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Research Article

Shape Preserving Properties for q-Bernstein-Stancu Operators

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We investigate shape preserving for q-Bernstein-Stancu polynomials $B_n^{q,\alpha}(f;x)$ introduced by Nowak in 2009. When $\alpha = 0$, $B_n^{q,\alpha}(f;x)$ reduces to the well-known q-Bernstein polynomials introduced by Phillips in 1997; when $q = 1$, $B_n^{q,\alpha}(f;x)$ reduces to Bernstein-Stancu polynomials introduced by Stancu in 1968; when $q = 1$, $\alpha = 0$, we obtain classical Bernstein polynomials. We prove that basic $B_n^{q,\alpha}(f;x)$ basis is a normalized totally positive basis on $[0,1]$ and q-Bernstein-Stancu operators are variation-diminishing, monotonicity preserving and convexity preserving on $[0,1]$.

1. Introduction

Let $q > 0$. For each nonnegative integer r, we define the q-integer $[r]_q$ as

$$[r]_q \equiv [r] := \begin{cases} \dfrac{(1-q^r)}{(1-q)}, & q \neq 1, \\ r, & q = 1, \end{cases} \tag{1}$$

we then define q-factorial $[r]!$ as

$$[r]_q! \equiv [r]! := [r][r-1]\cdots[1], \qquad [0]! = 1, \tag{2}$$

and we next define a q-binomial coefficient as

$$\begin{bmatrix} n \\ r \end{bmatrix}_q \equiv \begin{bmatrix} n \\ r \end{bmatrix} := \frac{[n][n-1]\cdots[n-r+1]}{[r]!} = \frac{[n]!}{[r]![n-r]!}, \tag{3}$$

for integers $n \geq r \geq 0$ and as zero otherwise. Also, we use the q-Pochhammer symbol defined as for any $c \in \mathbb{C}$

$$(c;q)_0 := 1, \qquad (c;q)_n := \prod_{k=0}^{n-1}\left(1-cq^k\right), \quad (n \geq 1),$$
$$(c;q)_\infty := \prod_{k=0}^{\infty}\left(1-cq^k\right), \quad (0 < q < 1). \tag{4}$$

For $f \in \mathbb{C}[0,1]$, $q > 0$, $\alpha \geq 0$, and each positive integer n, we will investigate the following q-Bernstein-Stancu operator introduced by Nowak in 2009 [1]:

$$B_n^{q,\alpha}(f;x) = \sum_{k=0}^{n} B_{n,k}^{q,\alpha}(x)\, f\left(\frac{[k]}{[n]}\right), \tag{5}$$

where

$$B_{n,k}^{q,\alpha}(x) = \begin{bmatrix} n \\ k \end{bmatrix} \frac{\prod_{i=0}^{k-1}(x+\alpha[i]) \prod_{s=0}^{n-k-1}(1-q^s x+\alpha[s])}{\prod_{i=0}^{n-1}(1+\alpha[i])}. \tag{6}$$

Note that empty product in (6) denotes 1.

In this case, when $\alpha = 0$, $B_n^{q,\alpha}(f;x)$ reduces to the well-known q-Bernstein polynomials introduced by Phillips [2] in 1997:

$$B_{n,q}(f;x) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} x^k \prod_{i=0}^{n-k-1}\left(1-q^i x\right) f\left(\frac{[k]}{[n]}\right). \tag{7}$$

When $q = 1$, $B_n^{q,\alpha}(f;x)$ reduces to Bernstein-Stancu polynomials introduced by Stancu [3] in 1968:

$$S_n(f;x) = \sum_{k=0}^{n} \binom{n}{r} \frac{\prod_{i=0}^{k-1}(x+\alpha i) \prod_{s=0}^{n-k-1}(1-x+s\alpha)}{\prod_{i=0}^{n-1}(1+i\alpha)} \times f\left(\frac{k}{n}\right). \tag{8}$$

When $q = 1$ and $\alpha = 0$, we obtain the classical Bernstein polynomials defined by

$$B_n(f;x) = \sum_{k=0}^{n} \binom{n}{r} x^k (1-x)^{n-k} f\left(\frac{k}{n}\right). \quad (9)$$

Now, we review and state some general properties of q-Bernstein-Stancu operators.

It follows directly from the definition that q-Bernstein-Stancu operators possess the endpoint interpolation property, that is,

$$B_n^{q,\alpha}(f;0) = f(0), \qquad B_n^{q,\alpha}(f;1) = f(1), \quad \forall q > 0 \text{ and all } n \in \mathbb{N}, \quad (10)$$

and leave invariant linear function:

$$B_n^{q,\alpha}(at+b) = ax+b, \quad \forall q > 0 \text{ and all } n \in \mathbb{N}. \quad (11)$$

They are also degree reducing on polynomials; that is, if $\mathscr{P}_m$ is a polynomial of degree m, then $B_n^{q,\alpha}(\mathscr{P}_m)$ is a polynomials of degree $\leq (m,n)$.

Taking $a = 0$, $b = 1$ in (11), we conclude that

$$\sum_{k=0}^{n} B_{n,k}^{q,\alpha}(x) = 1, \quad \forall n \in \mathbb{N}. \quad (12)$$

In 2009, Nowak proved that the q-Bernstein-Stancu operators can be expressed in terms of q-differences [1]:

$$B_n^{q,\alpha}(f;x) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} \triangle_q^k f_0 \prod_{s=0}^{k-1} \frac{x+\alpha[s]}{1+\alpha[s]}, \quad (13)$$

where

$$\triangle_q^k f_0 = \frac{[k]!}{[n]^k} q^{k(k-1)/2} f\left[0; \frac{1}{[n]}; \cdots; \frac{[k]}{[n]}\right]. \quad (14)$$

At the same time, he still showed that, for $0 < q < 1$, $\alpha \geq 0$,

$$B_n^{q,\alpha}(1;x) = 1, \qquad B_n^{q,\alpha}(t;x) = x,$$
$$B_n^{q,\alpha}(t^2;x) = \frac{1}{1+\alpha}\left(x(x+\alpha) + \frac{x(1-x)}{[n]}\right). \quad (15)$$

For a real-valued function f on an interval I, we define $S^-(f)$ to be the number of sign changes of f; that is,

$$S^-(f) = \sup S^-(f(x_0), \ldots, f(x_m)), \quad (16)$$

where the supremum is taken over all increasing sequence $(x_0, \ldots, x_m)$ in I, for all m. We say that L_n is variation-diminishing if

$$S^-(L_n(f)) \leq S^-(f). \quad (17)$$

Similarly, for a matrix $\mathbf{T}$, we say $\mathbf{T}$ is variation-diminishing if, for any vector $\mathbf{V}$ for which $\mathbf{TV}$ is defined, then $S^{-1}(\mathbf{TV}) \leq S^{-1}(\mathbf{V})$.

Let $(L_n)_{n\geq 1}$ be a sequence of positive linear operators on $\mathbb{C}[0,1]$. We say that L_n is monotonicity preserving if $L_n(f)$ is increasing (decreasing) for an increasing (decreasing) function f on $[0,1]$. We say that L_n is convexity preserving if $L_n(f)$ is convex (concave) for a convex (concave) function f on $[0,1]$.

Let $q \in (0,1)$, $x \in [0,1]$ and let $B_n^{q,\alpha} = (B_{n,0}^{q,\alpha}(x), B_{n,1}^{q,\alpha}(x), \ldots, B_{n,n}^{q,\alpha}(x))$ be the sequence of basic q-Bernstein-Stancu polynomials, and denote by Π_n the sequence of all polynomials of degree at most n; then $B_n^{q,\alpha}$ is a basis for Π_n (see [1]). Hence, there exists a nonsingular transformation matrix $S^{n,q_1,\alpha_1;q_2,\alpha_2}$ from $B_n^{q_1,\alpha_1}$ to $B_n^{q_2,\alpha_2}$ such that

$$B_{n,0}^{q_2,\alpha_2}(x) \;\vdots\; B_{n,n}^{q_2,\alpha_2}(x) = S^{n,q_1,\alpha_1;q_2,\alpha_2}\left(B_{n,0}^{q_1,\alpha_1} \;\vdots\; B_{n,n}^{q_1,\alpha_1}\right). \quad (18)$$

A matrix is said to be totally positivity (TP) if all its minors are nonnegative. It is well known that totally positivity matrix is various-diminishing. We say that a sequence $\phi(x) = (\phi_0(x), \ldots, \phi_n(x))$ of real-value function is TP on an interval I if, for any points $x_0 < x_1 \cdots < x_n$ in I, the collocation matrix $(\phi_j(x_i))_{i,j=0}^n$ is TP on I. If ϕ is TP on I and $\sum_{i=0}^n \phi_i(x) = 1$, $x \in I$, (so that its collocation matrix is stochastic), we say that ϕ is normalized totally positive system on I.

Theorem 1. *For $\alpha > 0$, $q \in (0,1)$, q-Bernstein-Stancu basis $B_n^{q,\alpha}$ is a normalized totally positivity basis on $[0,1]$.*

Theorem 2. *For $\alpha > 0$, $q \in (0,1)$, q-Bernstein-Stancu operators $B_n^{q,\alpha}(f;x)$ are variation-diminishing, monotonicity preserving, and convexity preserving.*

2. Proof of Theorems 1 and 2

Lemma 3 (see [4]). *A finite matrix is totally positive if and only if it is a product of 1-banded matrices with nonnegative elements, where a matrix $A = (a_{i,j})$ is called 1-banded matrix if, for some l, $a_{i,j} \neq 0$, implies $l \leq j - i \leq l + 1$.*

Lemma 4 (see [5]). *Let $\phi = (\phi_0(x), \ldots, \phi_n(x))$ and $\psi = (\psi_0(x), \ldots, \psi_n(x))$ be the base of Π_n and let S be the transformation matrix from ψ to ϕ; that is,*

$$\phi_0(x) \;\vdots\; \phi_n(x) = S\left(\psi_0(x) \;\vdots\; \psi_n(x)\right). \quad (19)$$

If S is a totally positive matrix and ψ is a totally positive system on $[0,1]$, so is ϕ.

Lemma 5 (see [6]). *If the sequence $\phi = (\phi_0(x), \ldots, \phi_n(x))$ is totally positivity on $[0,1]$, then, for any numbers $a_0, \ldots, a_n$,*

$$S^{-1}(a_0\phi_0(x) + \cdots + a_n\phi_n(x)) \leq S^{-1}(a_0, \ldots, a_n). \quad (20)$$

Proof of Theorem 1. We recall that the q-Bernstein-Stancu operators $B_n^{q,\alpha} : \mathbb{C}[0,1] \to \mathscr{P}$ are defined by

$$B_n^{q,\alpha}(x)(f;x) = \sum_{k=0}^{n} f\left(\frac{[k]}{[n]}\right) B_{n,k}^{q,\alpha}, \tag{21}$$

where

$$B_{n,k}^{q,\alpha}(x) = \begin{bmatrix} n \\ k \end{bmatrix} \frac{\prod_{j=0}^{k-1}(x+\alpha[j]) \prod_{j=0}^{n-k-1}\left(1-q^j x+\alpha[j]\right)}{\prod_{j=0}^{n-1}(1+\alpha[j])}. \tag{22}$$

Thus

$$\begin{aligned} B_{n,k}^{q,\alpha}(x) &= \begin{bmatrix} n \\ k \end{bmatrix} \frac{\prod_{j=0}^{k-1}(x+\alpha[j]) \prod_{j=0}^{n-k-1}\left(1-q^j x+\alpha[j]\right)}{\prod_{j=0}^{n-1}(1+\alpha[j])} \\ &= \begin{bmatrix} n \\ k \end{bmatrix} \frac{\prod_{j=0}^{n-k-1}(1+\alpha[j])}{\prod_{j=0}^{n-1}(1+\alpha[j])} \\ &\quad \times \prod_{j=0}^{k-1}(x+\alpha[j]) \prod_{j=0}^{n-k-1}\left(1-s_j x\right) \\ &= \begin{bmatrix} n \\ k \end{bmatrix} \prod_{j=n-k}^{n-1}(1+\alpha[j])^{-1} \prod_{j=0}^{k-1}\left(x+r_j\right) \prod_{j=0}^{n-k-1}\left(1-s_j x\right) \\ &= \begin{bmatrix} n \\ k \end{bmatrix} \prod_{j=n-k}^{n-1}(1+\alpha[j])^{-1} p_{n,k}^{q,\alpha}, \end{aligned} \tag{23}$$

where $r_j = \alpha[j]$, $s_j = (q^j/(1+\alpha[j]))$, and

$$p_{n,k}^{q,\alpha} = \prod_{j=0}^{k-1}\left(x+r_j\right) \prod_{j=0}^{n-k-1}\left(1-s_j x\right). \tag{24}$$

Clearly, from the definition we know that, for arbitrary positive numbers $a_0, \ldots, a_n$, if the sequence $(\phi_0(x), \ldots, \phi_n(x))$ is totally positive on $[0,1]$, then so is the sequence $(a_0\phi_0, \ldots, a_n\phi_n)$. We want to prove that $B_n^{q,\alpha} = (B_{n,0}^{q,\alpha}(x), B_{n,1}^{q,\alpha}(x), \ldots, B_{n,n}^{q,\alpha}(x))$ on $[0,1]$ is totally positive system, provided to prove that $P_n^{q,\alpha} = (P_{n,0}^{q,\alpha}(x), \ldots, P_{n,n}^{q,\alpha}(x))$ is totally positivity system on $[0,1]$; we use Heping Wang's methods (see [5]) to prove that $P_n^{q,\alpha} = (P_{n,0}^{q,\alpha}(x), \ldots, P_{n,n}^{q,\alpha}(x))$ is totally positivity system on $[0,1]$. For $0 \le i, k \le n$ and fixed $q \in (0,1)$, we define

$$\begin{aligned} {}^iR_k^n(x) &= \begin{cases} x^k \prod_{j=0}^{n-k-1}\left(1-s_j x\right), & n-k \le i, \\ x^k(1-x)^{n-k-i} \prod_{j=0}^{i-1}\left(1-s_j x\right), & n-k > i, \end{cases} \\ {}^iP_k^n(x) &= \begin{cases} \prod_{j=0}^{k-1}\left(x+r_j\right) \prod_{j=0}^{n-k-1}\left(1-s_j x\right), & k \le i, \\ x^{k-i} \prod_{j=0}^{i-1}\left(x+r_j\right) \prod_{j=0}^{n-k-1}\left(1-s_j x\right), & k > i, \end{cases} \end{aligned} \tag{25}$$

where s_j, r_j, $j = 0, \ldots, n$ are given in (24). Clearly, for $0 \le k \le n$,

$$\begin{aligned} {}^nP_k^n(x) &= P_{n,k}^{q,\alpha}(x), \\ {}^0P_k^n = {}^nR_k^n(x) &= x^k \prod_{j=0}^{n-k-1}\left(1-s_j x\right), \\ {}^0R_k^n(x) &= x^k(1-x)^{n-k}. \end{aligned} \tag{26}$$

For $0 \le i < n$, it follows from the definition of ${}^iR_k^n(x)$ that, for $k \ge n-i$,

$${}^{i+1}R_k^n(x) = {}^iR_k^n(x), \tag{27}$$

and, for $k < n-i$,

$$\begin{aligned} &{}^{i+1}R_k^n(x) \\ &\quad = x^k(1-x)^{n-k-i-1} \prod_{j=0}^{i}\left(1-s_j x\right) \\ &\quad = x^k(1-x)^{n-k-i}(1-x)^{-1}\left(1-s_j x\right) \prod_{j=0}^{i-1}\left(1-s_j x\right) \\ &\quad = x^k(1-x)^{n-k-i}(1-x)^{-1}\left((1-x)+\left(1-s_j\right)x\right) \\ &\qquad \times \prod_{j=0}^{i-1}\left(1-s_j x\right) \\ &\quad = x^k(1-x)^{n-k-i} \prod_{j=0}^{i-1}\left(1-s_j x\right) \\ &\qquad + x^{k+1}(1-x)^{n-k-i-1}\left(1-s_j\right) \prod_{j=0}^{i-1}\left(1-s_j x\right) \\ &\quad = {}^iR_k^n(x) + \left(1-s_i\right) {}^iR_{k+1}^n(x). \end{aligned} \tag{28}$$

Similarly, from the definition of ${}^iP_k^n(x)$, we get that, for $k \le i$,

$${}^{i+1}P_k^n = {}^iP_k^n(x), \tag{29}$$

and, for $k > i$,

$$\begin{aligned}
&{}^{i+1}P_k^n(x) \\
&= x^{k-i-1}\prod_{j=0}^{i}\left(x+r_j\right)\prod_{j=0}^{n-k-1}\left(1-s_jx\right) \\
&= x^{k-i-1}\left(x+r_i\right)\prod_{j=0}^{i-1}\left(x+r_j\right)\prod_{j=0}^{n-k-1}\left(1-s_jx\right) \\
&= x^{k-i-1}\left(r_i - r_is_{n-k}x + \left(1+r_is_{n-k}\right)x\right) \\
&\quad\times\prod_{j=0}^{i-1}\left(x+r_j\right)\prod_{j=0}^{n-k-1}\left(1-s_jx\right) \qquad (30)\\
&= r_ix^{k-i-1}\prod_{j=0}^{i-1}\left(x+r_j\right) \\
&\quad\times\prod_{j=0}^{n-k}\left(1-s_jx\right)+\left(1+r_is_{n-k}\right)x^{k-i} \\
&\quad\times\prod_{j=0}^{i-1}\left(x+r_j\right)\prod_{j=0}^{n-k-1}\left(1-s_jx\right) \\
&= r_i\,{}^{i}P_{k-1}^n(x)+\left(1+r_is_{n-k}\right){}^{i}P_k^n(x).
\end{aligned}$$

Hence, if we let

$$\begin{bmatrix} {}^{i+1}P_0^n(x) \\ \vdots \\ {}^{i+1}P_n^n(x) \end{bmatrix} = S^{(i)}\begin{bmatrix} {}^{i}P_0^n(x) \\ \vdots \\ {}^{i}P_n^n(x) \end{bmatrix}, \qquad (31)$$

$$\begin{bmatrix} {}^{i+1}R_0^n(x) \\ \vdots \\ {}^{i+1}R_n^n(x) \end{bmatrix} = T^{(i)}\begin{bmatrix} {}^{i}R_0^n(x) \\ \vdots \\ {}^{i}R_n^n(x) \end{bmatrix}, \qquad (32)$$

then

$$S^{(i)} = \begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & r_i & 1+r_iS_{n-i-1} & & \\ & & & \ddots & \ddots & \\ & & & & r_i & 1+r_iS_0 \end{pmatrix},$$

$$T^{(i)} = \begin{bmatrix} 1 & 1-S_i & & & & \\ & \ddots & \ddots & & & \\ & & 1 & 1-S_J & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}. \qquad (33)$$

From (26) and (31), we obtain

$$\begin{aligned}
&P_{n,0}^{q,\alpha} \vdots P_{n,n}^{q,\alpha}(x) \\
&= \begin{bmatrix} P_{n,0}^{q,\alpha}(x) \\ P_{n,1}^{q,\alpha}(x) \\ \vdots \\ P_{n,n}^{q,\alpha}(x) \end{bmatrix} = \begin{bmatrix} {}^{n}P_0^n(x) \\ {}^{n}P_{1,n}^n(x) \\ \vdots \\ {}^{n}P_{n,n}^n(x) \end{bmatrix} = S^{(n-1)}\begin{bmatrix} {}^{n-1}P_0^n(x) \\ {}^{n-1}P_{1,n}^n(x) \\ \vdots \\ {}^{n-1}P_{n,n}^n(x) \end{bmatrix} \\
&= S^{(n-1)}S^{(n-2)}\cdots S^{(1)}S^{(0)}\begin{bmatrix} {}^{0}P_0^n(x) \\ {}^{0}P_1^n(x) \\ \vdots \\ {}^{0}P_n^n(x) \end{bmatrix} \\
&= S^{(n-1)}S^{(n-2)}\cdots S^{(1)}S^{(0)}\begin{bmatrix} {}^{n}R_0^n(x) \\ {}^{n}R_1^n(x) \\ \vdots \\ {}^{n}R_n^n(x) \end{bmatrix} \\
&= S^{(n-1)}S^{(n-2)}\cdots S^{(1)}S^{(0)}T^{(n-1)}\begin{bmatrix} {}^{(n-1)}R_0^n(x) \\ {}^{(n-1)}R_1^n(x) \\ \vdots \\ {}^{(n-1)}R_n^n(x) \end{bmatrix} \qquad (34)\\
&= S^{(n-1)}S^{(n-2)}\cdots S^{(1)}S^{(0)}T^{(n-1)}T^{(n-2)}\begin{bmatrix} {}^{(n-2)}R_0^n(x) \\ {}^{(n-2)}R_1^n(x) \\ \vdots \\ {}^{(n-2)}R_n^n(x) \end{bmatrix} \\
&= S^{(n-1)}S^{(n-2)}\cdots S^{(1)}S^{(0)}T^{(n-1)}T^{(n-2)}\cdots T^{(1)}T^{(0)} \\
&\quad\times\begin{bmatrix} {}^{0}R_0^n(x) \\ {}^{0}R_1^n(x) \\ \vdots \\ {}^{0}R_n^n(x) \end{bmatrix} \\
&= S^{(n-1)}S^{(n-2)}\cdots S^{(1)}S^{(0)}T^{(n-1)}T^{(n-2)}\cdots T^{(1)}T^{(0)} \\
&\quad\times {}^{0}R_0^n(x) \vdots {}^{0}R_0^n(x).
\end{aligned}$$

Obviously, $S^{(i)}$, $T^{(i)}$, $i = 0, 1, \ldots, n-1$ are 1-banded matrixes with nonnegative elements. Since the sequence of functions,

$$\left((1-x)^n, x(1-x)^{n-1}, x^2(1-x)^{n-2}, \ldots, x^{n-1}(1-x), x^n\right), \qquad (35)$$

is totally positive on [0, 1], by (26), (34) and Lemmas 3 and 4, we obtain that $P_n^{q,\alpha}$ is a totally positive system on [0, 1]. The proof of Theorem 1 is complete. □

Proof of Theorem 2. The proof of Theorem 2 follows from Theorem 1. From Theorem 1, we know that q-Bernstein-Stancu basis

$$B_n^{q,\alpha} = \left(B_{n,0}^{q,\alpha}(x), \dots, B_{n,n}^{q,\alpha}\right) \tag{36}$$

is totally positive for $x \in [0,1]$. By Lemma 5 we obtain that

$$\begin{aligned} S^{-1}\left(B_n^{q,\alpha}(f;x)\right) &= S^{-1}\left(\sum_{r=0}^{n} f\left(\frac{[k]_q}{[n]_q}\right) B_{n,k}^{q,\alpha}(x)\right) \\ &\le S^{-1}\left(f\left(\frac{[0]_q}{[n]_q}\right), f\left(\frac{[1]_q}{[n]_q}\right), \dots, f\left(\frac{[n]_q}{[n]_q}\right)\right) \\ &\le S^{-1}\left(f(x)\right), \end{aligned} \tag{37}$$

which means that the q-Bernstein-Stancu operators $B_n^{q,\alpha}$ are variation-diminishing. Since q-Bernstein-Stancu polynomials reproduce linear functions, we get for any function f and any linear polynomial P,

$$\begin{aligned} S^{-1}\left(B_n^{q,\alpha}(f) - p\right) &= S^{-1}\left(B_n^{q,\alpha}(f-p)\right) \\ &\le S^{-1}(f-p). \end{aligned} \tag{38}$$

A standard reasoning based on (38) and endpoint interpolation property of $B_n^{q,\alpha}$ yields that $B_n^{q,\alpha}$ are monotonicity preserving and convexity preserving (see [7], pp. 287-288). Theorem 2 is proved. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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