function of design variables
B_c	Motor core flux density-assumed constant
B_g	Air-gap flux density-design variable
B_p	Main pole flux density-assumed constant
B_t	Armature teeth flux density-function of design variables
B_y	Stator or yoke flux density-assumed constant
D	Diameter of motor or armature design variable
D_n	Shaft bore or armature bore-assumed constant
d_s	Depth of armature slot-design variable
f_{cf}	Copper field winding space factor-assumed constant
h_f	Length of main field pole-function of variables
I_f	Electric current in field winding-function of variables
k_d	Derivative constant of PID controller
k_i	Integral constant of PID controller
k_m	Motor constant
k_p	Proportional constant of PID controller
L	Length of motor-design variable
L_{mtf}	Mean-turn length of field winding
L_{wa}	Length of armature conductors-design variable
L_{wf}	Length of main field winding conductor-design variable
n	Armature rotational speed-design parameter
p	Number of poles-design parameter
P_0	Output power of motor-design parameter
R	Armature winding resistance
S	Number of armature teeth-design parameter
T	Output torque of motor-design parameter
U	Motor efficiency-design parameter
V	Input armature voltage-design parameter

References

[1] Pien Y. P.& Chiao C. T., 2000, *Multiobjective optimal design of high speed brushless DC motor*, Dept. of Mech. Eng., Nat. Taiwan Univ., Taipei, Taiwan. Electric-Machines and Power Systems vol.28, no.1, Jan. 2000, p.13-30.

[2] Changzhi A. et al, 1996. *Electromagnetic analysis and optimal design of NdFeB PM DC motor for ocean robot*, Dept. of Electr. Eng, Shenyang Polytech. Univ., China. IEEE CEFC '96. The Seventh Biennial IEEE Conference on Electromagnetic Field Computation, IEEE, New York, NY, USA, 1996, 498pp.

[3] Seol S. S. et al, 1995, *Optimal design of a moving coil type linear DC motor*. Transactions of the Korean Institute of Electrical Engineers, vol.44, no.7, July 1995,p858-64.

[4] Kopp N.L. & Brown G.W., 1994, *Electric motor/controller design tradeoffs for noise, weight, and efficiency,* Ind. Drives Div., Kollmorgen, Radford, VA, USA, Proceedings of the 1994 Symposium on Autonomous Underwater Vehicle Technology (Cat. No. 94CH3393-6), IEEE, New York, NY, USA, 1994, xii+480pp.

[5] Hanitsch R. & Parspour N, 1992, *Contribution to brushless DC motor design for a heart assist device,* Tech. Univ. Berlin, Inst. Fur Elektrische Maschinen, Germany, Proceedings International Conference on Electrical Machines, Manchester Univ., Manchester, UK, 1992,3 vol. Xvi+1308 pp.

[6] Nelson R, 1992, *Permanent magnet DC motor selection method maximizes electric vehicle range*, Powerconversion & Intelligent Motion, vol. 18, no. 7, July 1992, p. 36-40.

[7] Jones D, 1990, *Praseodymium joins list of rare-earth magnetic material for brushless DC motors,* Powerconversion & Intelligent Motion, vol.16, no.7, July 1990, p25-9.

[8] Kaftanatii V.T., 1988, *Synthesised optimality criteria for magnetic systems and magnetically hard alloys*, Soviet Electrical Engineering vol. 59, no. 5, 1988, p. 66-9.

[9] Frielich S.D., 1987, *Computer modelling helps choice of DC servo motors*. Robotics World, vol. 5, no. 8, Aug. 1987, p 28-9.

[10] Mandryka O.R., *Optimising the air-gap of direct current traction motor*. Soviet Electrical Engineering, vol. 56, no. 10, 1985, p. 49-52.

[11] Savani A. & Fasola G., 1983, *Computer aided design of heavily saturated DC motors*, Istituto di Elettrotecnica. Univ. di Pavia, Italy, IEEE Transactions on Magnetics, vol. MAG-19, no. 5, Sept. 1983, p. 2088-90.

[12] Corbett A.E. & Roerig C.S., 1978, *Selecting permanent magnet materials for disc-armature DC motor*, Univ. of Warwick, Coventry, UK, Proceedings of the International Conference on Electrical Machines, Katholieke Univ. Leuven, Brussels, Belgium, 1978, 3 vol. (880+808+228) pp.
[13] Itoh H., 1981, *Optimum magnetic circuit design for small DC servomotors based on numerical analysis*, Yokosuka Electrical Communications Lab., NTT, Japan, Transactions of the Institute of Electronics and Communication Engineers of Japan, Section E (English vol. E64, no. 1, Jan. 1981, p. 44.
[14] Papalambros P. & Reyer J. AR, 1999, *Optimal design and control of an electric DC motor*, Proceedings of the 1999 ASME Design Engineering Technical Conferences, September 12-15, 1999, Las Vegas, Nevada.
[15] Burba et al, 1986, *Field weakening electric vehicle power train*, IEE Colloquium on 'Electric Vehicle Electronics and Control (Digest No. 123), IEE, London, UK, 1986, 44pp. P.3/1-3.
[16] Egami et al, 1985, Efficiency optimised speed control system synthesis method based on improved optimal regulator theory-applications to separately excited DC motor system, IEEE-Transactions on Industrial Electronics, vol IE-32, no. 4, Nov. 1985, p372-80.
[17] Hamdi E. S., 1994, *Design of Small Electrical Machines*, John Wiley & Sons Ltd

Table 1.2: A comparison between the results of this study and the previous one in optimising the weight of a 900-watt DC motor/controller system [14].

Symbol	Description	Optimal Design and Control (Current Study)	Single Pass Design First (SP-DF) (Previous Study)
W	Weight (kg)	3.36	7.9
J	Performance Index	39.0	69.04
L	Rotor length (m)	0.0879	0.0974
D	Rotor Diameter (m)	0.068	0.0689
Lwa	Armature wire length (m)	40	89.5
Lwf	Field wire length (m)	50	220
ds	Arm. lam. slot depth (cm)	0.0072	0.01
Kp	PID proportional coefficient	1.6	1.24
Ki	PID integral coefficient	26	6.89
Kd	PID derivative coefficient	9	0
Vt	Voltage (v)	45 / 24	45 / 28.2
Tmin	Torque (N.m)	6.0 / 5.59	6.0 / 5.6
Pmin	Power (kw)	900 / 860	900 / 640
n	Rotational speed (rev/sec)	45 / 26	45 / 24.6

Note: Top numbers in divided cells represent motor input values, bottom numbers in divided cells represent motor output values.

Table 1.3: A Comparison between active constraints in the current study and the previous one [14&17}

Constraint		Previous Method	Proposed Method
Number	Description		
g_2	UB magnetic saturation	■	
g_5	UB length to pole pitch ratio	■	
g_{13}	UB specific electrical loading	■	
g_{16}	Motor design torque	■	■

Note: UB: upper bound, LB: lower bound, shaded areas correspond to active constraints.