Setting the Processing Parameters in Injection Molding through Multiple Criteria Optimization: A Case Study

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Abstract—**In this work, a case study involving statistical characterization and multiple criteria optimization on injection molding is presented. This case study is the first application of a method previously described in the literature involving Data Envelopment Analysis geared towards setting design and process variables to meet several performance measures.**

***Index Terms*—Data Envelopment Analysis, Design of Experiments, Injection Molding, Multiple Criteria Optimization**

I. Introduction

Controlling the injection molding process of thermoplastics is critical due to the high dependency of the material behavior inside the mold and the final part's properties on the process variables. The aim of this work was to set the process variables in a thermoplastic injection molding operation in a local company considering multiple criteria in a simultaneous manner. The task was approached through the application of an optimization strategy based on Data Envelopment Analysis (DEA). This work is, in fact, the first reported practical case of this strategy proposed on previous publications [**1-4**].

II. Optimization Strategy

The optimization strategy proposed by Cabrera-Ríos, et al. [**3,4**] and Castro, et al [**5,6**] to find the best compromises among multiple performance measures in polymer processing was used in this work. The strategy comprises five steps schematically shown in **Figure 1**.

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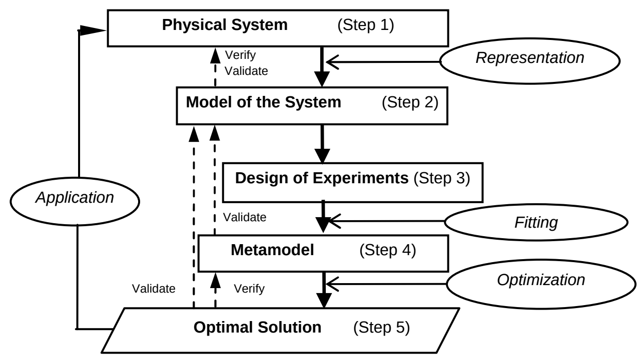

Figure 1. The Optimization Strategy

The strategy prescribes, as a first step, to define the physical system to be studied. This step includes identifying the phenomena of interest, the controllable and non controllable variables, the experimental region as well as the performance measures to be included in the study.

Once the physical system has been defined, the second step includes defining physics-based models (usually transport-phenomena models) that relate the controllable variables with the performance measures of interest.

In some cases, physics-based models might not exist or be too complex to handle either due to their functional form or due to the experimenters not having the right expertise. In these cases, however, it is possible to conduct an experimental design to elicit useful knowledge as indicated in the third step of the optimization strategy. Such knowledge will take the form of empirical models in the fourth step. These models are called metamodels from this point on. These metamodels are in general easier to use in an optimization problem.

The fifth and last step of the strategy consists on using Data Envelopment Analysis (DEA) to solve the multiple criteria optimization problem involving the resulting metamodels. Solving such a problem translates into finding the best compromises among all performance measures of interest.

The application in the case study of this optimization strategy is detailed in the following sections.

III. CASE STUDY

The optimization strategy outlined above was applied as an offline control project in a local manufacturing plant at their thermoplastic injection molding operations. The objective was to determine the process settings that accomplished the following for a particular line of rear automotive lamps: i) having a tighter control over critical dimensions (Significant Characteristics or SCs) and ii) meeting required optical properties.

The lamp under study had two components: (1) the body or housing (figure 2), and (2) the lens (figure 3). Both components are injection molded. Specifications for these components are determined considering industrial and safety standards as well as aesthetics.

Figure 2. Lamp Body

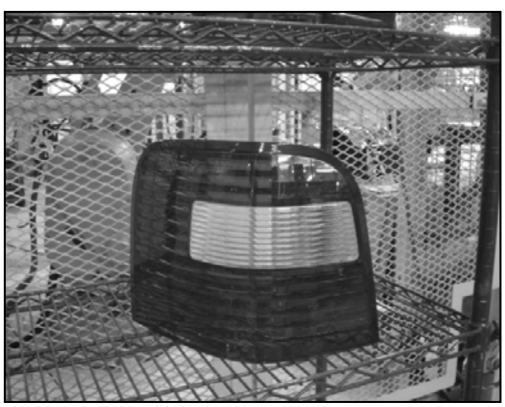

Figure 3. Lamp Lens

As it can be inferred, light reflection is important for the lens. This phenomenon is measured through photometry instruments and reported as a Reflex Index, which relates to the intensity of light reflected by the lamp. Light is reflected by the lamp through molded cubic features (figure 4). When perfectly filled, these cubes act as mirrors [**5**].

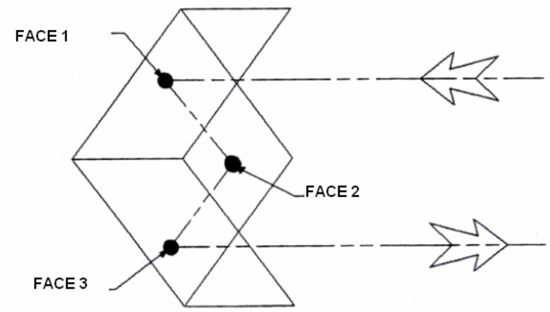

Figure 4. Injection Molded Cubes for Optical Properties

The general process to manufacture each lamp is shown in **Figure 5**. During the molding phase, parts are randomly chosen for inspection to measure their particular high impact characteristics (HICs).

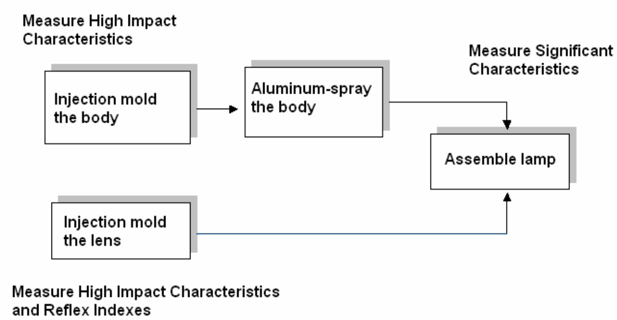

Figure 5. Schematic of Automotive Lamp Manufacturing Process

Reflex index measurements are additionally performed in the lenses from the side and the rear. These are different due to the lenses' geometry (**Figure 6**).

Following injection molding, the body goes through an aluminum thermal spraying process. Finally, both the body and the lens are assembled together. A final inspection will randomly select assemblies to measure specifications required by the particular customer called significant characteristics (SCs).

After analyzing the operation, it was decided to focus on the lenses, since they were experienced to show larger deflections than the bodies, and since the optical properties of the lamp came precisely from the material in this component (PMMA). The rationale was that it was necessary to map the dependency of the lenses' HICs and Reflex Indexes to the injection molding process variables, and then to map the dependency of the assemblies' SCs to the lenses´ HICs.

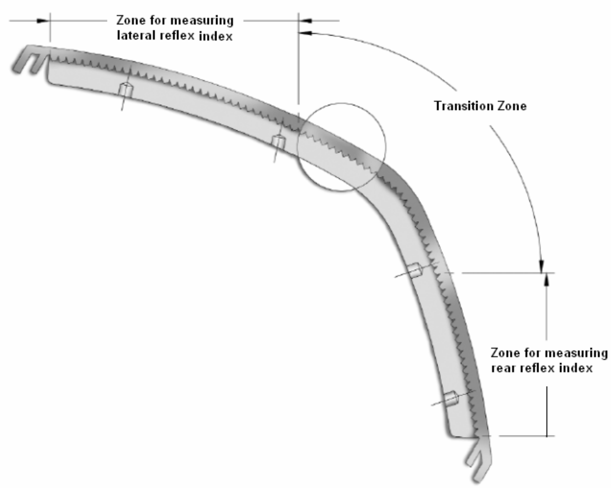

Figure 6. Shape of the Lamp Lens

IV. THE OPTIMIZATION STRATEGY IN THE CASE STUDY

A. *First Step: Physical System*

Emphasis on this first step is on structuring the details of the physical system under study. Controllable variables and their experimental range are chosen, as well as the performance measures of interest. In order to choose the controllable variables, it is helpful to gather knowledge either from technical expertise or from previous experience to improve the probability of these variables being significant to the performance measures.

Table 1 shows the performance measures chosen for this case study. These performance measures represent the SCs of the lamp under study. Larger reflex and mass values are preferred; therefore, it is required to maximize these performance measures. One of the starting hypothesis of this study is that the higher the mass, the better the part will resemble the mold cavity, which in turn follows customer's specifications.

Table 1. Peformance Measures to be optimized.

Performance Measure [units]	**Identifier**
Reflex Index - Rear Right [millicandelas/lux]	ReflexRR
Reflex Index - Rear Left [millicandelas/lux]	ReflexRL
Reflex Index - Lateral Right [millicandelas/lux]	ReflexLR
Reflex Index - Lateral Left [millicandelas/lux]	ReflexLL
Mass of Right Lens [Kilograms]	MassR
Mass of Left Lens [Kilograms]	MassL

Table 2 shows the seven controllable variables, chosen with the aid of the company's engineers, to be included in this study. Packing pressure (P_p) and packing time (t_p) were included because they can be used to compensate part shrinkage; valve temperature (T_v) and nozzle temperature (T_n) are important to keep the material liquid at the mold´s entry point and thereby allow for a better control of the flow; and injection speed (V) was chosen because of its effect on the part´s final quality: low speed could result in flow streaks and/or nonfills, and a high speed can result in material degradation and airtraps. Finally, mold temperatures such as the fixed half's (T_f) and the moving half (T_w) are critical for the part cooling, and therefore for cycle time and part warpage.

Table 2. Controllable Variables.

Controllable Variable [units]	**Identifier**
Nozzle Temperature [Kelvins]	T_n
Packing Pressure [Megapascals]	P_p
Packing Time [seconds]	t_p
Mold Temperature (Fixed half) [Kelvins]	T_f
Mold Temperature (Moving half) [Kelvins]	T_w
Injection Speed [meters/second]	V
Valve Temperature [Kelvins]	T_v

Experimental ranges for the controllable variables were determined based on the settings used by the company on their daily operations along with suppliers' data sheets; **Table 3** shows the experimental region and the three sampling levels per variable initially planned. Three levels are necessary to characterize nonlinear behavior in the performance measures of interest.

Table 3. The three levels per controllable variable used to explore the experimental region.

Coded Levels	T_n (K)	P_p (MPa)	t_p (s)	T_f (K)	T_w (K)	V ($m\times10^{-2}/s$)	T_v (K)
-1	499.82	5.17	7	302.59	305.37	3.302	305.37
0	510.93	5.52	8	310.92	310.92	3.556	310.927
1	522.04	5.86	9	319.26	316.48	3.810	316.48

B. Second Step: Modelling the Physical System

In this step physics-based models relating the controllable variables with the performance measures are defined. These models need to be a) verified, and b) validated, where a) refers to show that the model represents what it is supposed to represent, and b) means that the model represents the phenomenon to an adequate level of fidelity.

In this case study, there were no simple physics-based models to relate the controllable variables and the performance measures of interest, so this step did not apply.

C. Third Step: Run a Design of Experiments

In this step, a design of experiments must be carried out to measure the statistical effect of controllable variables in the performance measures. In this case study, the aim was to be able to control mass and reflex indexes.

A full factorial design is a common choice in this step. In a full factorial design, k factors each at m_i, i=1,…,k levels are sampled in an exhaustive enumeration fashion i.e. trying all $m_1 \times m_2 \times \ldots \times m_k$ combinations. In this case study, however, a full factorial was not an option because even when considering only one replicate, it would have required manufacturing 3^7=2187 lenses. With three replicates, the total would have been 6561 lenses. This was clearly too expensive in terms of time and money.

The choice, then, was a D-Optimal design [**6**]. This design comprises a subset of the experimental runs from the full factorial design, with a number of runs decided *a priori*. The runs are chosen to minimize the variance of the coefficients of the metamodels described in step four. A total of 40 runs with three replicates per run were finally required (120 lenses) according to the experimentation time and the material that would be used. The experiment was constrained to be run during normal production time.

During the execution, several compromises had to be made along the way. Although the experiment was planned to be run in a random order, due to a malfunction in the injection molding machine it required a lot longer than the usual 15 min to get the injection molding machine to steady state for each combination, therefore it was decided to organize the runs as to have the fewest possible changes between them. Also because an urgent order came in requiring a different mold, the experiment's time was cut short, making it possible to run only 21 runs. This resulted in sacrificing the estimability of some of our metamodels´ terms as well as not fully using three levels in all variables. The effects of these compromises are assessed in the next section, along with the explanation on how the metamodels were obtained.

D. Fourth Step: Obtaining the metamodels.

The data generated in the previous step was used to create empirical expressions called metamodels. These expressions relate the performance measures to the controllable variables with statistical basis. Among the best known metamodels one can find the regression models and the artificial neural networks.

Following the optimization strategy, the metamodels would be used to, first, perform an analysis of variance and then to predict values for the performance measures. In this case study, the metamodels are solely used for the first objective.

Six linear regression metamodels were created in this case study, one per performance measure of interest (**Table 1**). **Table 4** shows these metamodels with their statistically significant terms at a confidence level of 95%.

Besides checking for the adequate fit of the regression metamodels through the R^2 value, special care was given to the behavior of the prediction errors (residuals). These must always be checked to follow a normal distribution, and have a mean of 0 and constant variance.

Having obtained an R^2 value larger than 93% for all but one performance measure in presence of replicates showed that the terms that could not be estimated could have accounted for no more than 7% of the total variation in the experiment, alleviating their omission. In the only case where this does not happen, the fit, at a value larger than 80%, is still considered satisfactory for a replicated experiment Furthermore, having checked the independence of the residuals helped to verify that a major systematic error was not induced when decided to run the experiment in a non random order.

The term "cavity" in Table 4 refers to a blocking factor not considered initially. Because the mold used in the experiment was a multicavity one, it was assumed to produce identical parts in all cavities, however, the analysis of variance showed that this assumption did not hold. This finding led to including this term in the model to characterize its effect on the performance measures.

Further exploration of **Table 4** can help uncover how different performance measures are affected in conflicting directions by changes in particular variables. One example of this is the effect of T_w on the mass values and the reflex index values, where it would be desirable that all measures changed in the same direction.

E. Fifth Step: Optimize

In order to conclude this study, the performance measures that resulted from the experiment were evaluated to assess which process settings gave the best compromises among them. This entailed a multiple objective optimization task, solved here through Data Envelopment Analysis (DEA) as prescribed by the optimization strategy. One of the advantages of using DEA is that it relies in the solution of a series of linear optimization problems even when the objective functions involved are nonlinear.

DEA is able to find the efficient frontier or envelope of a set of candidate solutions. The solutions in this envelope are in turn the best compromises described previously, and are formally called Pareto-efficient solutions or efficient solutions for simplicity. In this case, the candidate solutions were the averages of the performance measures of the three lenses produced under each particular combination of process settings tried in the experiment. A total of 21 candidate solutions were analyzed.

The DEA formulations in their so called BCC model forms, named after their authors Banks, Charnes and Cooper [**7,8**], are given as follows:

$$
\begin{aligned}
&\text{Find } \mathbf{v}, \boldsymbol{\mu}, \mu_0^+, \mu_0^- \text{ to} \\
&\text{Maximize } \boldsymbol{\mu}^T \mathbf{Y}_0^{\max} + \mu_0^+ + \mu_0^- \\
&\text{Subject to} \\
&\mathbf{v}^T \mathbf{Y}_0^{\min} = 1 \\
&\boldsymbol{\mu}^T \mathbf{Y}_j^{\max} - \mathbf{v}^T \mathbf{Y}_j^{\min} + \mu_0^+ + \mu_0^- \le 0 \qquad j = 1,\dots,n \\
&\boldsymbol{\mu}^T \ge \varepsilon \cdot \mathbf{1} \\
&\mathbf{v}^T \ge \varepsilon \cdot \mathbf{1} \\
&\mu_0^+, \mu_0^- \ge 0
\end{aligned}
\qquad (1)
$$

$$
\begin{aligned}
&\text{Find } \mathbf{v}, \boldsymbol{\mu}, \nu_0^+, \nu_0^- \text{ to} \\
&\text{Minimize } \mathbf{v}^T \mathbf{Y}_0^{\min} + \nu_0^+ + \nu_0^- \\
&\text{Subject to} \\
&\boldsymbol{\mu}^T \mathbf{Y}_0^{\max} = 1 \\
&\mathbf{v}^T \mathbf{Y}_j^{\min} - \boldsymbol{\mu}^T \mathbf{Y}_j^{\max} + \nu_0^+ + \nu_0^- \ge 0 \qquad j = 1,\dots,n \\
&\boldsymbol{\mu}^T \ge \varepsilon \cdot \mathbf{1} \\
&\mathbf{v}^T \ge \varepsilon \cdot \mathbf{1} \\
&\nu_0^+, \nu_0^- \ge 0
\end{aligned}
\qquad (2)
$$

where $\boldsymbol{\mu}$ and $\mathbf{v}$ are column vectors of multipliers to be determined along with variables μ_0^+ and μ_0^- in the first case and ν_0^+ and ν_0^- in the second case; $\mathbf{Y}_j^{\min}$ and $\mathbf{Y}_j^{\max}$ are column vectors containing the values of the j^{th} combination of performance measures to be minimized and maximized respectively; finally ε is a scalar typically set to a value of $1\text{x}10^{-6}$.

Model (1) is called the input-oriented BCC model and model (2) is the output-oriented BCC model. Both models must be applied to each of the *n* candidate solutions. The particular solution with an optimal objective function value equal to 1 in both models is considered a Pareto-efficient solution, and therefore, it belongs to the envelope of the solution set. A more detailed discussion regarding DEA can be found in [**1-4,7**].

V. Results

Table 5 and **Figure 7** show the seven combinations of settings found to be efficient through DEA along with their predicted performance values. With this information, the decision maker can quantitatively assess the compromise implied by the choice of a particular solution over the others.

For example, in reference to **Table 5** and **Figure 7**, the decision maker can opt for high Reflex Index values with their associated mass values or, if the dimensions are critical, the maximum mass values can be sought after with their associated Reflex index values.

The advantage of this part of the optimization strategy is precisely the objectivity of the final analysis, where only efficient solutions are presented for the decision maker to choose from. This has to do with the decision maker's preference being revealed *a posteriori* and not *a priori*.

In this particular case, the choice was to pick the option with the lowest combined mass (Option 6), to allow for material savings. These settings still kept the Reflex Index values to acceptable levels, i.e. larger than 4.5 mcd/lx and less than 300 mcd/lx.

VI. Conclusions and Future Work

Controlling an injection molding process to obtain conforming parts is complicated by the fact that the behavior of the material inside the mold depends heavily on the process parameters. Such behavior affects several key performance measures simultaneously, not all of them in a positive manner, thereby imposing different compromises. Adequately setting these parameters require a deep understanding of the phenomena involved in the process and their variation as functions of the processing parameters.

In this work, an optimization strategy previously proposed in references [**1-4**] was demonstrated through a case study to gain knowledge about the injection molding process, as well as to competitively set the controllable variables to result in the best compromises among several performance measures concurrently. This is the first reported practical case of the above mentioned strategy.

Future work in this research line will include the formulation of efficient injection molding process windows through a finer characterization of the efficient set of solutions.

VII. Acknowledgement

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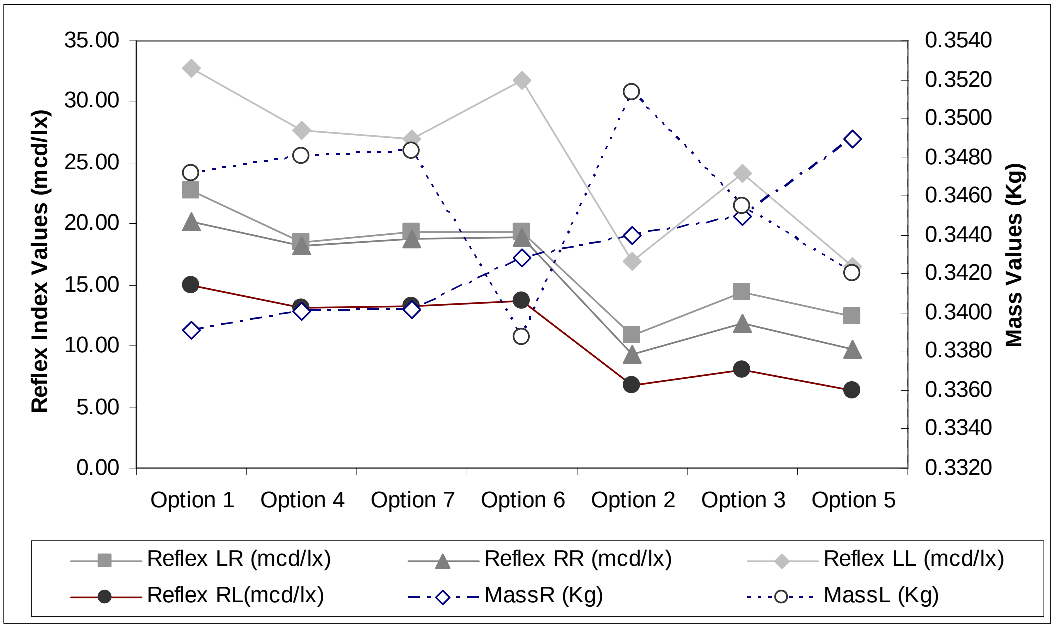

Figure 7. The seven efficient solutions in terms of their performance measure values.

Table 4. Six metamodels created with their significant terms at a confidence level of 95%.

Metamodel Term	MassR	Reflex LR	Reflex RR	MassL	Reflex LL	Reflex RL
Constant	**0.347099**	**15.9453**	**14.6645**	**0.349265**	**30.2467**	**10.4766**
Cavity (Block)	**0.007355**	**1.0196**	**0.4940**	**-0.011118**	**1.5192**	0.0704
Tn	-0.000750	**-0.9124**	**-0.7762**	-0.000772	-0.7553	**-0.5441**
Pp	**-0.001846**	0.3383	0.2490	**-0.002139**	-0.5849	0.2136
V	-0.000281	0.1762	0.0665	-0.000423	-0.4661	0.0920
Tv	0.000558	0.4277	**0.5031**	0.000646	1.1283	**0.2715**
Tf	-0.000464	**-1.1636**	**-0.4949**	-0.000719	**-3.7669**	**-0.4568**
Tp	-0.000682	**-0.6821**	**-0.6525**	-0.000737	**-1.1610**	**-0.3339**
Tw	**0.002107**	**-2.5428**	**-3.1097**	**0.001340**	**-3.8072**	**-2.4608**
Tn*Tn	0.000574	**1.9937**	**2.4183**	0.001148	**4.1486**	**1.5194**
Pp*Pp	-0.000735	**-1.3309**	**-1.5143**	-0.001305	**-4.0818**	**-1.2816**
Tp*Tp	-0.000522	0.9444	0.6506	-0.000617	-0.9205	0.6555
Tw*Tw	0.000017	0.4650	0.0888	-0.000290	0.7561	0.0988
Tn*Pp	-0.000073	**0.6598**	**0.5198**	-0.000433	0.9824	**0.2594**
Tn*V	0.000725	**0.8729**	**1.1083**	0.001164	**1.9293**	**0.7589**

Tn*Tv	0.000155	0.2171	0.0575	-0.000098	-0.8940	-0.0670
Tn*Tf	0.000411	**2.0182**	**2.1603**	0.000436	**3.0806**	**1.4868**
Tn*Tp	-0.000328	**-1.5385**	**-1.7094**	-0.000591	**-2.8001**	**-1.1881**
Tn*Tw	-0.000812	0.4442	**0.4904**	-0.001067	-0.4022	0.2340
Pp*V	-0.000075	**-1.7040**	**-2.2320**	-0.000414	**-3.6213**	**-1.4985**
Pp*Tv	-0.000031	-0.0217	0.3229	0.000103	0.3635	0.1832
Pp*Tp	0.000027	0.0792	**0.3117**	0.000410	0.3800	0.2508
R-Sq	93.7%	95.5%	98.8%	96.4%	88.6%	97.8%
R-Sq(adj)	90.4%	93.3%	98.2%	94.5%	82.8%	96.7%

Table 5. Best compromises identified through Data Envelopment Analysis for the combinations of processing conditions.

	Controllable Variables							Performance Measures					
Identifier	Nozzle Temperature (K)	Packing Pressure (MPa)	Injection Speed ($m\times10^{-2}/s$)	Valve Temperature (K)	Mold Temperature (Fixed Half) (K)	Packing Time (s)	Mold Temperature (Moving Half) (K)	Mass Left Lens (kg)	Reflex Index – Rear Left (mcd/lx)	Reflex Index – Lateral Left (mcd/lx)	Mass Right Lens (kg)	Reflex Index Rear Right (mcd/lx)	Reflex Index Lateral Right (mcd/lx)
Option 1	522.04	5.17	3.30	316.48	319.26	7	330.37	0.3391	22.78	20.23	0.3471	32.70	15.00
Option 2	522.04	5.17	3.30	305.37	302.59	7	302.59	0.3440	10.82	9.26	0.3513	16.97	6.77
Option 3	499.82	5.17	3.30	305.37	319.26	7	302.59	0.3449	14.40	11.80	0.3455	24.20	8.10
Option 4	499.82	5.17	3.30	305.37	302.59	7	330.37	0.3400	18.54	18.20	0.3481	27.67	13.10
Option 5	510.93	5.17	3.30	305.37	302.59	9	302.59	0.3489	12.48	9.70	0.3421	16.53	6.40
Option 6	499.82	5.17	3.81	316.48	302.59	7	330.37	0.3428	19.29	18.87	0.3387	31.80	13.67
Option 7	522.04	5.17	3.81	316.48	302.59	9	330.37	0.3402	19.35	18.73	0.3483	27.00	13.23