

## Research Article

# On the Study of Global Solutions for a Nonlinear Equation

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The well-posedness of global strong solutions for a nonlinear partial differential equation including the Novikov equation is established provided that its initial value  $v_0(x)$  satisfies a sign condition and  $v_0(x) \in H^s(R)$  with  $s > 3/2$ . If the initial value  $v_0(x) \in H^s(R)$  ( $1 \leq s \leq 3/2$ ) and the mean function of  $(1 - \partial_x^2)v_0(x)$  satisfies the sign condition, it is proved that there exists at least one global weak solution to the equation in the space  $v(t, x) \in L^2([0, +\infty), H^s(R))$  in the sense of distribution and  $v_x \in L^\infty([0, +\infty) \times R)$ .

## 1. Introduction

Recently, Wu [1] obtained the existence of local solutions in the space  $C([0, T]; H^s(R)) \cap C^1([0, T]; H^{s-1}(R))$  with  $s > 3/2$  for the following nonlinear equation:

$$\begin{aligned} v_t - v_{txx} + kv^m v_x + (m+3)v^{m+1}v_x \\ = (m+2)v^m v_x v_{xx} + v^{m+1}v_{xxx} + \lambda(v - v_{xx}), \end{aligned} \quad (1)$$

where  $m \geq 0$  is a natural number,  $k \geq 0$ , and  $\lambda$  is a constant. Letting  $m = 0$  and  $\lambda = 0$ , (1) becomes the Camassa-Holm equation [2]. If  $m = 1$ ,  $k = 0$ , and  $\lambda = 0$ , (1) reduces to the Novikov equation [3].

A lot of works have been carried out to study various dynamic properties for the Camassa-Holm and the Novikov equations. Xin and Zhang [4] proved that there exists a global weak solution for the Camassa-Holm equation in the space  $H^1(R)$  without the assumption of sign conditions on the initial value. Coclite et al. [5] investigated the global weak solutions for a generalized hyperelastic rod wave equation or a generalized Camassa-Holm equation. It is shown in Constantin and Escher [6] that the blowup occurs in the form of breaking waves; namely, the solution remains bounded but its slope becomes unbounded in finite time. After wave breaking, the solution can be continued uniquely either as a global conservative weak solution [7] or a global dissipative solution [8–10]. The periodic and the nonperiodic

Cauchy problems for the Novikov equation were discussed by Grayshan [11] in the Sobolev space. Using the Galerkin-type approximation method, Himonas and Holliman [12] established the well-posedness for the Novikov model in the Sobolev space  $H^s(R)$  with  $s > 3/2$  on both the line and the circle. The scattering theory was employed in Hone et al. [13] to find nonsmooth explicit soliton solutions with multiple peaks for the Novikov equation. Wu and Zhong [14] proved the existence of local strong and weak solutions for a generalized Novikov equation.

The objective of this work is to study (1) with  $k = 0$ . Namely, we investigate the problem

$$\begin{aligned} v_t - v_{txx} + (m+3)v^{m+1}v_x \\ = (m+2)v^m v_x v_{xx} + v^{m+1}v_{xxx} + \lambda(v - v_{xx}), \quad (2) \\ v(0, x) = v_0(x), \end{aligned}$$

where  $m$ ,  $k$ , and  $\lambda$  are described in (1). Assuming that the initial value  $v_0(x)$  satisfies a sign condition and  $v_0(x) \in H^s(R)$ ,  $s > 3/2$ , we will show that there exists a unique global strong solution in the Sobolev space  $C([0, \infty); H^s(R)) \cap C^1([0, \infty); H^{s-1}(R))$ . If the initial value  $v_0(x) \in H^s(R)$  ( $1 \leq s \leq 3/2$ ) and the mean function of  $(1 - \partial_x^2)v_0(x)$  satisfies the sign condition, it is shown that there exists at least one global weak solution to the equation in the space  $v(t, x) \in$

$L^2([0, +\infty), H^s(R))$  in the sense of distribution and  $v_x \in L^\infty([0, +\infty) \times R)$ .

The structure of this paper is as follows. The main results are given in Section 2. Several lemmas are given in Section 3. Section 4 establishes the proof of the main results.

### 2. Main Results

We define

$$\phi(x) = \begin{cases} e^{1/(x^2-1)}, & |x| < 1, \\ 0, & |x| \geq 1, \end{cases} \quad (3)$$

and let  $\phi_\varepsilon(x) = \varepsilon^{-1/4} \phi(\varepsilon^{-1/4}x)$  with  $0 < \varepsilon < 1/4$ . For the convolution  $v_{\varepsilon 0} = \phi_\varepsilon * v_0$ , we know that  $v_{\varepsilon 0} \in C^\infty$  for any  $v_0 \in H^s$  with  $s > 0$ . Notation  $(1 - \partial_x^2)v \in N^+(R)$  (or equivalently  $(1 - \partial_x^2)v \in N^-(R)$ ) means that the mean function of  $(1 - \partial_x^2)v$  is nonnegative; namely,  $(1 - \partial_x^2)v * \phi_\varepsilon \geq 0$  (or equivalently  $(1 - \partial_x^2)v * \phi_\varepsilon \leq 0$ ) for an arbitrary sufficiently small  $\varepsilon > 0$ . For  $T > 0$  and nonnegative number  $s$ , we let  $C([0, T]; H^s(R))$  denote the Frechet space of all continuous  $H^s$ -valued functions on  $[0, T)$  and write  $\Lambda = (1 - \partial_x^2)^{1/2}$ .

We state the result of global strong solutions for problem (2).

**Theorem 1.** *Let  $v_0(x) \in H^s(R)$ ,  $s > 3/2$ , and  $(1 - \partial_x^2)v_0 \geq 0$  for all  $x \in R$  or  $(1 - \partial_x^2)v_0 \leq 0$  for all  $x \in R$ . Then problem (2) has a unique strong solution satisfying*

$$v(t, x) \in C([0, \infty); H^s(R)) \cap C^1([0, \infty); H^{s-1}(R)). \quad (4)$$

**Definition 2.** A function  $v(t, x) \in L^2([0, +\infty), H^s(R))$  is called a global weak solution to problem (2) if for every  $T > 0$  and all  $\varphi(t, x) \in C_0^\infty([0, T] \times R)$ , it holds that

$$\int_0^T \int_R [v_t - v_{txx} + (m+3)v^{m+1}v_x - (m+2)v^m v_x v_{xx} - v^{m+1}v_{xxx} - \lambda(v - v_{xx})] \varphi(t, x) dx dt = 0 \quad (5)$$

with  $v(0, x) = v_0(x)$ .

Now we give the main result of global weak solution for problem (2).

**Theorem 3.** *Let  $v_0(x) \in H^s(R)$ ,  $1 \leq s \leq 3/2$ ,  $(1 - \partial_x^2)v_0 \in N^+(R)$  (or equivalently  $(1 - \partial_x^2)v_0 \in N^-(R)$ ). Then problem (2) has a unique global weak solution  $v(t, x) \in L^2([0, +\infty), H^s(R))$  in the sense of distribution and  $v_x \in L^\infty([0, +\infty) \times R)$ .*

### 3. Several Lemmas

**Lemma 4** (see [1]). *Let  $v_0(x) \in H^s(R)$  with  $s > 3/2$ . Then the Cauchy problem (2) has a unique local solution*

$$v(t, x) \in C([0, T]; H^s(R)) \cap C^1([0, T]; H^{s-1}(R)), \quad (6)$$

where  $T > 0$  depends on  $\|v_0\|_{H^s(R)}$ .

Using the first equation of system (2) derives

$$\frac{d}{dt} \int_R (v^2 + v_x^2) dx = 2\lambda \int_R (v^2 + v_x^2) dx, \quad (7)$$

which yields the conservation law

$$\int_R (v^2 + v_x^2) dx = \int_R (v_0^2 + v_{0x}^2) dx \quad (8)$$

$$+ 2\lambda \int_0^t \int_R (v^2 + v_x^2) dx dt.$$

**Lemma 5** (see [1]). *Let  $s > 3/2$  and the function  $v(t, x)$  is a solution of problem (2) and the initial data  $v_0(x) \in H^s$ . Then the following inequalities hold:*

$$\begin{aligned} \|v\|_{H^1}^2 &\leq \int_R (v^2 + v_x^2) dx \leq \int_R (v_0^2 + v_{0x}^2) dx, \quad \text{if } \lambda \leq 0. \\ \|v\|_{H^1}^2 &\leq \int_R (v^2 + v_x^2) dx \leq e^{2\lambda t} \int_R (v_0^2 + v_{0x}^2) dx, \quad \text{if } \lambda > 0. \end{aligned} \quad (9)$$

For  $q \in (0, s - 1]$ , there is a constant  $c$  such that

$$\begin{aligned} &\int_R (\Lambda^{q+1}v)^2 dx \\ &\leq \int_R (\Lambda^{q+1}v_0)^2 dx \\ &+ c \int_0^t \|v\|_{H^{q+1}}^2 (|\lambda| + (\|v\|_{L^\infty}^{m-1} + \|v\|_{L^\infty}^m) \|v_x\|_{L^\infty} \\ &\quad + \|v\|_{L^\infty}^{m-1} \|v_x\|_{L^\infty}^2) d\tau. \end{aligned} \quad (10)$$

For  $q \in [0, s - 1]$ , there is a constant  $c$  such that

$$\begin{aligned} \|v_t\|_{H^q} &\leq c \|v\|_{H^{q+1}} (|\lambda| + (\|v\|_{L^\infty}^{m-1} + \|v\|_{L^\infty}^m) \|v\|_{H^1} \\ &\quad + \|v\|_{L^\infty}^m \|v_x\|_{L^\infty} + \|v\|_{L^\infty}^{m-1} \|v_x\|_{L^\infty}^2). \end{aligned} \quad (11)$$

Consider the differential equation

$$\begin{aligned} p_t &= v^{m+1}(t, p), \quad t \in [0, T), \\ p(0, x) &= x, \end{aligned} \quad (12)$$

where  $v(t, x)$  is the solution of problem (2) and  $T$  is the maximal existence time of the solution.

**Lemma 6.** *Let  $v_0 \in H^s(R)$ ,  $s \geq 3$ , and let  $T > 0$  be the maximal existence time of the solution to problem (2). Then system (12) has a unique solution  $p(t, x) \in C^1([0, T) \times R)$ . Moreover, the map  $p(t, \cdot)$  is an increasing diffeomorphism of  $R$  with  $p_x(t, x) > 0$  for  $(t, x) \in [0, T) \times R$ .*

*Proof.* From Lemma 4, we know that there exists a unique solution

$$v(t, x) \in C([0, T]; H^s(R)) \cap C^1([0, T]; H^{s-1}(R)). \quad (13)$$

The Sobolev imbedding theorem derives  $H^s(R) \in C^1(R)$ . This means that two functions  $v(t, x)$  and  $v_x(t, x)$  are bounded, Lipschitz in space and  $C^1$  in time. Using the existence and uniqueness theorem of ordinary differential equations, we derive that problem (12) has a unique solution  $p(t, x) \in C^1([0, T) \times R)$ .

Differentiating (12) with respect to  $x$  gives rise to

$$\begin{aligned} \frac{d}{dt} p_x &= (m + 1) v^m v_x(t, p) p_x, \quad t \in [0, T), \\ p_x(0, x) &= 1, \end{aligned} \tag{14}$$

from which we obtain

$$p_x(t, x) = \exp\left(\int_0^t (m + 1) v^m v_x(\tau, p(\tau, x)) d\tau\right). \tag{15}$$

For every  $T' < T$ , applying the Sobolev imbedding theorem results in

$$\sup_{(\tau, x) \in [0, T') \times R} |v_x(\tau, x)| < \infty. \tag{16}$$

Therefore, we know that there exists a constant  $M > 0$  such that  $p_x(t, x) \geq e^{-Mt}$  for  $(t, x) \in [0, T) \times R$ . The proof is completed.  $\square$

**Lemma 7.** Let  $v_0 \in H^s$  with  $s \geq 3$ , and let  $T > 0$  be the maximal existence time of the problem (2); it holds that

$$y(t, p(t, x)) p_x^2(t, x) = y_0(x) e^{\int_0^t (mv^m v_x + \lambda) d\tau}, \tag{17}$$

where  $(t, x) \in [0, T) \times R$  and  $y := v - v_{xx}$ .

*Proof.* We have

$$\begin{aligned} &\frac{d}{dt} [y(t, p(t, x)) p_x^2(t, x)] \\ &= y_t p_x^2 + 2y p_x p_{xt} + y_x p_t p_x^2 \\ &= y_t p_x^2 + 2y(m + 1) v^m v_x p_x^2 + v^{m+1} y_x p_x^2 \\ &= [y_t + (m + 2) v^m v_x y + y_x v^{m+1}] p_x^2 + mv^m v_x y p_x^2 \\ &= [v_t - v_{txx} + (m + 2) v^m v_x (v - v_{xx}) \\ &\quad + v^{m+1} (v_x - v_{xxx}) - \lambda (v - v_{xx})] p_x^2 \\ &\quad + (mv^m v_x + \lambda) y p_x^2 \\ &= [v_t - v_{txx} + (m + 3) v^{m+1} v_x - (m + 2) v^m v_x v_{xx} \\ &\quad - v^{m+1} v_{xxx} - \lambda (v - v_{xx})] p_x^2 \\ &\quad + (mv^m v_x + \lambda) y p_x^2 \\ &= (mv^m v_x + \lambda) y p_x^2, \end{aligned} \tag{18}$$

from which we have

$$y(t, p(t, x)) p_x^2(t, x) = p_x(0, x) y_0(x) e^{\int_0^t (mv^m v_x + \lambda) d\tau}. \tag{19}$$

Using  $p_x(0, x) = 1$  completes the proof.  $\square$

**Lemma 8.** If  $v_0 \in H^s(R)$ ,  $s \geq 3/2$ ,  $(1 - \partial_x^2)v_0 \geq 0$  or  $(1 - \partial_x^2)v_0 \leq 0$ , then the solution of problem (2) satisfies

$$\|v_x\|_{L^\infty} \leq \|v\|_{L^\infty}. \tag{20}$$

*Proof.* We only need to prove this lemma for the case  $v_0 - v_{0xx} \geq 0$  since the proof of the other case  $(1 - \partial_x^2)v_0 \leq 0$  is similar. It follows from Lemmas 6 and 7 that  $v - v_{xx} \geq 0$ . Letting  $\xi(t, x) = v - v_{xx}$ , we have

$$v = \frac{1}{2} e^{-x} \int_{-\infty}^x e^\eta \xi(t, \eta) d\eta + \frac{1}{2} e^x \int_x^\infty e^{-\eta} \xi(t, \eta) d\eta, \tag{21}$$

which derives

$$\begin{aligned} \partial_x v(t, x) &= -\frac{1}{2} \left( e^{-x} \int_{-\infty}^x e^\eta \xi(t, \eta) d\eta + e^x \int_x^\infty e^{-\eta} \xi(t, \eta) d\eta \right) \\ &\quad + e^x \int_x^\infty e^{-\eta} \xi(t, \eta) d\eta \\ &= -v(t, x) + e^x \int_x^\infty e^{-\eta} \xi(t, \eta) d\eta \\ &\geq -v(t, x). \end{aligned} \tag{22}$$

On the other hand, we have

$$\begin{aligned} \partial_x v(t, x) &= \frac{1}{2} \left( e^{-x} \int_{-\infty}^x e^\eta \xi(t, \eta) d\eta + e^x \int_x^\infty e^{-\eta} \xi(t, \eta) d\eta \right) \\ &\quad - e^{-x} \int_{-\infty}^x e^\eta \xi(t, \eta) d\eta \\ &= v(t, x) - e^{-x} \int_{-\infty}^x e^\eta \xi(t, \eta) d\eta \\ &\leq v(t, x). \end{aligned} \tag{23}$$

The inequalities (22) and (23) derive that inequality (20) is valid.  $\square$

**Lemma 9.** For  $s > 0$ ,  $u \in H^s(R)$ , and  $u_\varepsilon = \phi_\varepsilon * u$ , it holds that

$$\begin{aligned} \|u_{\varepsilon x}\|_{L^\infty} &\leq c \|u_x\|_{L^\infty}, \\ \|u_\varepsilon\|_{H^q} &\leq c, \quad \text{if } q \leq s, \\ \|u_\varepsilon\|_{H^q} &\leq c \varepsilon^{(s-q)/4}, \quad \text{if } q > s, \end{aligned} \tag{24}$$

$$\|u_\varepsilon - u\|_{H^q} \leq c \varepsilon^{(s-q)/4}, \quad \text{if } q \leq s,$$

$$\|u_\varepsilon - u\|_{H^s} = o(1),$$

where  $c$  is a constant independent of  $\varepsilon$ .

The proof of this lemma can be found in [15, 16].  
From Lemma 4, it derives that the Cauchy problem

$$\begin{aligned} v_t - v_{txx} &= -(m+3)v^{m+1}v_x + (m+2)v^m v_x v_{xx} \\ &\quad + v^{m+1}v_{xxx} + \lambda(v - v_{xx}) \\ &= -\frac{m+3}{m+2}(v^{m+2})_x + \frac{1}{m+2}\partial_x^3(v^{m+2}) \\ &\quad - (m+1)\partial_x(v^m v_x^2) + v^m v_x v_{xx} + \lambda(v - v_{xx}), \\ v(0, x) &= v_{\varepsilon 0}(x), \end{aligned} \tag{25}$$

has a unique solution  $v$  depending on the parameter  $\varepsilon$ . We write  $v_\varepsilon(t, x)$  to represent the solution of problem (25). Using Lemma 4 derives that  $v_\varepsilon(t, x) \in C^\infty([0, T], H^\infty(R))$  since  $v_{\varepsilon 0}(x) \in C_0^\infty(R)$ .

**Lemma 10.** *Provided that  $v_0 \in H^s(R)$ ,  $1 \leq s \leq 3/2$ , and  $(1 - \partial_x^2)v_0 \in N^+(R)$  (or equivalently  $(1 - \partial_x^2)v_0 \in N^-(R)$ ), then there exists a constant  $c > 0$  independent of  $\varepsilon$  and  $t$  such that the solution of problem (25) satisfies*

$$\|v_{\varepsilon x}\|_{L^\infty} \leq ce^{ct}. \tag{26}$$

*Proof.* Using Lemmas 5 and 9, if  $v_0 \in H^s(R)$  with  $1 \leq s \leq 3/2$ , we have

$$\|v_\varepsilon\|_{L^\infty(R)} \leq c\|v_\varepsilon\|_{H^1(R)} \leq ce^{ct}\|v_{\varepsilon 0}\|_{H^1(R)} \leq ce^{ct}, \tag{27}$$

where  $c$  is independent of  $\varepsilon$  and  $t$ .  
From Lemma 8, we have

$$\|v_{\varepsilon x}\|_{L^\infty(R)} \leq \|v_\varepsilon\|_{L^\infty(R)}, \tag{28}$$

which completes the proof. □

#### 4. Proof of Main Results

*Proof of Theorem 1.* Since  $\|v\|_{L^\infty(R)} \leq c\|v\|_{H^1(R)} \leq ce^{ct}$  and taking  $q + 1 = s$  in inequality (10), we have

$$\|v\|_{H^s}^2 \leq \|v_0\|_{H^s}^2 + c \int_0^t e^{c\tau} \|v\|_{H^s}^2 (\|v_x\|_{L^\infty} + \|v_x\|_{L^\infty}^2) d\tau, \tag{29}$$

from which we obtain

$$\|v\|_{H^s} \leq \|v_0\|_{H^s} e^{c \int_0^t e^{c\tau} (\|v_x\|_{L^\infty} + \|v_x\|_{L^\infty}^2) d\tau}. \tag{30}$$

Applying Lemma 8 yields

$$\|v\|_{H^s} \leq \|v_0\|_{H^s} ce^{e^{ct}}, \tag{31}$$

from which we complete the proof of Theorem 1. □

Provided that  $1 \leq s \leq 3/2$ , for problem (25), applying Lemmas 5, 8, and 10, and the Gronwall's inequality, we obtain the inequalities

$$\begin{aligned} \|v_\varepsilon\|_{H^1} &\leq \|v_{\varepsilon 0}\|_{H^1} \leq ce^{ct}, \\ \|v_\varepsilon\|_{H^q} &\leq c\|v_{\varepsilon 0}\|_{H^q} \exp \left[ \int_0^t (\|v_{\varepsilon x}\| + \|v_{\varepsilon x}\|_{L^\infty}^2) d\tau \right] \leq ce^{e^{ct}}, \\ \|u_{\varepsilon t}\|_{H^r} &\leq c\|u_\varepsilon\|_{H^{r+1}} (1 + e^{ct}) \leq c(1 + e^{ct}), \end{aligned} \tag{32}$$

where  $q \in (0, s]$ ,  $r \in [0, s-1]$ , and  $c$  is a constant independent of  $t$  and  $\varepsilon$ . Using the Aubin compactness theorem, we know that there is a subsequence  $\{v_{\varepsilon_n}\}$  of  $\{v_\varepsilon\}$  such that  $\{v_{\varepsilon_n}\}$  and their temporal derivatives  $\{v_{\varepsilon_n t}\}$  converge weakly to a function  $v(t, x)$  and its derivative  $v_t$  in the space  $L^2([0, T], H^s(R))$  and  $L^2([0, T], H^{s-1}(R))$ , respectively, where  $T$  is an arbitrary fixed positive number. In addition, for any real number  $M_1 > 0$ ,  $\{v_{\varepsilon_n}\}$  converges strongly to the function  $v$  in the space  $L^2([0, T], H^q(-M_1, M_1))$  for  $q \in (0, s]$  and  $\{v_{\varepsilon_n t}\}$  converges strongly to  $v_t$  in the space  $L^2([0, T], H^r(-M_1, M_1))$  for  $r \in [0, s-1]$ .

*Proof of Theorem 3.* For an arbitrary fixed  $T > 0$ , using Lemma 10, we know that  $\{v_{\varepsilon_n x}\}$  ( $\varepsilon_n \rightarrow 0$ ) is bounded in the space  $L^\infty$ . Therefore, we derive that the sequences  $\{v_{\varepsilon_n}\}$ ,  $\{v_{\varepsilon_n x}\}$ ,  $\{v_{\varepsilon_n x}^2\}$ , and  $\{v_{\varepsilon_n x}^3\}$  converge weakly to  $v$ ,  $v_x$ ,  $v_x^2$ , and  $v_x^3$  in  $L^2([0, T], H^r(-R_1, R_1))$  for any  $r \in [0, s-1]$ , separately. Applying the identity  $v^m(v_x^2)_x = (v^m v_x^2)_x - (v^m)_x v_x^2$ , we conclude that  $v$  satisfies the equation

$$\begin{aligned} & - \int_0^T \int_R v(\varphi_t - \varphi_{xxt}) dx dt \\ &= \int_0^T \int_R \left[ \left( \frac{m+3}{m+2} v^{m+2} + (m+1)v^m v_x^2 \right) \varphi_x \right. \\ &\quad \left. - \frac{1}{m+2} v^{m+2} \varphi_{xxx} - \frac{1}{2} v^m v_x^2 \varphi_x \right. \\ &\quad \left. - \frac{m}{2} v^{m-1} v_x^3 \varphi + \lambda v(\varphi - \varphi_{xx}) \right] dx dt, \end{aligned} \tag{33}$$

where  $\varphi(t, x) \in C_0^\infty([0, T] \times R)$ . We know that  $Y = L^1([0, T] \times R)$  is a separable Banach space and  $\{v_{\varepsilon_n x}\}$  is a bounded sequence in the dual space  $Y^* = L^\infty([0, T] \times R)$  of  $Y$ . Thus, there exists a subsequence of  $\{v_{\varepsilon_n x}\}$ , still denoted by  $\{v_{\varepsilon_n x}\}$ , weakly star convergent to a function  $u$  in  $L^\infty([0, T] \times R)$ . Since  $\{v_{\varepsilon_n}\}$  weakly converges to  $v_x$  in  $L^2([0, T] \times R)$ , it derives that  $v_x = u$  almost everywhere. Therefore, we obtain  $v_x \in L^\infty([0, T] \times R)$ . Since  $T > 0$  is an arbitrary number, we complete the proof of existence of global weak solutions to problem (2). □

#### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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