Research Journal of Applied Sciences, Engineering and Technology 5(23): 5478-5483, 2013
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: December 08, 2012 Accepted: January 19, 2013 Published: May 28, 2013

Numerical Study on the Influence of Different Waving Bottom Form on the Fluid Surface Wave

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Abstract: In the present study, the effect of waving bottom on the surface wave is studied. Basing on the fundamental equations of potential flow theory and boundary conditions, using the multiple scales perturbation method to derive the first-order and the second-order approximate equation which the fluid surface waves satisfied in the presence of waving bottom. Under the second-order approximation, the fluid surface waveform in first-order approximate equation is numerically simulated with MATLAB in the presence of different waving bottom form. The results show that: the fluid surface waveform is composed of a harmonic wave which has the same frequency with waving bottom and a pair of KdV solitary waves that spread to both the right and the left side when the waving bottom wave is a harmonic wave; and when the waving bottom is a solitary wave packet, it consists of a solitary wave which is closely related to the specific form of waving bottom and a couple of KdV solitary waves. With the development of time, three waves in fluid surface do not affect each other and they propagate independently. Thus it can be seen the waving bottom is effective for maintaining surface wave energy balance income and expenditure in the spreading process.

Keywords: Multiple scales perturbation method, numerical simulation, surface wave, waving bottom

INTRODUCTION

In the theoretical study of fluid surface waves, some considered the flow with a fixed bottom boundary and some considered the bottom boundary with spatial variation, however very few ones studied the time-varying bottom boundary. But in practical problems, there are many phenomena associated with the waving bottom, such as the vibrational wall in the experiment of the non-propagating solitary wave (Cui *et al*., 1988, 1991), the undulating substrate in the coating industry, a tsunami caused by seabed earthquake, the platform vibration in marine engineering and the movement of seabed sediment (Zhong and Yan, 1992; Jiang *et al*., 2004), etc. The basements of these phenomena often change with time. In addition, the bottom changes with time also exist in elastic bottom. The understanding of the free vibration characteristics of the fluid-structure interaction plays a significant role in various branches of engineering, for example, the propellant in space vehicles can be free from resonance, large-capacity oil containers in the petrochemical industry can be kept from the damage of earthquakes, very large floating oil storage tanks, ships and submarines can be avoided or reduced localized vibrations (Mohapatra and Sahoo, 2011). Therefore, the theoretical and numerical studies of the issue are of great practical significance.

There was a substantial growth in the theoretical and numerical studies of the fluid surface wave in the past decades. Madsen and Mei (1969) have numerical calculated the solitary waves propagating on the uneven bottom and found splitting phenomenon of solitary wave. Johnson (1973) researched solitary wave propagation over slowly varying bottom and obtained the height formula surface rised when a solitary wave was split into N solitary waves. Davies (1982) used perturbation theory to study the reflection problem of the surface water wave energy which was caused by seabed fluctuations. Matsuno (1993) theoretically studied the nonlinear evolution of two-dimensional, incompressible, inviscid, irrotational fluid surface gravity waves in any depth uneven bottom. Mallard *et al*. (1977) discussed the wave propagation over a flexible elastic bottom and found that the wave characteristic was significantly affected by shear modulus of elastic bed. Dawson (1978) analyzed the wave motion over an elastic bed including the effect of soil inertia. Davies and Heathershaw (1984) studied the propagation of surface wave over sinusoidally varying topography and found that between topography and surface-wave exists resonant interaction. Zhang and Zhu (1997) studied resonant transcritical flow over a wavy bed, for a finite-extent bed, it is shown that the linear model is valid and a steady state exists even within the resonant regime, with an “upstream influence”. Wu *et al*. (2006a, b) used the perturbation method to derive the fKdV equation which nonlinear

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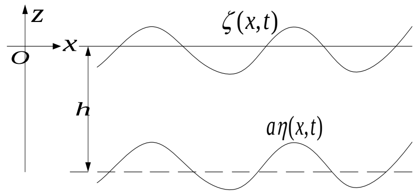


Fig. 1: Schematic representation of the flow geometry

surface wave met, numerically simulated by MATLAB software combined with pseudo-spectral method and analyzed the impact of varying bed and waving bed on the nonlinear surface wave. Zhong and Yan (1992) studied the influence of waving bed on the surface solitary wave theoretically. Mohapatra and Sahoo (2011) developed a hydroelastic model to deal with surface gravity wave interaction with an elastic bed based on the small amplitude water wave theory and plate deflection in finite water depth. In all the studies mentioned in the brief review above, they focuses on the influence of the fixed bottom and varying bottom on the fluid surface wave and their research methods are diverse. But the extension to the fluid surface wave propagating on the waving bottom has, to our knowledge, been considered very few. With this present situation, the study will discuss the effect of waving bottom on the surface wave. Assume that the waving bottom wave is long wave and small amplitude.

In this study, the surface wave propagation over a waving bottom is analyzed. Using the multiple scales perturbation method to derive the first-order and the second-order approximate equation fluid surface waves satisfied in the presence of waving bottom. The fluid surface waveform is simulated with the MATLAB software in the presence of different waving bottom form.

GOVERNING EQUATION AND PERTURBATION SOLVING

Physical model: In the study, we consider the two-dimensional motion of irrotational, inviscid, incompressible ideal fluid and bottom boundary is waving bottom. The physical model is shown in Fig. 1. The depth of the fluid is h without waving bottom. $\zeta(x, t)$ represents the free surface elevation, $a\eta$ (x, t) is the function of waving bottom (where α is the amplitude of waving bottom), $\varphi = (x, z, t)$ is the disturbed velocity potential of the fluid.

Assuming that the waving bottom and the fluid surface displacement are of small amplitude. Introducing small parameter ε:

$$\varepsilon = \left|\zeta\right|_{\max}/h \tag{1}$$

The waving bottom boundary is given as:

$$z_0 = -h + \varepsilon ha\eta(x,t) \tag{2}$$

Governing equation: Ideal fluid flow velocity potential and surface displacement satisfy the basic equations and boundary conditions (The basic equations and boundary conditions ideal fluid flow velocity potential and surface displacement satisfied can be shown) as follows:

$$\varphi_{xx} + \varphi_{zz} = 0 \qquad z_0 < z < \zeta \tag{3a}$$

$$\varphi_z - \varepsilon ah(\varphi_x\eta_x + \eta_t) = 0 \qquad z = z_0 \tag{3b}$$

$$\zeta_t + \varphi_x\zeta_x - \varphi_z = 0 \qquad z = \zeta \tag{3c}$$

$$\varphi_t + g\zeta + \frac{1}{2}(\varphi_x^2 + \varphi_z^2) = 0 \qquad z = \zeta \tag{3d}$$

where, g is the acceleration of gravity.

Dimensionless equation: φ was expanded into Taylor series at z = -h in Eq. (3b) and introduce the following dimensionless variables:

$$z^* = z/h,\ x^* = \sqrt{\varepsilon}\,x/h,\ t^* = \sqrt{\varepsilon g/h}t,\ \phi = h\sqrt{\varepsilon gh}\phi^*,$$
$$\zeta = \varepsilon h\zeta^*$$

So the Eq. (3a)-(3d) can be transformed into the following (the asterisk* is omitted):

$$\varepsilon\varphi_{xx} + \varphi_{zz} = 0 \quad -1 < z < \zeta \tag{4a}$$

$$\varphi_z + \varepsilon a(\varphi_{zz}\eta - \varepsilon\varphi_x\eta_x - \eta_t) + \frac{1}{2}\varepsilon^2 a^2\varphi_{zzz}\eta^2 + O(\varepsilon^3) = 0 \quad z = -1 \tag{4b}$$

$$\varepsilon\zeta_t + \varepsilon^2\varphi_x\zeta_x - \varphi_z = 0 \qquad z = \varepsilon\zeta \tag{4c}$$

$$\varphi_t + \zeta + \frac{1}{2}(\varepsilon\varphi_x^2 + \varphi_z^2) = 0 \qquad z = \varepsilon\zeta \tag{4d}$$

Processing of the velocity potential ϕ: Since ϕ is an analytic function, it can be expanded to the power series of coordinates in the vertical direction.

$$\phi(x,z,t) = \sum_{n=0}^{\infty}(z+1)^n\varphi_n(x,t) \tag{5}$$

Using the Eq. (4a), (4b) and (5), we can obtain the expression of the velocity potential when the error is $O(\varepsilon^3)$:

$$\phi=\phi_0+(z+1)\left(\varepsilon^2 a\phi_{0xx}\eta+\varepsilon^2 a\phi_{0x}\eta_x+\varepsilon a\eta_t\right) -(z+1)^2\frac{1}{2}\varepsilon\phi_{0xx}+(z+1)^3\frac{1}{3!}\varepsilon^2 a\eta_{xxt} +(z+1)^4\frac{1}{4!}\varepsilon^2\phi_{0xxxx}+O\left(\varepsilon^2\right) \tag{6}$$

Solving by perturbation method: Introducing the slowly varying time scale: $\tau = \varepsilon t$ and asymptotically expanding ζ and $\emptyset_{0x}$ by the small parameter ε:

$$\zeta=\zeta_0+\varepsilon\zeta_1+\varepsilon^2\zeta_2+\cdots \tag{7a}$$

$$\phi_{0x}=u_0+\varepsilon u_1+\varepsilon^2 u_2+\cdots \tag{7b}$$

Substituting τ and Eq. (7a-7b) into the Eq. (4c-4d), the following approximate equations of each order can be obtained with the perturbation method.

The first order approximate equation can be given as:

$$\zeta_{0t}+u_{0x}-a\eta_t=0 \tag{8a}$$

$$u_{0t}+\zeta_{0x}=0 \tag{8b}$$

The second order approximate equation has the form of:

$$\zeta_{1t}+u_{1x}=-\left[\zeta_{0\tau}+\left(u_0\zeta_0\right)_x-\frac{1}{6}u_{0xxt}\right] +au_{0x}\eta+au_0\eta_x+\frac{1}{2}a\eta_x xt \tag{9a}$$

$$u_{1t}+\zeta_{1x}=-[u_{0\tau}+\frac{1}{2}(u_0^2)_x-\frac{1}{2}u_{0xxt}]-a\eta_{xtt} \tag{9b}$$

- **When the waving bottom is a harmonic wave:** $\alpha\eta(x, t) = \alpha \cos b (x - c_0 t)$, where b and c_0 are the wave number and the phase velocity of the solitary wave respectively, the solutions of first order approximation equations are:

$$\zeta_0=\frac{ac_0^2}{c_0^2-1}\cos b(x-c_0 t) + f_1(x-t,\tau)+f_2(x+t,\tau) \tag{10a}$$

$$u_0=\frac{ac_0}{c_0^2-1}\cos b(x-c_0 t) + f_1(x-t,\tau)-f_2(x+t,\tau) \tag{10b}$$

When the waving bottom is solitary wave packet: $\alpha\eta(x, t) = \alpha \sec h^2 b (x - c_0 t)$, the solutions of first order approximation equations are:

$$\zeta_0=\frac{ac_0^2}{1-c_0^2}\sec h^2 b(x-c_0 t) + f_1(x-t,\tau)+f_2(x+t,\tau) \tag{11a}$$

$$u_0=\frac{ac_0}{1-c_0^2}\sec h^2 b(x-c_0 t) + f_1(x-t,\tau)-f_2(x+t,\tau) \tag{11b}$$

Substituting the solutions of first order approximation equations Eq. (10a-10b) or Eq. (11a-11b) into the Eq. (9a-9b), according to the multi-scale perturbation method, the following equations are obtained from the conditions for no secular term to appear in second order approximate equations:

$$2f_{1\tau}+3ff_{1r}+\frac{1}{3}f_{1rrr}=0 \tag{12a}$$

$$-2f_{2\tau}+3ff_{2l}+\frac{1}{3}f_{2lll}=0 \tag{12b}$$

where, r = x - t, l = x + t, Eq. (12a-12b) are two standard KdV equations, the solution can be obtained by the traveling wave method, f_1 and f_2 represents the KdV solitary wave which spreads respectively towards the right and the left side.

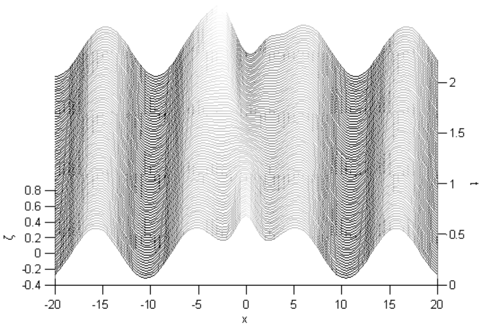
In the next section, under the second-order approximation, the fluid surface waveform is simulated with MATLAB in the presence of different waving bottom form and we analyze the influence of waving bottom on the fluid surface wave.

NUMERICAL SIMULATION RESULTS AND DISCUSSION

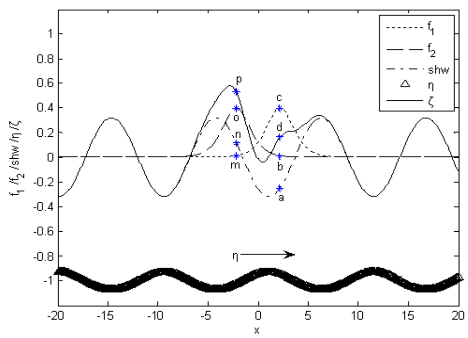
In the study, we give the following parameters assignments: $\alpha \pm 0.8$, b = 0.6, $c_0 = 0.53$, ε 0.09, then we get specific surface wave function. The simulation results for a harmonic waving bottom wave, a convex-waving bottom wave and a concave-waving bottom wave in different time are shown as follows.

The waving bottom bed is a harmonic wave form: The waving bottom bed is a harmonic wave form: $a\eta(x, t) = a \cos b (x - c_0 t)$, the simulation results are shown in Fig. 2.

Figure 2a and c show the waterfall of fluid surface waveform in different time scales. It can be seen from the figures that the fluid surface waveform is composed of a simple harmonic wave and a pair of KdV solitary waves. Over time, part of the superimposed waveform of the harmonic wave and solitary waves change and the amplitude of surface wave appears to the maximum. After superposition, the harmonic wave of fluid surface recovers back to the original shape. It suggests that they



(a)



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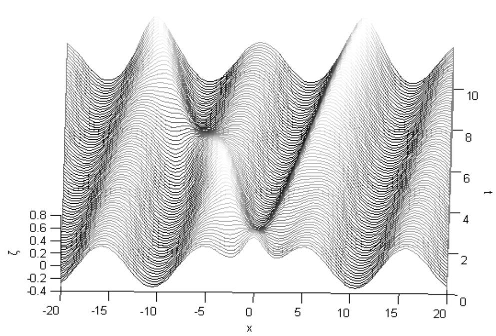

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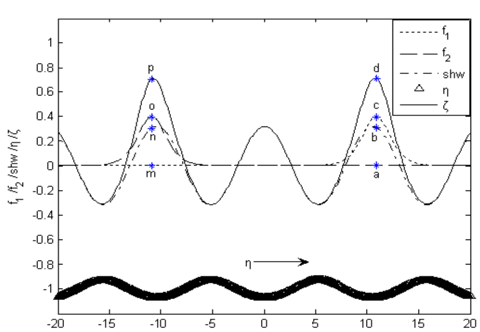
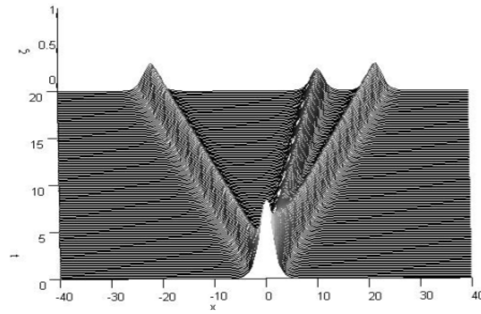
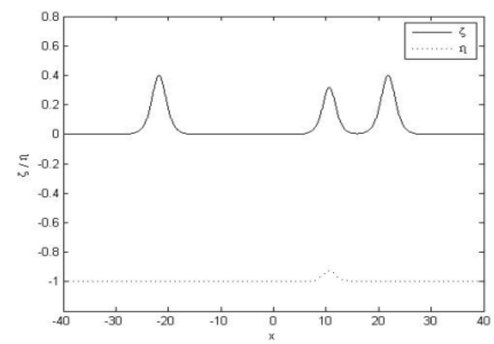


(d)

Fig. 2: The simulation results for a harmonic waving bottom wave, (a) Waterfall of fluid surface waveform; (b) Fluid surface waveform at 2, (c) Waterfall of fluid surface waveform, (d) Fluid surface waveform at 10



(a)



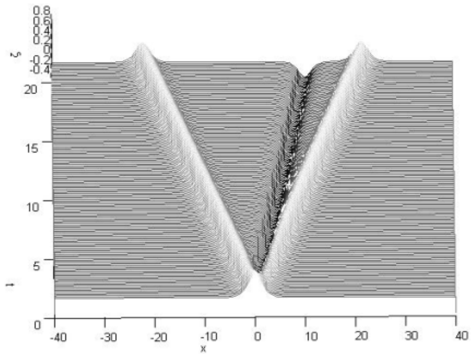
(b)

Fig. 3 : Simulation results for a convex-waving bottom wave, (a) Waterfall of fluid surface waveform, (b) Fluid surface waveform at 20

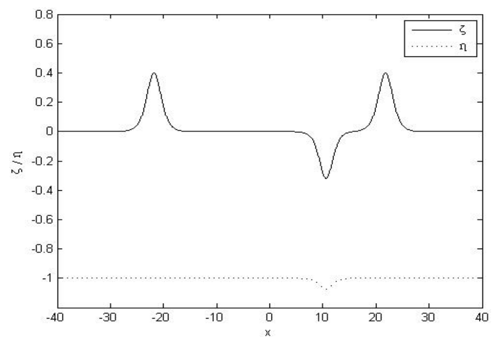
propagate independently, not interfere with each other. In Fig. 2b and d surface wave elevation ζ in the presence of a waving bottom bed is plotted at 2 and 10 respectively. The fluid surface wave is also composed of simple harmonic wave (shw) and a pair of KdV solitary waves (f_1, f_2) and they propagate independently, not interfere with each other. It can be known that superposition of simple harmonic wave and the solitary wave propagate forward which is clear from the points (a + b + c = d, m + n + o = p) in the diagram. And superimposed waveform changed, the other was done not. It is observed that the amplitude of the waves in the fluid surface is much larger compared to the waves in the waving bottom bed.

The waving bottom bed is a solitary wave packet form: The waving bottom bed is a solitary wave packet form: $a\eta(x, t) = a \cos h^2 b (x - c_0 t)$, The simulation results for the convex-waving bottom wave and the concave-waving bottom wave in different times are shown in Fig. 3 and 4 respectively.

Figure 3a shows the waterfall of fluid surface waveform when the waving bottom bed is a convex solitary wave packet. It can be observed that the wave of the fluid surface is composed of three parts, the upstream is the KdV solitary wave spreads rightward,



(a)



(b)

Fig. 4 : Simulation results for a concave-waving bottom wave, (a) Waterfall of fluid surface waveform, (b) Fluid surface waveform at 20

the downstream is the KdV solitary wave spreads leftward, the middle one is the solitary wave propagates to the right which is closely related to the waving bottom. At the initial moment, three solitary waves superposed with each other and made the amplitude of surface waveform occurs the maximum and then they separated and spread independently. When the waving bottom bed is a concave solitary wave packet, the simulated result is shown in Fig. 4a; the waveform of the fluid surface also consists of three solitary waves, which includes two KdV solitary waves and one concave-soliton. It is clear from Fig. 3b and 4b that the amplitudes of the waves in the fluid surface are much larger than the waves in the waving bottom. However, compare Fig. 3a with Fig. 4a, it can be observed that the former surface wave appears as convex solitary wave and the latter one appears as concave solitary wave. It seems that the emergence of convex solitary wave and concave solitary wave is closely related to the specific form of waving bottom. Along with time, the amplitude of each solitary wave does not changed. So it suggests that the waving bottom is effective for maintaining surface wave energy balance income and expenditure in spreading process.

CONCLUSION

In the present study, the effect of waving bottom on the fluid surface wave was investigated. The first-order and the second-order approximate equation which the fluid surface waves satisfied in the presence of waving bottom are obtained by the multiple scales perturbation method. Under the second-order approximation, the solution of first-order approximate equation was obtained and then was simulated by MATLAB Software. Simulation results shows that: the fluid surface wave consists of a harmonic wave which has the same frequency with waving bottom and a pair of KdV solitary waves when the waving bottom wave is a harmonic wave; and the fluid surface waveform is composed of three parts when the waving bottom is a solitary wave packet, the upstream is the KdV solitary wave spreads rightward, the downstream is the KdV solitary wave spreads leftward, the middle is the solitary wave propagates to the right which is closely related to the waving bottom. Along with time, the amplitude of each wave does not changed. It suggests that three waves do not affect each other and they propagate independently. So it seems that the waving bottom is effective for maintaining surface wave energy balance income and expenditure in spreading process. We also found that the specific form of waving bottom plays a significant role in the evolution of fluid surface wave. For the second order case, the effect of waving bottom on the fluid surface wave will be discussed separately.

ACKNOWLEDGMENT

This study is supported by Specialized Research Fund for Doctoral Program of Colleges in China (20110036110009).

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