

THE EFFICACY OF TERM STRUCTURE ESTIMATION TECHNIQUES: A MONTE CARLO STUDY

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It is well-recognized that the term structure of default-free interest rates is not directly observable in a market where government obligations of various maturities bear coupons at different rates, and where ordinary income and capital gains are subject to unknown and varying effective tax rates. (See, for example, Jordan [1984], Livingston [1979], McCulloch [1975], Robichek and Niebuhr [1970], Ronn [1987], Schaefer [1982], and Torous [1985].) At the same time, accurate knowledge of the term structure of spot rates and the underlying term structure of forward rates is essential for financial research and practice.

This information cannot be obtained from the yield curve of Treasury strips, because those obligations are traded in a separate and distinct market. The strips market is dominated by a unique clientele of U.S. tax-exempt institutions and Japanese investors who have a tax incentive to hold long-term principal (versus coupon) strips. There are unknown differences in effective tax rates between the two markets, and a difference in liquidity in favor of the wider market for standard coupon bonds. The term structure underlying the coupon bond market must therefore be estimated from bonds traded in that market.

Such estimates are required for the management of fixed-income security portfolios and for pricing interest rate-contingent claims such as fixed-income securities, options, and futures (see Ho and Lee [1986] and Kishimoto [1989]). They are also an essential input of Monte Carlo simulations used for valuing complex claims such as mortgage-backed securities (see Dattatreya and Fabozzi [1989]). Finally, term structure estimates are used in testing theories about the term structure itself. See, for example, Brennan and Schwartz [1979], Brown and

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Dybvig [1986], Cox, Ingersoll, and Ross [1981], Langetieg [1980], and Vasicek [1977].

There are numerous methods for estimating forward rates, as well as many studies testing the accuracy of those methods. Some not referred to so far include Carleton and Cooper [1976] and Chambers, Carleton, and Waldman [1984]. Accuracy depends on knowing the true underlying forward rates, or, at minimum, the true distribution of errors associated with those rates. In the case of empirical data, however, the true distribution of errors is unknown, so statistical tests may be biased.

This study departs from previous research in two ways. First, we use a Monte Carlo simulation instead of empirical data to circumvent the empirical difficulties. One advantage of Monte Carlo simulation is that it allows definition of a set of "true" forward rates with a known distribution of errors for comparison with the accuracy of various methods of estimating those rates. Another advantage is that the data generated are free of unknown biases inherent in any estimation based on real data, such as those attributable to tax effects.

The second innovation of this study is that, consistent with user objectives, we measure accuracy in estimating forward *rates* rather than in explaining bond prices. While other studies show that all methods are good at explaining variations in bond prices, we find that there are considerable differences in their ability to estimate forward rates.

Methods developed for estimating the term structure can be classified as empirical or theoretical, according to their likely use. Empirical methods seek maximum accuracy in describing a complex reality; theoretical ones emphasize parsimony, choosing elegance over empirical fidelity. Because our objective is to offer a new methodology for comparing empirical methods of estimating the term structure, we limit ourselves to a sample of only the better-known methods.

Two of the methods we use are empirical, the OLS method proposed by Carleton and Cooper [1976] and the recursive method used by practitioners (Haugen [1986]). The third one is theoretical, the exponential polynomial (EP) method developed by Chambers, Carleton, and Waldman [1984]. We avoid the popular exponential spline method for reasons discussed later.

Our comparison of errors in forward rate estimates of the three methods reveals several patterns:

- For reasons unclear to us, the recursive and OLS methods generate virtually indistinguishable results.
- Estimates of the recursive and OLS methods are gener-

ally more accurate than those of the EP method, a relationship holding with or without heteroscedasticity.

- The recursive and OLS methods dominate the EP method over the entire term structure when forward rates follow a complex pattern (which is consistent with a smooth term structure of spot rates). This is caused by the inclination of the EP method to generate a smooth function even when the true set of forward rates is unsmooth. In such a case, the difference in accuracy is likely to be substantial.
- In terms of mean absolute error, all three methods are more accurate in estimating the short end of the term structure than the long end. When measured by standard deviation, accuracy is not affected by the term to maturity.
- The recursive and the OLS methods are more accurate than the EP method in estimating the short end of the term structure when forward rates follow a simple pattern, but less accurate in estimating the long end. This feature is not affected by the presence of heteroscedasticity.
- The absolute performance of all three methods deteriorates with a decrease in the number of observations per period, typically occurring toward the long end of the term structure, but the relative performance remains the same.

BACKGROUND

The empirically-based methods of estimating the term structure can be classified as continuous or discrete; the theoretically-based ones are all continuous. The continuous empirical methods include a few versions of the polynomial spline method and the exponential spline method.

McCulloch [1971, 1975] introduces the method of fitting by polynomial spline a discount function that produces a continuous term structure of spot rates. Although McCulloch's discount function is cubic, the model itself is linear in the discount function, so that an OLS regression can be used.

Langetieg [1980] and Langetieg and Smoot [1981, 1988] modify McCulloch's method by fitting cubic splines to the spot rates rather than the discount function, and varying the location of the spline knots. Non-linear estimation is required in this method.

The various spline methods share two shortcomings. Shea [1984, 1985] demonstrates that the term structure estimated by those methods tends to bend sharply

upward or downward toward the long maturities, rather than leveling off. This seems to be a most unlikely property of a true term structure, suggesting that those methods are of limited usefulness in predicting rates of long maturities. The spline methods also produce estimates sensitive to arbitrary assumptions about the number and location of knots separating the splines or to an arbitrary choice of an algorithm in search of the best configuration of knots.

These shortcomings are avoided by the discrete empirical methods. Carleton and Cooper [1976] assume that bond payments occur on a discrete set of specified dates, and that discount factors corresponding to those dates are independent, conforming to no specific pattern. Discount factors are estimated as coefficients in a linear regression, using bond payments as independent variables and the bond price as the dependent variable. The resulting term structure of spot rates is discrete, a feature shared by the recursive method (see Haugen [1986]).

Forward rates in the recursive method, which is essentially non-stochastic, are derived one at a time rather than simultaneously, starting with the shortest term to maturity. The forward rate of each incremental period is found by solving the discount function for one incremental unknown.

A third discrete empirical method, proposed by Schaefer [1982] and Ronn [1987], uses linear programming to estimate the term structure of interest rates faced by investors of different tax brackets, under the assumption that rational investors choose the coupon bonds that maximize their post-tax return.

Of the theoretical methods, three are cited frequently. Under a method proposed by Chambers, Carleton, and Waldman [1984], the term structure of spot rates is approximated by a single function, an exponential polynomial, in lieu of a chain of cubic splines. Like the spline methods, this one produces term structures with explosive tendencies toward the end of the fitted maturity range.

A second method, offered by Brown and Dybvig [1986], is based on a model proposed by Cox, Ingersoll, and Ross [1981]. Empirical tests by these authors indicate a misspecified model, overestimating short-term rates.

Nelson and Siegel [1987] propose a method where the term structure of spot rates is a three-parameter Laguerre function. According to their own findings, this model performs well for U.S. Treasury bills but substantially overestimates bond prices of longer maturities.

In selecting the methods for our comparison, we have been guided by several principles: 1) any method

used should be free of underlying theoretical assumptions that, if violated, might change the results; 2) there should be no arbitrary parameters or assumptions; 3) the methods should give a reasonable estimation of the term structure in all maturities, even when the true term structure is not a simple shape; and 4) the methods should be reasonable with all realistic phenomena, such as tax effects.

Of the theoretical methods, the Cox-Ingersoll-Ross method is tailored to a specific term structure theory, and therefore relies on specific theoretical assumptions. The Nelson-Siegel method is designed for theoretical parsimony, not accuracy. Of the empirical methods, the linear programming methods rely on a behavioral assumption and lose their edge in the absence of tax effects. We have already noted that the various spline methods require arbitrary empirical assumptions. Our screening criteria leave us with the exponential polynomial method (EP) from the theoretical group, and the recursive and OLS methods from the empirical group.

ESTIMATION METHODS

We use the notation below for all procedures:

- N_T = number of bonds maturing in period T ;
- T_i = maturity period for bond i , $T = 1, \dots, 20$ (20 six-month periods);
- $P_{i,T}$ = price per one dollar face value of bond i maturing T periods hence;
- $C_{i,t}$ = cash flow per one dollar face value of bond i to be paid t periods hence ($t = 1, 2, \dots, T$); note that $C_{i,T}$ includes the repayment of one dollar face value;
- $Y_{i,T}$ = yield to maturity of bond i ;
- $r_{i,t}$ = forward interest rate on bond i over period $t - 1$ to t ;
- r_t = the arithmetic average of all forward rates over period $t - 1$ to t (for bonds $i = 1, \dots, N$);
- $R_{i,t}$ = spot interest rate on bond i over period 0 to t ,

where $(1 + R_{i,t})^t = (1 + r_{i,1})(1 + r_{i,2}) \dots (1 + r_{i,t})$.

The ex-coupon price of bond i maturing at T is

$$P_{i,T} = \sum_{t=1}^T \frac{C_{i,t}}{(1 + Y_{i,T})^t} \quad (1)$$

which can be restated as a function of the spot interest rates implying a set of period 2 forward rates

$$P_{i,T} = \sum_{t=1}^T \frac{C_{i,t}}{(1+R_{i,t})^t} \quad (2)$$

$$r_{i,2} = \frac{C_{i,2}}{(1+r_1) \left[P_{i,2} - \frac{C_{i,1}}{(1+r_1)} \right]} - 1.$$

or a function of the forward interest rates

$$P_{i,T} = \sum_{t=1}^T \frac{C_{i,t}}{\prod_{s=1}^t (1+r_{i,s})} \quad (3)$$

Note that $Y_{i,T} = R_{i,1} = R_{i,2} = \dots = R_{i,T} = r_{i,1} = r_{i,2} = \dots = r_{i,T}$ only in the special case of a flat yield curve.

The Recursive Method

The set of r_t is derived from Equation (3) in a recursive manner, starting with r_1 . For each bond i ($i = 1, 2, \dots, N_T$) maturing at $T = 1$,

$$P_{i,1} = \frac{C_{i,1}}{(1+r_{i,1})} = \frac{C_{i,1}}{(1+R_{i,1})}$$

which implies

$$r_{i,1} = \frac{C_{i,1}}{P_{i,1}} - 1.$$

We define r_1 by

$$r_1 = \frac{\sum_{i=1}^{N_1} r_{i,1}}{N_1}.$$

For each bond i maturing at $T = 2$,

$$P_{i,2} = \frac{C_{i,1}}{(1+r_{i,1})} + \frac{C_{i,2}}{(1+r_{i,1})(1+r_{i,2})}.$$

Substitution of the average rate r_1 derived above yields for every bond maturing at $T = 2$,

$$P_{i,2} = \frac{C_{i,1}}{(1+r_1)} + \frac{C_{i,2}}{(1+r_1)(1+r_{i,2})}.$$

These rates are averaged over the set of bonds $i = 1, 2, \dots, N_2$ and then substituted in the price expression for three-period bonds, and so on. In general, the forward rate for period t is given by

$$r_t = \frac{\sum_{i=1}^{N_t} r_{i,t}}{N_t}$$

where

$$r_{i,t} = \frac{C_{i,t}}{\prod_{s=1}^{t-1} (1+r_s) \left[P_{i,t} - \sum_{s=1}^{t-1} \frac{C_{i,s}}{\prod_{q=1}^s (1+r_q)} \right]} - 1.$$

The OLS Method

Following Carleton and Cooper [1976], we define the discount function

$$D_{i,t} = \frac{1}{\prod_{s=1}^t (1+r_{i,s})}$$

which is the present value of one dollar paid on bond i at time t . Substitution of the discount function into Equation (3) yields

$$P_{i,T} = \sum_{t=1}^T C_{i,t} D_{i,t}$$

which is augmented by a disturbance term to give the linear estimation equation

$$P_{i,T} = \sum_{t=1}^T C_{i,t} D_{i,t} + e_i. \quad (4)$$

The $D_{i,t}$ coefficients are estimated subject to the assumptions $E(e_i) = 0$ and $E(e_i, e_j) = 0$ for all $i \neq j$. The

forward rates are then derived by $r_t = (D_{t-1}/D_t) - 1$. The OLS method assumes that bond payments occur only at a discrete set of specified dates, and that bond payments are independent. Independence is achieved by ensuring that at least one bond matures in each time period.

The Exponential Polynomial Method

The OLS method requires that bonds mature in each and every period. If this condition is not met, the discount function can be estimated by the EP method proposed by Chambers, Carleton, and Waldman [1984]. According to the Weierstrass Theorem, a continuously differentiable function can be approximated over a given interval to within an arbitrarily small error by some polynomial defined over the same interval. Therefore, let

$$R_t = \sum_{j=1}^J x_j t^{j-1}$$

where

$$\begin{aligned} x_j &= \text{the } j\text{th polynomial coefficient; and} \\ J &= \text{the order of the polynomial.} \end{aligned}$$

Assuming continuous compounding, the discount function is

$$D_t = \exp\left(-\sum_{j=1}^J x_j t^j\right)$$

and the regression equation

$$P_{i,T} = \sum_{t=1}^T C_{i,t} + \exp\left(-\sum_{j=1}^J x_j t^j\right) + \epsilon_i \quad (5)$$

is estimated using non-linear least squares.

SIMULATION METHODOLOGY

Overview

In order to compare the three chosen estimation procedures, we use Monte Carlo simulation with 100 trials. Each trial has several steps.

First, we generate four profiles of forward rates roughly matching the four kinds of yield curves observed in practice. Second, we create several series of bonds with various coupon rates and maturities, and various numbers of bonds in each series type. Third, we use the "actual"

forward rates and cash flows to calculate the price of the bonds. To mimic reality, we add a random component to the calculated price. Fourth, we use the cash flows and prices to estimate the forward rates with each of the three methods. Finally, we compare the precision of the three estimation methods in revealing the predetermined forward rates.

Creating the Forward Rates

The term structure shapes observed empirically — rising, declining, flat, and humped — are simulated with rising and declining logarithmic curves, a straight line, and a random walk. This is not an exhaustive list of all possible shapes, but our interest is to compare estimation methods in a manner that preserves the salient features of actual term structures. (For example, with a rising yield curve, the rates at first increase rapidly and then level off, as does the logarithm curve.) The random walk is included as a catch-all of many possible shapes that may or may not be predeterminable.

In order to generate curves that are as realistic as possible, we want annual forward rates roughly in the range of 0% to 10%. To do this, we pick an arbitrary initial forward rate of 2.5% (5.1% annual) for each of the methods, and then create subsequent rates as follows:

(a) For the increasing term structure,

$$r_t = r_1 + \frac{\ln t}{100}; \quad t = 2, \dots, 20.$$

(b) For the decreasing term structure,

$$r_t = r_1 + \frac{\ln t}{100}; \quad t = 2, \dots, 20.$$

(c) For the flat term structure,

$$r_t = r_1 \quad t = 2, \dots, 20.$$

(d) For the random walk,

$$r_t = r_{t-1} + r_{t-1}\phi,$$

where $\phi \sim U(0, -0.5 \text{ to } 0.5)$; $t = 2, \dots, 20$.

The coefficient 1/100 in (a) and (b) is merely a scaling factor to assure feasible rates. Similarly, the range of the uniform distribution in (d) is chosen experimentally to maximize realism.

Note that so far all but the random walk are smooth functions. Smoothness is appropriate for spot rates, which average forward rates, but would be a restric-

tive assumption for forward rates. Therefore, we perturb the forward rates in (a) through (c) by

$$r_t^* = r_t + \tau, \text{ where } \tau \sim N(0.001, 1).$$

Again, the mean of the normal distribution is chosen for realism.

Coupon Cash Flows

We arbitrarily choose a range of 5% to 10% for the annual coupon rates, but randomly assign a coupon rate to each issue within that range. Specifically, the coupon rate of bond *i* is determined as

$$C_i \sim U(0.025, 0 \text{ to } 0.05); i = 1, \dots, N,$$

where *U* is a uniform distribution, with mean 0.025 and range 0 to 0.05.

Maturity and the Number of Bonds

In order to include in the study short- and long-term bonds without burdening the results, we choose maturities of six months to ten years, at six-month intervals. The number of bonds is set two ways: 1) five bonds for each maturity, and 2) a random number of bonds, which declines over time. The latter alternative is closer to reality where the number of bonds varies across maturities, typically declining as the time to maturity increases. This is accomplished somewhat arbitrarily by

$$N_t = \text{int}\left(\frac{20\phi}{t}\right) + 1.$$

Prices

The price of bond *i* maturing in period *T* is calculated as the present value of all cash flows discounted by the appropriate “true” forward rates, and then perturbed to simulate market pricing errors. These pricing errors might be homoscedastic or heteroscedastic, so we use both patterns.

In the heteroscedastic case, our aim is to reflect reality by increasing the errors with maturity at a decreasing rate. Formally,

$$P_{i,t} = \frac{\sum_{t=1}^T C_{i,t}}{\prod_{s=1}^t (1 + r_s^*)} + \epsilon_i$$

where $\epsilon \sim N(0, \sigma)$ and

$$\sigma = 1, \quad \text{if } \epsilon_i \text{ homoscedastic,}$$

$$1 + 0.5 \ln(\text{duration}), \quad \text{if } \epsilon_i \text{ heteroscedastic,}$$

where 0.5 is again a “realism” scaling factor, and duration is defined according to Macaulay.

RESULTS

Two types of Monte Carlo results are reported. The mean absolute error (MAE) in estimating given sets of forward rates is calculated by the three competing methods and compared in Figures 1 through 3 and Figure 6. Statistical efficacy of the three methods is compared according to the number of standard deviations (SD) by which the estimates differ from the predetermined sets of forward rates. These results are displayed in Figures 4 and 5.

A summary of the results in the Table, divided into homoscedastic versus heteroscedastic simulated data, reveals that:

- The results of the recursive and OLS methods are virtually indistinguishable, yet they can be substantially different from those of the EP method.
 - The presence of heteroscedasticity reduces the accuracy of all three estimation methods as measured by the MAE, with the least effect seen for the EP method. When measured by the SD, the effect of heteroscedasticity on accuracy may be in the opposite direction.
 - Overall, the recursive and OLS methods appear to be at least as accurate as the more sophisticated EP method. This relationship holds both economically and statistically, with or without heteroscedasticity.
 - All three methods show a similar average SD under a flat term structure with homoscedasticity (Panel A), and a similar average MAE under an increasing term structure with heteroscedasticity (Panel B). This parity is replaced by superiority of the recursive and OLS methods under all other scenarios tested.
- Figures 1 through 6 reveal further detail about the performance of the three estimation methods. Figures 1 and 2 display the MAE in estimating rates with and without homoscedasticity. From the figures we conclude:
- Consistent with the summary findings reported in the Table, results obtained using the recursive and OLS methods are indistinguishable at the level of the individual forward rate.
 - All three methods are generally more accurate in esti-

TABLE ■ Forecast Error in Forward Rates Averaged Over Twenty Periods

Shape	Mean Absolute Error(%)			Number of Standard Deviations From True Forward Rates		
	Recursive	OLS	Polynomial	Recursive	OLS	Polynomial
A. Homoscedastic Errors						
Flat	0.081	0.080	0.143	0.082	0.082	0.073
Increasing	0.098	0.098	0.160	0.120	0.119	1.560
Decreasing	0.074	0.074	0.151	0.075	0.076	1.500
Random	0.067	0.666	0.613	0.083	0.083	23.86
B. Heteroscedastic Errors						
Flat	0.139	0.138	0.144	0.088	0.088	0.104
Increasing	0.175	0.174	0.164	0.048	0.048	1.177
Decreasing	0.111	0.111	0.160	0.093	0.093	0.967
Random	0.124	0.124	0.615	0.060	0.060	17.29

inating the short end of the term structure than the long end.

- The recursive and OLS methods are more accurate than the EP method in estimating the term structure, except when forward rates follow a simple time pattern and are heteroscedastic. Comparison of Figures 1 and 2 indicates that all these features are present with or without homoscedasticity.

To help interpret the last feature under the more complex random walk pattern, we present in Figure 3A the sets of forward rates and their estimates underlying the errors displayed in Figure 1D. The set of random forward rates in Figure 3A, generated by lognormal distribution, is visually indistinguishable from the sets of rates estimated by the recursive and OLS methods.

The EP method, by contrast, generates a set of estimated forward rates along a smooth curve, resulting in large errors. The feasibility of such a scenario is confirmed by evidence in Figure 3B that the complex pattern of forward rates in Figure 3A is consistent with a perfectly realistic smooth pattern of spot rates.

Figures 4 and 5 provide a statistical comparison of the performance of the three methods. They report the number of standard deviations in estimating each of twenty known forward rates with and without homoscedasticity. These Figures support several statements:

- The close similarity between the recursive and OLS methods is reconfirmed.
- Unlike the MAE, the statistical error measured by SD does not systematically increase for estimation of forward rates lying farther into the future under any of the three methods. On the contrary, the error diminishes somewhat at the long end under the EP method (Figures 4B-D, 5B-D).

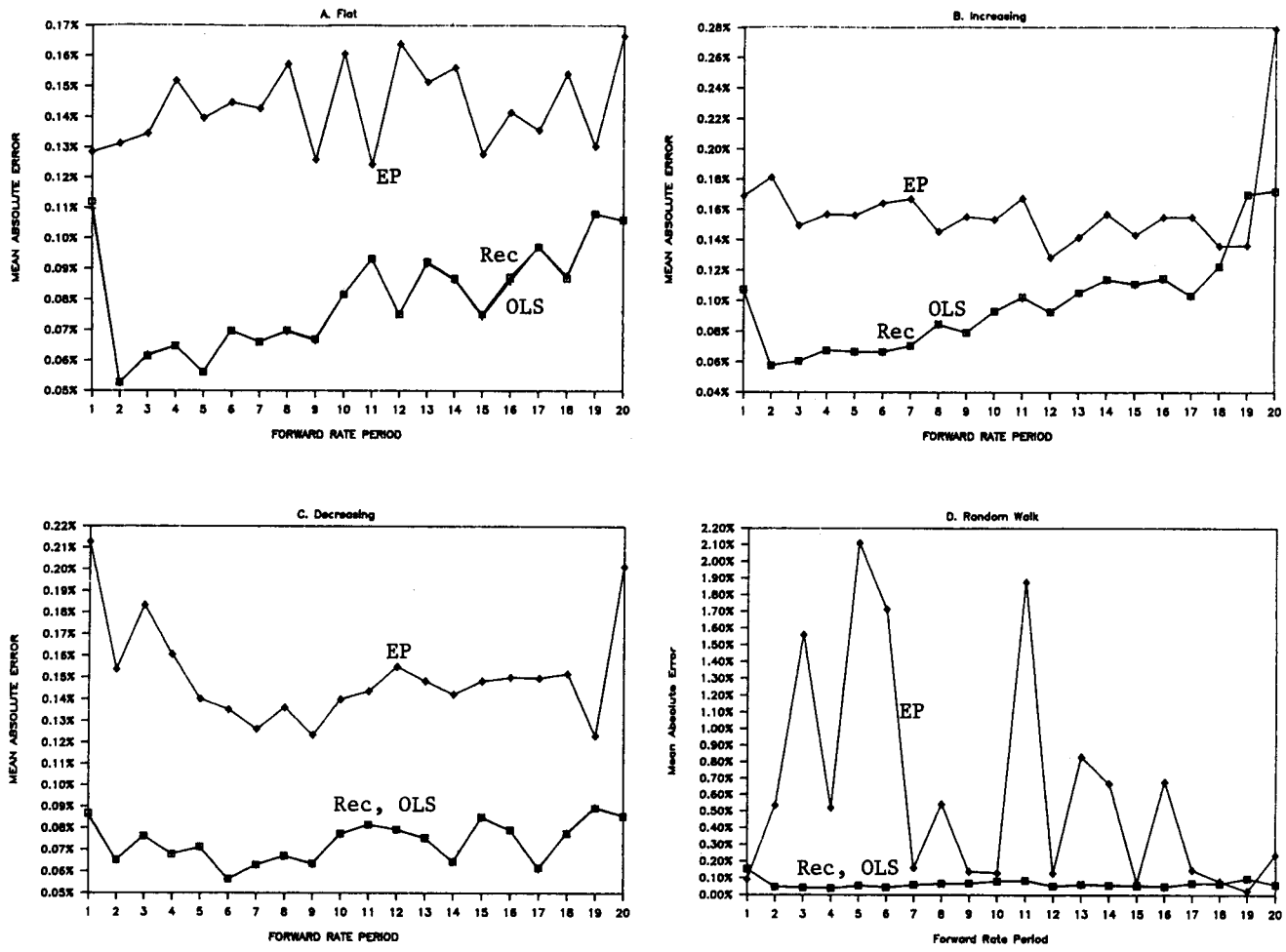
- Except in the case of a flat term structure, where the three methods perform equally well (Figures 4A, 5A), the recursive and OLS methods are generally more accurate. Like the MAE, the advantage in accuracy measured by the SD is most dramatic when the term structure of forward rates does not follow a simple pattern (Figures 4D, 5D).
- The presence of heteroscedasticity does not affect the relative performance of the three methods but, somewhat unexpectedly, improves their absolute performance (Figures 4 and 5).

All comparisons discussed so far assume the same number of bonds for each maturity. This assumption is not realistic in the case of the market for U.S. Treasury securities, where there are more short-term bonds than long-term bonds because of issuing patterns and “stripping” at the long end. This distribution of maturities is consistent with a uniform distribution of *new* issues across maturities, because in time all new issues become short-term bonds.

To determine the effect of this characteristic on the relative performance of the three methods, we recalculate the MAE under upward-sloping and random walk sets of forward rates, monotonically and randomly decreasing the number of observations with increases in the term to maturity. These results, displayed in Figures 6A and 6B, should be compared with those of Figures 1B and 1D, which are based on homoscedastic data and similar time profiles of forward rates.

Inspection of the four figures suggests that a decrease in the number of observations reduces the absolute accuracy of estimating forward rates as maturity increases under all three methods, but does not appreciably change the relative accuracy of those methods.

FIGURE 1 ■ Mean Absolute Forecast Error with Homoscedastic Errors



CONCLUSION

The major conclusion emerging from our tests is that the recursive and OLS methods are generally not inferior and may be superior to the EP method in estimating the term structure of interest rates. Superiority of the former methods is generally present when the term structure of forward rates has a complex shape. With only five parameters, the EP method is not designed to follow all the “wiggles” of a complex function, but to capture its basic shape.

While previous empirical studies have shown that all estimation methods are effective in explaining bond prices, this conclusion in itself is not directly relevant for someone concerned with accurate estimation of forward rates. As the methods that we compare represent two extremes in terms of the number of parameters used, we would expect the accuracy of the various spline methods to fall between the extremes defined by our results. In view of the small improvement in accuracy likely, we question the use of more expensive estimation methods based upon arbitrary assumptions.

FIGURE 2 ■ Mean Absolute Forecast Error with Heteroscedastic Errors

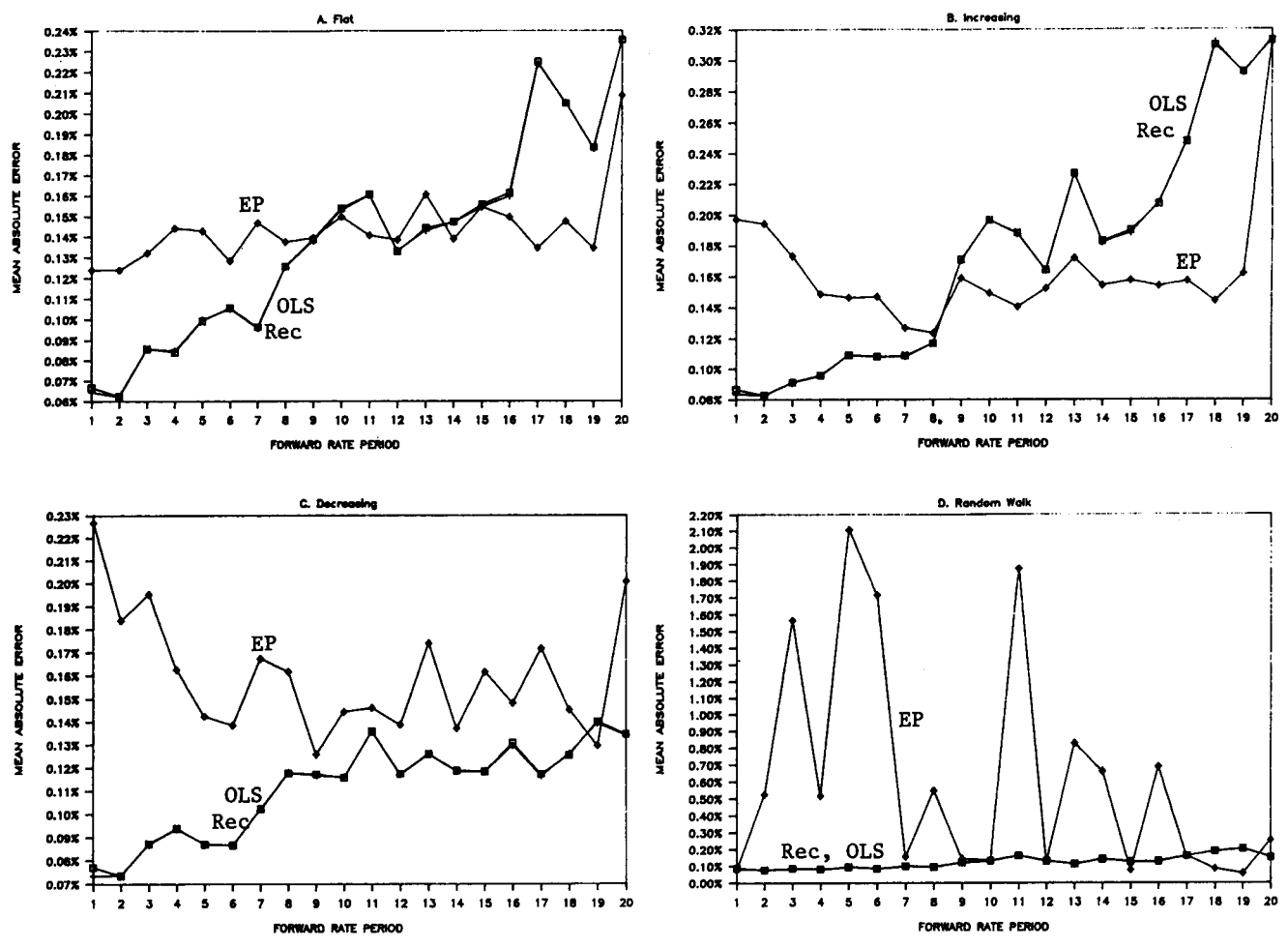


FIGURE 3 ■ Forward and Spot Rates Consistent with Figure 1

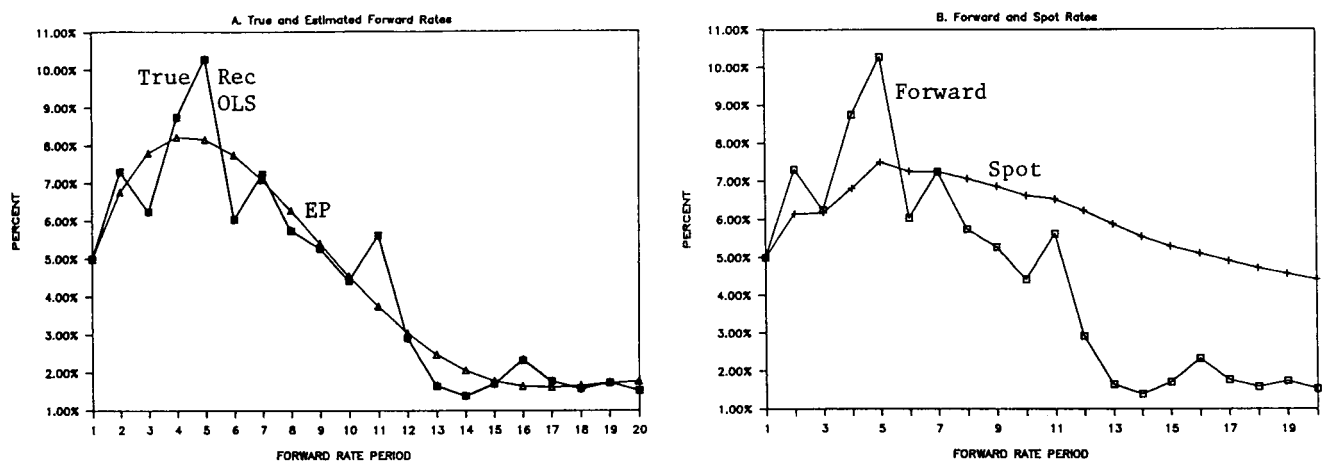


FIGURE 4 ■ Number of Standard Deviations of Forecast Error with Homoscedastic Errors

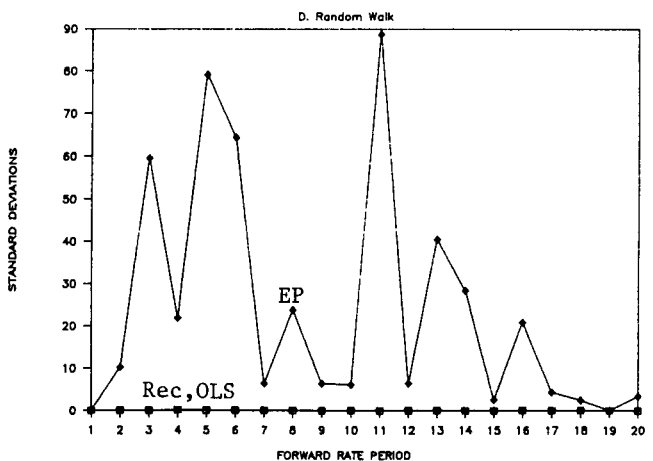
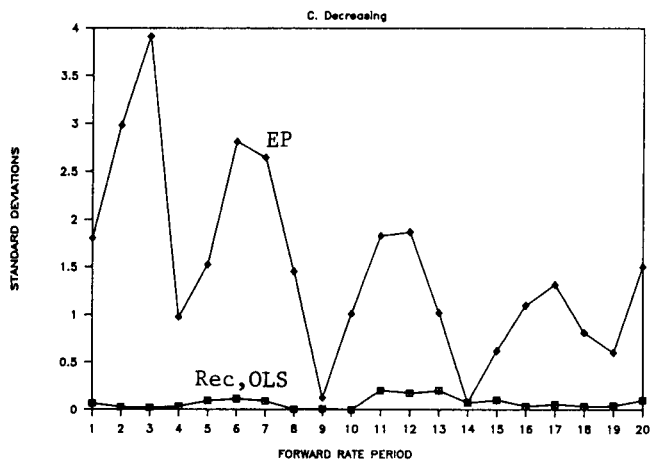
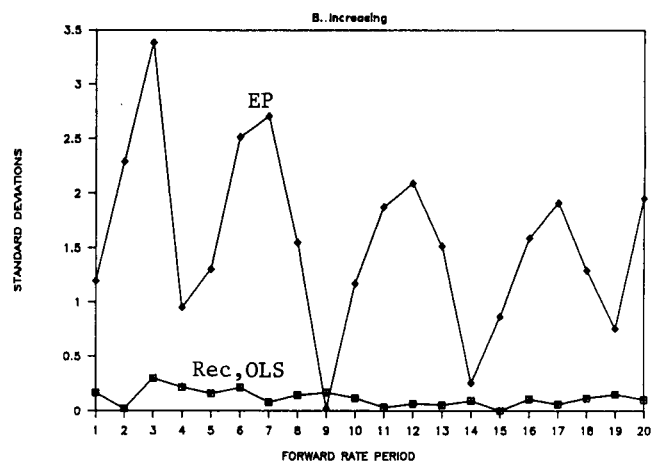
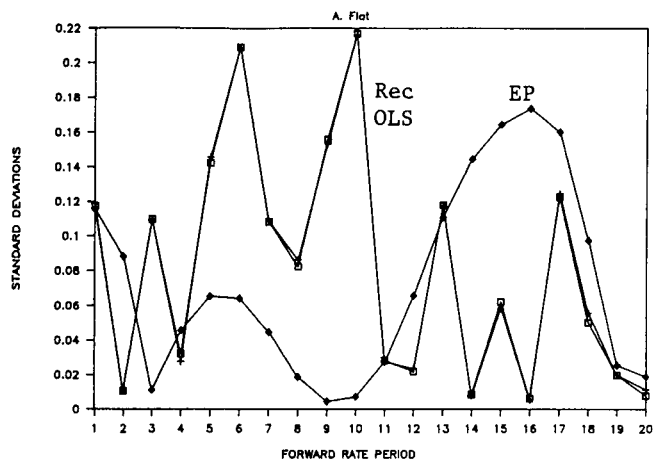


FIGURE 5 ■ Number of Standard Deviations of Forecast Error with Heteroscedastic Errors

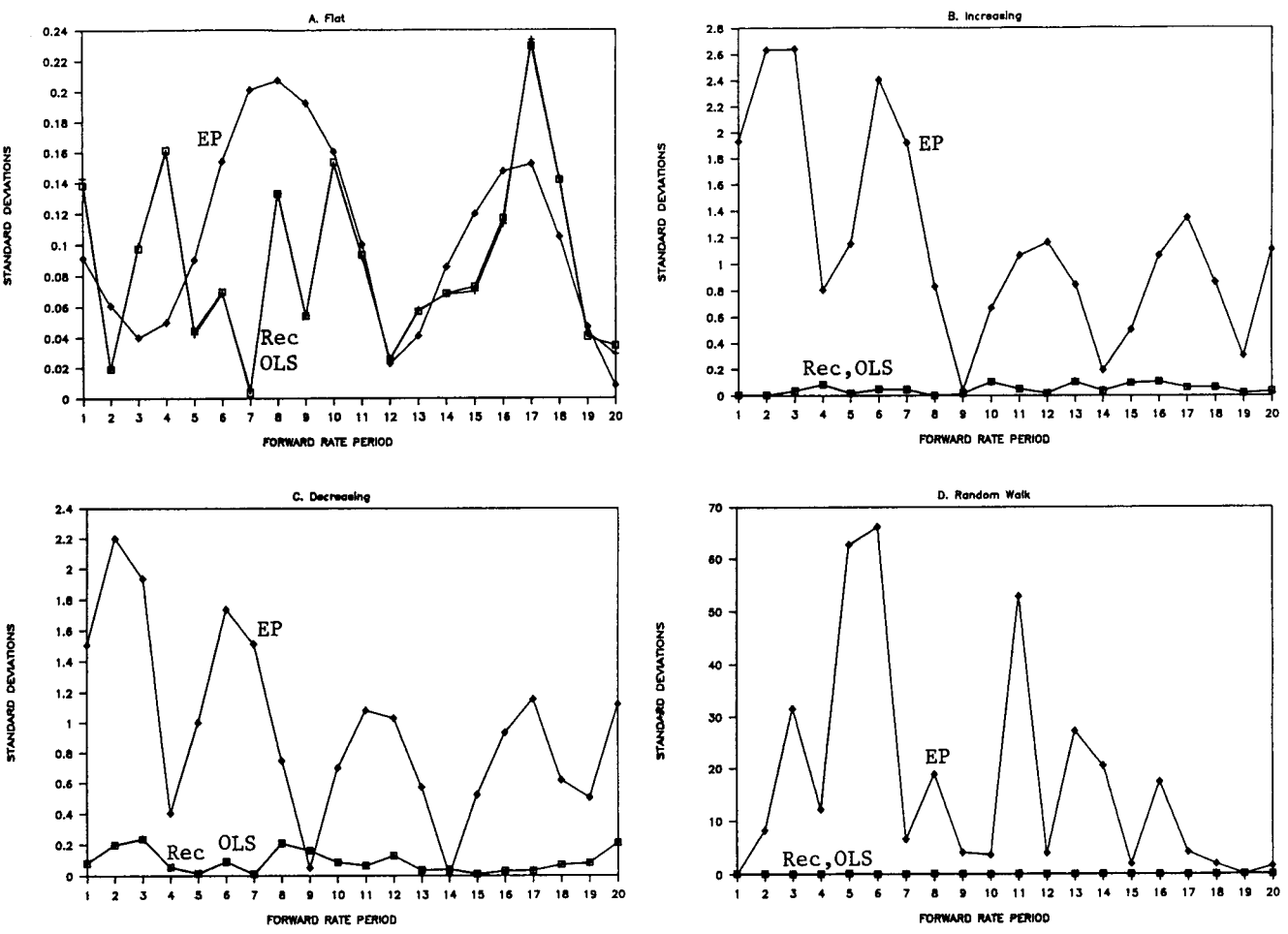
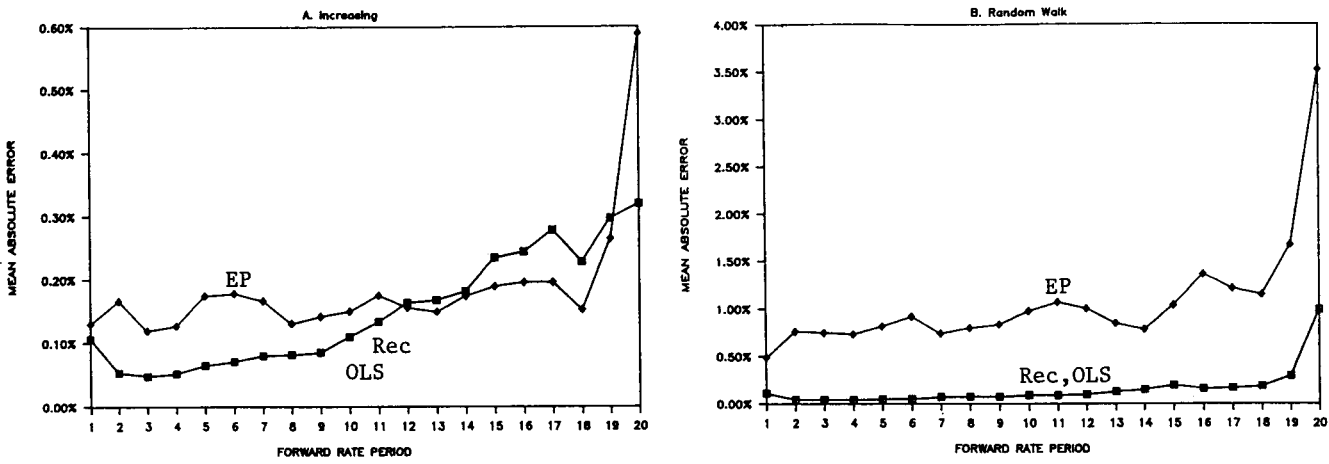


FIGURE 6 ■ Mean Absolute Forecast Error with Homoscedastic Errors and Diminishing Observations



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