Minimizing Energy and Maximizing Network Lifetime Multicasting in Wireless Ad Hoc Networks

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***Abstract*— Most mobile nodes in a wireless ad hoc network are powered by energy limited batteries, the limited battery lifetime imposes a constraint on the network performance. Therefore, energy efficiency is paramount of importance in the design of routing protocols for the applications in such a network, and efficient operations are critical to enhance the network lifetime.**

In this paper we consider energy-efficient routing for the minimizing energy and maximizing network lifetime multicast problem in ad hoc networks. We aim to construct a multicast tree rooted at the source and spanning the destination nodes such that the minimum residual battery energy (also referred to the network lifetime) among the nodes in the network is maximized and the total transmission energy consumption is minimized. Due to the NP-hardness of the concerned problem, all previously proposed algorithms for it are heuristic algorithms, and there is little known about the analytical performance of these algorithms in terms of approximation ratios. We here focus on devising approximation algorithms for the problem with provably guaranteed approximation ratios. Specifically, we present an approximation algorithm for finding a multicast tree such that the total transmission energy consumption is no more than γ times of the optimum, under the constraint that the network lifetime is no less than β times of the optimum, where γ is either $4 \ln K$ or $O(K^{\epsilon})$, depending on whether the network is symmetric or not, ϵ and β are constants with $0 < \epsilon, \beta \leq 1$, and K is the number of destination nodes in a multicast session.

I. Introduction

In recent years, the multi-hop wireless ad hoc network has been receiving significant attention due to its potential applications in civil and military domains. A multi-hop wireless ad hoc network consists of a collection of mobile nodes forming a temporary network dynamically. In such a network each mobile node serves as a router to transmit and receive messages from other mobile nodes within its transmission range. The communication between two mobile nodes can be either in a single hop transmission in which case each of the nodes is within the transmission range of the other or in multi-hop transmission where the message is relayed by intermediate mobile nodes. It is well known that wireless communications consume significant amounts of battery power [10], and the limited battery lifetime imposes a constraint on the network performance. Energy efficient operations are critical to enhance the lifetime of such a network. Multicast is a fundamental problem in any telecommunication network including the wireless ad hoc network. Multicasting is an efficient means of one to many communication, and is typically implemented by creating a multicast tree. Because of the severe battery power and transmission bandwidth limitations in wireless ad hoc networks, multicast routing can significantly improve the network performance. In this paper we will focus on energy-efficient multicast routing by constructing energy-efficient multicast trees with multiple optimization objectives.

A. Related Work

A number of energy-aware broadcast and multicast routing algorithms have been proposed [1], [2], [5], [8], [15], [20], [21], which aim to minimize the total energy consumption. It has been proved independently by Liang [15] and Cagalj et al [2] that the problem of finding a minimum-energy broadcast tree is NP-hard. Particularly in [2], they have shown that it is unlikely to have an approximation algorithm for the problem with approximation ratio better than $\Omega(\log n)$ unless P=NP, where n is the number of nodes in the network. This implies that the minimum-energy multicast tree problem is NP-hard too. However, in many practical applications in wireless ad hoc networks, the performance measure of actual interest is not only to optimize the overall energy consumption but also to maximize the lifetime of each node in the network, because the failure of a node may lead to the network partitioned, and any further service will be interrupted. Thus, the *network lifetime* is defined as the time of the first node failure [7]. Several studies of prolonging a network lifetime have been conducted by researchers. For example, Chang and Tassiulas [6], [7] proposed to maximize the network lifetime through avoiding to use low power nodes and choosing a shortest path in terms of the energy consumption if the packet rate in the network is given in advance. Gupta and Kumar [9] discussed the critical power at which a node needs to transmit in order to ensure that the network is connected. Li et. al [13] considered a residual power routing that aims to maximize the remaining battery energy at each node, while keeping the total energy consumption for the routing is bounded. Liang and Yang [16] recently provided an improved solution for this problem. Toh [19] proposed a *conditional max-min battery capacity routing schema* which adopts different strategies for different stages of battery energy consumption. Kang and Poorvendran [11] considered maximizing network lifetime broadcast tree problem in both symmetric and asymmetric networks with the objective to keep the broadcast tree alive as long as possible until the power energy in a node expires.

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In real life, to prolong a network lifetime through each multicast session, it is required not only to minimize the energy consumption at each individual node but also to minimize the overall transmission energy consumption for the session. This is the problem that we will deal with in this paper. In fact, both the minimum energy multicast tree and the maximizing network lifetime multicast tree problems are the special cases of this general setting, by optimizing one of the two objectives and ignoring another. Recently, there are a few studies about a special case of this problem, the minimizing energy and maximizing network lifetime broadcast problem, for which Wieselthier et al [22] introduced a heuristic algorithm called BIP(β) which aims to discourage low residual energy nodes participating the broadcast session by defining a cost $w(u, v)$ for each link (u, v) as $w(u, v) = e_{u,v}(C(v)/rc(v))^{\beta}$, where $C(v)$ is the initial battery capacity at v, $rc(v)$ is the residual battery capacity before the current broadcast session, and β is a parameter that reflects the importance we assign to the impact of the residual energy, and $e_{u,v} = d^{\alpha}_{u,v}$ is the energy needed to send a unit message from node u to node v along the link $\langle u, v\rangle$. Kang and Poorvendran [11] dealt with the broadcast problem using the similar heuristics. Although these heuristic algorithms were evaluated by experimental simulations, little is known about their analytical performance in terms of approximation ratios.

B. Contributions

In this paper we study energy-efficient multicast routing with two opposite optimization objectives. One is to maximize the minimum residual battery energy among the nodes in the network, and another is to minimize the total energy consumption for the multicast session. Due to the NP hardness of this problem, we aim to provide approximation solutions for it. Specifically, for a given constant $0 < \beta \leq 1$, we aim to find such a multicast tree that the total transmission energy consumption is minimized under the constraint that the minimum residual battery energy among the nodes is no less than β times of the optimum. Specifically, if the network is symmetric, there is an approximation algorithm for the problem with an approximation ratio of $4 \ln K$; otherwise, there is an approximation algorithm with an approximation ratio of $O(K^{\epsilon})$, where K is the number of destination nodes, β and ϵ are constants with $0 < \beta, \epsilon \leq 1$.

The remainder of this paper is organized as follows. Section II introduces the wireless ad hoc network model and the problem definition. Section III proposes an algorithm for the maximizing network lifetime multicast problem and reproduces approximation algorithms for finding minimum-energy multicast trees, which will be employed in this paper. Section IV proposes approximation algorithms for the problem in both symmetric and asymmetric wireless ad hoc networks.

II. Preliminaries

A. Wireless communication model

We consider source-initiated multicast sessions. The wireless ad hoc network is modeled by a graph $M = (N, A)$, where N is the set of nodes with $|N| = n$, and there is a directed edge $\langle u, v\rangle$ in A if node v is within the transmission range of u. For the sake of simplicity, we assume the nodes in M are stationary. We also assume that each node in M is equipped with omnidirectional antennas. In this model each node has a number of power levels, which can be chosen to transmit a message. In other words, assume that there are l_i adjustable power levels at node $v_i \in N$. Let $w_{i,l}$ be the power at its power level l, $1 \leq l \leq l_i$. Assume that $w_{i,j_1} \leq w_{i,j_2}$ if $j_1 < j_2$, $1 \leq j_1, j_2 \leq l_i$. Among the l_i power levels, one is the minimum operational power level with power $p_{\min}(v_i)$, and another is the maximum operational power level with power $p_{\max}(v_i)$, $1 \leq i \leq n$. For a transmission from node u to node v, separated by a distance $d_{u,v}$, to guarantee that v can receive the message from u directly, the transmission power at node u is modeled to be proportional to $d^{\alpha}_{u,v}$, assuming that the proportionality constant is 1 for notational simplicity, where α is a parameter that typically takes on a value between 2 and 4, depending on the characteristics of the communication medium. A wireless ad hoc network is *a symmetric network* if there is always a corresponding power level with the same amount of power for each of the two neighboring nodes u and v, i.e., there is a power level with power $d^{\alpha}_{u,v}$ at u and v respectively if u and v are the neighbors. Otherwise, the network is an *asymmetric network*.

B. The minimizing energy and maximizing network lifetime multicast problem

Given a wireless ad hoc network $M = (N, A)$, a source node s, and a destination set D ($\subset N$), the *minimizing energy and maximizing network lifetime multicast problem* is to construct a multicast tree rooted at the source and spanning the nodes in D such that the minimum residual battery energy among the nodes is maximized and at the same time the sum of transmission energies at non-leaf nodes is minimized. The problem involves not only the choice of transmission nodes and the transmitter-power level at every chosen transmission node, but also maximizing the minimum residual battery energy among the nodes in the network.

III. Maximizing Network Lifetime or Minimum-Energy Multicast Tree Problem

In this section we first present an optimal algorithm for finding a multicast tree with the objective to maximize the network lifetime. We then reproduce existing approximation algorithms for finding a multicast tree with minimizing the total transmission energy consumption. These algorithms will be used as subroutines later.

A. Maximizing network lifetime multicast problem

To maximize the network lifetime for a multicast session, we start by showing the maximizing network lifetime broadcast problem (MNLB), a special case of the *maximizing network lifetime multicast problem*, is polynomially solvable as follows.

Let $N_{opt}(B)$ be the optimum solution for the MNLB problem and $rc(v)$ the residual energy at node v before the current broadcast session starts. Initially, $rc(v) = C(v)$ and $C(v)$ is the battery energy at v. We construct a directed graph $G' = (N, A', w)$ for a wireless ad hoc network M as follows. There is a directed edge from u to v in A' if v is within the transmission range of u and the weight assigned to this edge is $w(u,v) = rc(u) - d^{\alpha}_{u,v}$, which is the amount of residual battery energy at u if u uses this link to transmit a unit message to v directly, where $d_{u,v}$ is the distance between them. Note that we here assume that each broadcast session only broadcasts a unit length message. If the broadcast session broadcasts a message of τ units, then we can define $w(u,v) = rc(u) - \tau d^{\alpha}_{u,v}$ instead. Let T be a directed spanning tree in G' rooted at the source and $S(v)$ the set of children of v in T. Then, the residual energy energy at v after realizing the broadcast session is $rc'(v) = \min_{u \in S(v)}\{w(v,u)\}$. Let $ME(T)$ be the weight of the minimum weighted edge in a tree T. Define $rc(T) = \min_{v \in N}\{rc'(v)\}$, then $rc(T) = ME(T)$. Let $\mathcal{T}$ be the collection of all directed spanning trees in G' rooted at the source. Then, the MNLB problem is to find a spanning tree T from $\mathcal{T}$ to maximize $rc(T)$. In other words, $N_{opt}(B) = \max_{T \in \mathcal{T}}\{rc(T)\} = \max_{T \in \mathcal{T}}\{ME(T)\}$. Thus, the directed spanning tree T can be constructed as follows, using the Prim's algorithm.

Let V_s be the set of vertices in the tree including the source s and $V' = N - V_s$. Initially $V_s = \{s\}$. The algorithm first picks up a directed edge $e = \langle u, v \rangle \in A' \subseteq V_s \times V'$ from u to v and includes the edge to the tree if the weight $w(e)$ of e is the maximum one among the directed edges from $u \in V_s$ to $v \in V'$. It then updates the sets $V_s = V_s \cup \{v\}$ and $V' = V' - \{v\}$. This procedure continues until $V' = \emptyset$. As a result, a directed spanning tree T is obtained. It is easy to show that $N_{opt}(B) = rc(T)$.

Let T_D be the resulting multicast tree rooted at the source by pruning those leaf-vertices in T that are not in D until no such vertices in the resulting tree exist, and let $N_{opt}(M)$ be the optimal solution for the multicast version. We now show that $N_{opt}(M) = \min_{v \in T_D}\{rc'(v)\}$, i.e., the maximizing network lifetime multicast problem is also polynomially solvable, by the following lemma.

Lemma 1: Let T^*_D be the optimal multicast tree in G' and $\langle u, v \rangle$ the minimum weighted edge in it. Let $\langle a, b \rangle$ be the minimum weighted edge in T_D and T_D be the tree derived from T, then $w(a,b) = w(u,v)$, i.e., $rc(T^*_D) = w(u,v) = rc(T_D)$.

Proof: What we need is to show that $w(a,b) \geq w(u,v)$. Assume that $w(a,b) < w(u,v)$. Let $V(T_a)$ and $V(T_b)$ be the sets of nodes in the two subtrees including a and b after the removal of the directed edge $\langle a, b \rangle$ from T. We further assume that the source is in $V(T_a)$. If $u \in V(T_a)$ and $v \in V(T_b)$, then $w(a,b) \geq w(u,v)$, because otherwise, the directed edge $\langle u, v \rangle$ instead of $\langle a, b \rangle$ will be included in T, following the construction of T. The claim thus holds. Otherwise, both u and v are in either $V(T_a)$ or $V(T_b)$. We assume that they both are in $V(T_a)$. Because $\langle a, b \rangle$ is a directed edge in T_D, there must be at least one node d in D such that $d \in V(T_b)$, and there is a unique directed path P in T^*_D from s to d. Thus, there must exist at least a directed edge $\langle u', v' \rangle$ in P such that $u' \in V(T_a)$ and $v' \in V(T_b)$. Obviously, $w(u',v') \leq w(a,b)$, otherwise, a new spanning tree T' rooted at the source is obtained by adding $\langle u', v' \rangle$ to and removing $\langle a, b \rangle$ from T, and $rc(T') > rc(T)$. This contradicts the fact that T has the maximum $rc(T)$ in all directed spanning trees in G' rooted at the source. Therefore, $w(u',v') \leq w(a,b)$. However, $\langle u', v' \rangle$ is an edge in T^*_D, and $w(u',v') \leq w(a,b) < w(u,v)$, which contradicts that $\langle u, v \rangle$ is the minimum weighted edge in T^*_D. The lemma then follows. ■

B. The minimum-energy multicast tree problem

It has been shown that it is unlikely to have an approximation algorithm for the minimum-energy multicast tree problem in polynomial time, which delivers a solution that is better than $\Omega(\log K W_{opt})$ unless $P = NP$, where W_{opt} is the minimum-energy consumption for the multicast session and K is the number of destination nodes. Instead, for asymmetric networks, an approximate solution within $O(K^{\epsilon})$ times of the optimum has been achieved [15], and for symmetric networks, an approximate solution within $4 \ln K$ times of the optimum has also been proposed in [14], where ϵ is constant with $0 < \epsilon \leq 1$. For the sake of completeness, we outline them as follows.

1) Algorithm for symmetric networks: Given the wireless ad hoc network $M = (N, A)$ with finitely adjustable power at every node, an auxiliary, node-weighted, undirected graph $G = (V, E, \omega)$ is constructed as follows. To distinguish the nodes in the original network M and the auxiliary graph G, the nodes in G are called the *vertices*. For each node $v_i \in N$, let $w_{i,1}, w_{i,2} \ldots, w_{i,l_i}$ be the amounts of power of the l_i adjustable power levels at v_i with $w_{i,j_1} < w_{i,j_2}$, $1 \leq j_1 < j_2 \leq l_i$. A widget $G_i = (V_i, E_i)$ for v_i is built and shown in Fig. 1, $V_i = \{s_i, u_{i,1}, u_{i,2}, \ldots, u_{i,l_i}\}$ and $E_i = \{(s_i, u_{i,j}) \mid 1 \leq j \leq l_i\}$, where s_i represents node v_i, $u_{i,j}$ represents node v_i working at its transmission power level j and the weight assigned to $u_{i,j}$ is power $w_{i,j}$. An edge $(s_i, u_{i,j})$ between s_i and $u_{i,j}$ represents the jth power level of v_i, $1 \leq j \leq l_i$. For the sake of convenience, the vertex s_i is called *mobile vertex*, the vertex $u_{i,j}$ is called *power vertex*, and $u_{i,j}$ is *derived from* s_i, $1 \leq j \leq l_i$ and $1 \leq i \leq n$. The graph G is then constructed as follows. $V = \cup_{i=1}^{n} V_i$,

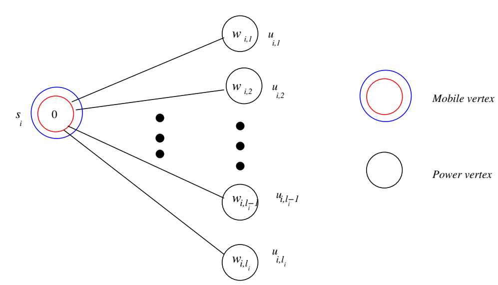

Fig. 1. The widget $G_i = (V_i, E_i)$ for node v_i

$E = \cup_{i=1}^{n} E_i \cup E_{dist}$, and $\omega : V \mapsto \mathbb{R}$, where E_{dist} is the set of edges between the vertices of different widgets, which is defined as follows. Given two nodes v_i and v_j in N with $i \neq j$, if v_i works at power level l to broadcast and v_j is within its transmission range or *vice versa*, then there is an edge $(u_{i,l}, s_j)$ in E_{dist}, $1 \le i \le n$ and $1 \le l \le l_i$. For each power vertex $u_{i,l} \in V_i \subset V$, $\omega(u_{i,l}) = w_{i,l}$, and for each mobile vertex $s_i \in V$, $\omega(s_i) = 0$, $1 \le l \le l_i$ and $1 \le i \le n$. Having G, without loss of generality we assume that vertex s_1 corresponds to the source, and $s_2, s_3, \ldots, s_{K+1}$ correspond to the nodes in D. The objective is to find a minimum node-Steiner tree in G rooted at s_1 spanning the vertices in $S = \{s_i \mid 2 \le i \le K+1\}$.

The approximate solution for the minimum-energy multicast tree problem in a wireless ad hoc network M can be found by first finding an approximate, minimum node-Steiner tree in G, and then transforming the tree into a valid multicast tree through a series of transformations. However, it is well known that the problem of finding a minimum node-Steiner tree in a graph is NP hard. Instead, an approximate solution is expected. Let T_{app} be an approximate, minimum node-Steiner tree in G rooted at s_1 spanning the vertices in S, obtained by applying the algorithm in [12]. But T_{app} may not be a valid multicast tree because (i) for a given node $v_i \in N$, it is possible that l_i power vertices derived from v_i are included in T_{app}, while each transmission node in a valid multicast tree only uses one of its power levels for transmission; (ii) Every power vertex in a valid multicast tree is derived from one of its neighboring mobile vertices, while a power vertex in T_{app} may not be derived from any of its neighboring mobile vertices; (iii) There may be several such tree edges in T_{app} between a mobile vertex and multiple power vertices that are not derived from it. A valid multicast tree can be obtained by the removal of the above (i), (ii), and (iii) through a series of transformations to T_{app}. Thus, we have the following lemma.

Lemma 2: [14] Given a symmetric wireless network $M(N, A)$ with adjustable power at every node, a source, and a set D of destination nodes, there is an approximation algorithm for the minimum-energy multicast tree problem, which delivers a solution within $4 \ln K$ times of the optimum. The algorithm takes $O(kK^2n^2)$ time, where $K = |D|$, $k = \max_{i=1}^{n}\{l_i\}$ and l_i is the number of power levels at node v_i in M, $1 \le i \le n$.

2) Algorithm for asymmetric networks: Given an asymmetric wireless ad hoc network $M(N, L)$ with l_i adjustable power levels at node $v_i \in N$, an auxiliary, edge-weighted, directed graph $G = (V, E, \omega_1)$ is constructed as follows.

A widget $G_i = (V_i, E_i)$ for each node $v_i \in N$ is built, shown in Fig 2, $V_i = \{s_i, v_{i,1}, v_{i,2}, \ldots, v_{i,l_i}\}$ and $E_i = \{\langle s_i, v_{i,l}\rangle \mid 1 \le l \le l_i\}$, where s_i represents node v_i, $v_{i,l}$ represents node v_i working at transmission power level l, the directed edge from s_i to $v_{i,l}$ represents the lth power level and the weight assigned to it is the power $w_{i,l}$, $1 \le l \le l_i$. The graph G is then obtained. $V = \cup_{i=1}^{n} V_i$, $E = \cup_{i=1}^{n} E_i \cup E_{dist}$, and $\omega_1 : E \mapsto \mathbb{R}$, where E_{dist} is the set of edges between the nodes of different widgets, which is defined as follows. For two nodes v_i and v_j with $i \ne j$, if v_i works at level l to broadcast a message and v_j is within the transmission range of v_i, then there is a directed edge $\langle v_{i,l}, s_j\rangle$ in E_{dist} with weight zero, $1 \le i \le n$ and $1 \le l \le l_i$. Now, consider the weight assignment of the edges in G. For each $\langle s_i, v_{i,l}\rangle \in E_i \subset E$, $\omega_1(s_i, v_{i,l}) = w_{i,l}$, and for each $\langle v_{i,l}, s_j\rangle \in E_{dist}$, $\omega_1(v_{i,l}, s_j) = 0$. The rest is to find a directed Steiner tree in G rooted at s_1 including each terminal node in $S = \{s_i \mid 2 \le i \le K+1\}$. Let T_{app} be an approximate, minimum directed Steiner tree in G rooted at s_1 and spanning the nodes in S. Note that T_{app} may not be a valid multicast tree due to the fact that it may contain multiple directed edges derived from a single node. It is easy to transform T_{app} into a valid multicast tree by merging these edges into one. Thus, we have the following lemma.

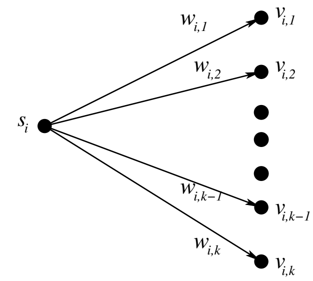

Fig. 2. The widget $G_i = (V_i, E_i)$ for node v_i

Lemma 3: [15] Given an asymmetric wireless network $M(N, L)$ with l_i adjustable power at each node $v_i \in N$, a source node, and a set D of terminal nodes, there is an approximation algorithm for the minimum-energy multicast tree problem. The total transmission energy consumption is $O(K^{\epsilon})$ times of the optimum, where, $K = |D|$, $1 \le i \le n$ and ϵ is constant with $0 < \epsilon \le 1$.

IV. Approximation Algorithms

In the concerned problem, for a given multicast session, there are two opposite optimization objectives involved. One is the bottleneck optimization which aims to maximize the minimum residual battery energy $N(M)$ among the nodes in the network, another is the additive optimization which aims to minimize the total transmission energy consumption $W(M)$. Let $N_{opt}(M)$ and $W_{opt}(M)$ be the optimal values of $N(M)$ and $W(M)$. It is usually impossible to optimize both $N(M)$ and $W(M)$ simultaneously. Thus, we aim to optimize $W(M)$ under the constraint that the network lifetime is no less than $\beta N_{opt}(M)$ where β is constant with $0 < \beta \le 1$. In particular, we focus on finding a multicast tree such that the total energy consumption is no more than γ times of the optimum while the network lifetime is no less than β times of the optimum, where γ is either $4 \ln K$ or $O(K^{\epsilon})$, depending on whether the network is symmetric or not, and ϵ and β are constants with $0 < \epsilon, \beta \le 1$.

A. Algorithm for symmetric networks

The proposed algorithm consists of two stages. Stage 1 is to remove those potential links, which will result in the network lifetime less than β times of the optimum if they are used to transfer the multicast message. Stage 2 is to apply the algorithm for finding an approximate, minimum-energy multicast tree in the subnetwork [14]. Clearly, the network lifetime in the subnetwork is no less than β times of the optimum in the original network. The approximation algorithm is presented as follows.

Algorithm Balan_Symm_Multicast_Tree (N, s, D, β)

Step 1. Find $N_{opt}(M)$ using the auxiliary directed graph $G' = (N, A', w)$ and the algorithm in Section III.

Step 2. Construct a node-weighted, auxiliary graph $G = (V, E, \omega')$, where $\omega'(v) = rc(u) - d^{\alpha}_{u,v}$ for each power vertex v derived from a mobile vertex u,.

Step 3. An auxiliary graph $G_1 = (V, E', \omega')$ is induced from G by removing those power vertices v and the edges incident to them if $\omega'(v) < \beta N_{opt}(M)$.

Step 4. Another auxiliary graph $G_2 = (V, E', \omega'')$ is constructed whose topology is identical to G_1, and each power vertex $v \in V$ in G_2 derived from u is assigned a new weight $\omega''(v) = rc(u) - \omega'(v) = d^{\alpha}_{u,v}$.

Step 5. Find an approximate, minimum node-Steiner tree T_{app} in G_2 using the algorithm in [12].

Step 6. Transform T_{app} into a valid multicast tree using the approach for finding minimum-energy multicast trees in [14].

Theorem 1: Given a symmetric wireless ad hoc network $M = (N, A)$, a source, a destination set D, and a given constant β with $0 < \beta \leq 1$, there is an approximation algorithm for the minimizing energy and maximizing network lifetime multicast problem, which delivers such a solution that the total energy consumption in the tree is within $4 \ln K$ times of the optimum, while the minimum residual battery energy among the nodes in M is no less than β times of the optimum.

B. Algorithm for asymmetric networks

The minimizing energy and maximizing network lifetime multicast problem in an asymmetric wireless ad hoc network can be dealt similarly. Assume that $N_{opt}(M)$ is given already. Instead of working in a node-weighted, undirected auxiliary graph G, we will use an edge-weighted, directed auxiliary graph G by assigning each directed edge from a mobile vertex u to its power vertex v a weight $\omega'(u, v) = rc(u) - d^{\alpha}_{u,v}$. A subgraph G_1 of G is then obtained after the removal of each of those edges $\langle u, v\rangle$ and the power vertex v adjacent to it if $\omega'(u, v) < \beta N_{opt}(M)$. Another subgraph G_2 is constructed, whose topology is identical to G_1 but each directed edge is assigned a new weight $\omega''(u, v) = d^{\alpha}_{u,v}$. An approximate solution for the minimum-energy multicast problem in G_2 is found by applying Liang's algorithm [15]. The detailed algorithm is given as follows.

Algorithm Balan_Asymm_Multicast_Tree (N, s, D, β)

Step 1. Find $N_{opt}(M)$ using $G' = (N, A', w)$;

Step 2. Construct an edge-weighted, directed auxiliary graph $G = (V, E, \omega')$, where $\omega'(u, v) = rc(u) - d^{\alpha}_{u,v}$ for each directed edge $\langle v, u\rangle$ derived from a mobile vertex u.

Step 3. $G_1 = (V, E', \omega')$ is induced from G by removing those directed edges $\langle u, v\rangle$ if $\omega'(u, v) < \beta N_{opt}(M)$.

Step 4. Another auxiliary graph $G_2 = (V, E', \omega'')$ whose topology is identical to G_1 is constructed. Assign each directed edge $\langle u, v\rangle$ in G_2 a new weight $\omega''(u, v) = d^{\alpha}_{u,v}$.

Step 5. Find an approximate, minimum directed Steiner tree T_{app} in G_2 rooted at the source using the algorithm in [4].

Step 6. Transform T_{app} into a valid multicast tree using the approach for finding minimum-energy multicast trees in [15].

Theorem 2: Given an asymmetric ad hoc wireless network $M(N, A)$, a destination set D, and a constant β with $0 < \beta \leq 1$, there is an approximation algorithm for finding a directed multicast tree rooted at the source and spanning the nodes in D. The solution delivered is $O(|D|^{\epsilon})$ times of the optimum under the constraint that the minimum residual battery energy energy among the nodes is no less than β times of the optimum, where ϵ is constant with $0 < \epsilon \leq 1$.

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