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Interactive Bi-Level Multi-Objective Integer Non-linear Programming Problem

O. E. Emam

Department of Information Systems
Faculty of Computer Science and information
Helwan University, P.O. Box 11795, Egypt
emam_o_e@yahoo.com

Abstract

This paper presents a bi-level multi-objective integer non-linear programming (BLMINP) problem with linear or non-linear constraints and an interactive algorithm for solving such model. At the first phase of the solution algorithm to avoid the complexity of non convexity of this problem, we begin by finding the convex hull of its original set of constraints using the cutting-plane algorithm to convert the BLMINP problem to an equivalent bi-level multi-objective non-linear programming (BLMNP) problem. At the second phase the algorithm simplifies an equivalent (BLMNP) problem by transforming it into separate multi-objective decision-making problems with hierarchical structure, and solving it by using ε-constraint method to avoid the difficulty associated with non-convex mathematical programming. In addition, the author put forward the satisfactoriness concept as the first-level decision-maker preference. Finally, an illustrative numerical example is given to demonstrate the obtained results.

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(1) Introduction

Bi-level programming (BLP) is a subset of the multi-level programming problem which identified as a mathematical programming problem that solves decentralized planning problems with two decision makers (DMs) in a two- level or hierarchical organization ([3], [4], [5], [6], [7], [9]). An algorithm for the interactive multi-level non-linear multi-objective decision-making problem is presented in many searches (Osman et al. [7] and Shi and Xia [10]).

The interactive algorithm uses the concepts of satisfactoriness to multi-objective optimization at every level until a preferred solution is reach. Based on (Shi and Xia [10]) satisfactory solution concepts, the proposed solution method proceed from the first-level decision-maker (FLDM) to the second-level decision-maker (SLDM). The FLDM gets the preferred or satisfactory solutions that are acceptable in rank order to the SLDM. The SLDM will search for the preferred solution of the FLDM until the preferred solution is reached. Integer multi-objective programming has attracted the attention of many researchers in the past. The main reason for interest in linear or nonlinear programming stems from the fact that programming models could better fit the real problems if we consider optimization of economic quantities ([1], [8]). This paper is organized as follows: we start in Section 2 by formulating the model of bi-level multi-objective integer non-linear programming problem with the solution concept is introduced. In Section 3, Definitions and Theorems is carried out. In Section 4, an interactive model for BLMINP problem is presented. In Section 5, an interactive algorithm for BLMINP problem is presented. In Section 6, an example is provided to illustrate the developed results. Finally, in Section 7, some open points are stated for future research work in the area of interactive multi-level integer programming optimization problems.

(2) Problem Formulation and Solution Concept

Let $x_i \in R^{n_i}, (i=1,2)$ be a vector variables indicating the first decision level's choice, the second decision level's choice. Let the FLDM and SLDM have N_1 and N_2 objective functions, respectively. And M is the set of feasible choices $\{(x_1, x_2)\}$.

So the BLMINP problem may be formulated as follows:

[1st Level] $\underset{x_1}{Max}\, F_1(x_1,x_2) = \underset{x_1}{Max}\left(f_{11}(x_1,x_2),\ldots, f_{1N_1}(x_1,x_2)\right),$ (1)

where x_2 solves

[2nd Level] $\underset{x_2}{Max}\, F_2(x_1,x_2) = \underset{x_2}{Max}\left(f_{21}(x_1,x_2),\ldots, f_{2N_2}(x_1,x_2)\right),$ (2)

Subject to

$$M = \{(x_1,x_2) \mid g_i(x_1,x_2) \le 0, i = 1,2,\ldots,m,.\quad x_j \ge 0 \text{ and integer, j} = 1,2 \} \quad (3)$$

Where M is a non-convex constraint set, F_1 and F_2 are non-linear functions. The decision mechanism of BLMINP problem is that the FLDM and SLDM adopt the two-planner Stackelberg game. According to the two-planner Stackelberg game and mathematical programming, the definitions of solution for the model of BLMINP problem are given as follows .

Definition 1. For any $x_1\left(x_1 \in M_1 = \{x_1 | (x_1,x_2) \in M\}\right)$ given by FLDM, if the decision-making variable $x_2\left(x_2 \in M_1 = \{x_2 | (x_1,x_2) \in M\}\right)$ is the non-inferior solution of the SLDM, then (x_1, x_2) is a feasible solution of BLMINP problem.

Definition 2. If $(x_1^*, x_2^*)$is a feasible solution of the BLMINP problem; no other feasible solution $(x_1, x_2) \in M$ exists, such that $f_{1j}(x_1^*, x_2^*) \leq f_{1j}(x_1, x_2)$, with at least one j $(j = 1, 2, ..., N_1)$; so (x_1^*, x_2^*) is the preferred solution of the BLMINP problem.

In what follows, an equivalent bi-level multi-objective nonlinear programming (BLMNP) problem associated with problem (1)-(3) can be stated with the help of cutting-plane technique ([1], [2], [8]) together with Balinski algorithm [2]. This equivalent BLMNP problem can be written in the following form:

$$\text{(FLDM)} \quad \underset{x_1}{Max}\, F_1(x_1, x_2) = \underset{x_1}{Max}\left(f_{11}(x_1, x_2), ..., f_{1N_1}(x_1, x_2)\right), \tag{4}$$

where x_2 solves

$$\text{(SLDM)} \quad \underset{x_2}{Max}\, F_2(x_1, x_2) = \underset{x_2}{Max}\left(f_{21}(x_1, x_2), ..., f_{2N_2}(x_1, x_2)\right), \tag{5}$$

Subject to

$$x \in [M], \tag{6}$$

where $[M]$is the convex hull of the feasible region M defined by (3) earlier. This convex hull is defined by:

$$[M] = M_R^{(s)} = \{x \in R^n \mid A^{(s)} x \leq b^{(s)}, x \geq 0\} \tag{7}$$

and in addition,

$$A^{(s)} = \begin{bmatrix} A \\ . \\ a_1 \\ . \\ . \\ a_s \end{bmatrix} \quad \text{and} \quad b^{(s)} = \begin{bmatrix} b_1 \\ . \\ b_m \\ c_1 \\ . \\ c_s \end{bmatrix} \tag{8}$$

are the original constraint matrix A and the right-hand side vector b , respectively, with *s*-additional constraints each corresponding to an efficient cut in the form $a_i x \leq c_i$. By an efficient cut, we mean that a cut which is not redundant.

(3) Definitions and Theorems

We will obtain the solution of the equivalent BLMNP problem of the BLMINP problem by solving FLDM and SLDM problems each one separately. In this way, we can quantitatively present satisfactoriness and the preferred solution in view of singular-level multi-objective decision-making problem, and introduce several theorems with the help of the quality of ε-constraint method to provide a theoretical basis for upper-level multi-objective decision-making.
Consider a multi-objective decision-making problem as follows:

$$\underset{x}{Max} \ (f_1(x),\ldots, f_n(x)), \tag{9}$$

subject to

$$h_j(x) \geq 0 , \quad j = 1,2,\ldots,q .$$

Where $x = (x_1 , x_2)$, $x \in R^{n_1+n_2}$ denotes the decision-making variable and $f_i(x), (i = 1,2)$ denotes the objective function of the multi-objective decision-making problem. Let $\Omega = \{x \,|\, h_j(x) \geq 0 , \ j = 1,2,\ldots,q\}$ and $a_i = \underset{x\in\Omega}{Min} \ f_i(x)$, $b_i = \underset{x\in\Omega}{Max} \ f_i(x)$. On $u_i = [a_i , b_i]$ define $A_i \in f(u_i)$, whose membership function $\mu_{A_i}(f_i(x))$ meet (i) and (ii) as below :

(i) When the objective value $f_i(x)$ approaches or equals the decision-maker's ideal value, $\mu_{A_i}(f_i(x))$ approaches or equals 1.Otherwise, 0 .

(ii) If $f_i(x) > f_i(x^*)$, then $\mu_{A_i}(f_i(x)) \geq \mu_{A_i}(f_i(x^*))$, $i = 1,2,\ldots,n.$

Definition 3. If x^* is a non-inferior solution, then $\mu_{A_i}(f_i(x^*))$ is defined as the satisfactoriness of x^* to objective $f_i(x)$.

Definition 4. $\mu(x^*) = \underset{1\leq i\leq n}{Min} \mu_{A_i}(f_i(x^*))$ is defined as the satisfactoriness of non-inferior solution x^* to all the objectives.

Definition 5. With a certain value s_0 given in advance by the decision-maker, if non-inferior solution x^* satisfies $\mu(x^*) \geq s_0$, then x^* is the preferred solution corresponding to the satisfactoriness s_0 .

We give membership function $\mu_{A_i}(f_i(x))$ as below:

$$\mu_{A_i}(f_i(x)) = \frac{f_i(x) - a_i}{b_i - a_i} \tag{10}$$

It is decided according to the decision-maker's requirements. Obviously, (10) meets the two requirements (i) and (ii) for $\mu_{A_i}(f_i(x))$.

The ε-constraint method is effective for solving multi-objective decision-making problems. The formalization of $P(\varepsilon_{-1})$ is as follows:

$$Max \ f_1(x), \tag{11}$$

subject to
$f_i(x) \le \varepsilon_i, \quad i = 2, ..., n,$
$x \in \Omega.$
Assume
$\varepsilon_{-1} = (\varepsilon_2, ..., \varepsilon_n),$
$X'(\varepsilon_{-1}) = \{x \mid f_i(x) \le \varepsilon_i, \quad i = 2, ..., n, \quad x \in \Omega\},$
and
$\mathrm{E}_1 = \{\varepsilon_{-1} \mid x'(\varepsilon_{-1}) \ne \phi \ (empty\ set)\}.$

Theorem 1. If $\varepsilon_{-1} = (\varepsilon_2, \varepsilon_3, ..., \varepsilon_n) \in \mathrm{E}_1$, then the optimal solution to $P(\varepsilon_{-1})$ exists and includes the non-inferior solution of (9) (see Shi and Xia [10]).

Corollary 1. If x^1 is the only optimal solution to $P(\varepsilon_{-1})$, then x^1 is the non-inferior solution of (9).

The ε-constraint problem including satisfactoriness is as follows:

$$Max \ f_1(x), \tag{12}$$

subject to
$f_i(x) \ge \delta_i, \quad i = 2, ..., n,$
$x \in \Omega.$

Theorem 2. If $P(\varepsilon_{-1}(s))$ has no solution or has the non-inferior solution $\bar{x}$ and $f_1(\bar{x}) \le \delta_1$, then no non-inferior solution x^* exists, such that $\mu(x^*) \ge s$.

Proof:
If x^* is a non-inferior solution of (9), such that $\mu(x^*) \ge s$, namely, $\mu_{A_i}(f_i(x)) \ge s, \quad (i = 1, 2, ..., n)$. Then x^* is a feasible solution of $P(\varepsilon_{-1}(s))$, and $f_1(x^*) \ge \delta_1$, therefore, $P(\varepsilon_{-1}(s))$ has a non-inferior solution $\bar{x}$, such that $f_1(\bar{x}) \ge f_1(x^*) \ge \delta_1$, which is in contradiction to the hypothesis.

Theorem 3. Assume $s < s_1$ if there is no preferred solution to s, then go to s_1.[10]

Theorem 4. Assume $\bar{x}$ is a non-inferior solution of $P(\varepsilon_{-1}(s))$ and $f_i(\bar{x}) \ge \delta_i, (i = 1, 2, ..., n)$. Let $f_i(\bar{x}) = \varepsilon_i \ (i = 1, 2, ..., n)$, and let $\varepsilon_{-1} = (\varepsilon_2, \varepsilon_3, ..., \varepsilon_n)$. Then $\bar{x}$ is still an optimal solution of $P(\varepsilon_{-1})$.
If $\bar{x}$ is the only optimal solution of $P(\varepsilon_{-1})$, then $\bar{x}$, is a non-inferior solution.
If other optimal solution x' of $P(\varepsilon_{-1})$ exists, and $L \in \{1, 2, ..., n\}$ exists, such that $f_L(x') \ge \varepsilon_L$, then $\bar{x}$ is inferior solution.

Proof

(a) $\varepsilon_i \geq \delta_i$, $(i = 1,2,...,n)$, namely, $\varepsilon_{-1} = (\varepsilon_2, \varepsilon_3, ..., \varepsilon_n) \geq (\delta_2, \delta_3, ..., \delta_n)$; $x'(\varepsilon_{-1}) \subset x'(\varepsilon_{-1}(s))$, let $\bar{x}$ to be a non-inferior solution of $P(\varepsilon_{-1}(s))$, and $\bar{x} \in x'(\varepsilon_{-1})$, then

$$f_1(\bar{x}) = \underset{x \in x'(\in_{-1}(s))}{Max} f_1(x) = \underset{x \in x'(\in_{-1})}{Max} f_1(x)$$

Therefore, $\bar{x}$ is a non-inferior solution of $P(\varepsilon_{-1})$ and (a) is proven by Corollary 1.

(b) $f_1(x') \geq f_1(\bar{x})$, and $f_i(x') \geq f_i(\bar{x}) = \varepsilon_i$, which "$\succ$" holds when $i = L$, therefore, $\bar{x}$ is inferior solution .

(4) An interactive model for BLMINP problem

To solve the BLMINP problem by adopting the two-planner Stackelberg game, first we have to retransfer set of constraints M to its equivalent [M], so we will obtain an equivalent BLMNP problem then the FLDM gives the preferred or satisfactory solutions that are acceptable in rank order to the SLDM, and then the SLDM will seek the solutions by ε-constraint method , and to arrive at the solution that gradually approaches the preferred solution or satisfactory solution to the FLDM. Finally, the FLDM decide the preferred solution of the BLMINP problem according to the satisfactoriness.

4.1-The First-Level Decision-Maker (FLDM) Problem

The first-level decision-maker problem of the (BLMINP) problem is as follows:

$$\underset{x_1}{Max}\, F_1(x_1, x_2) = \underset{x_1}{Max}\left(f_{11}(x_1, x_2), ..., f_{1N_1}(x_1, x_2)\right), \tag{13}$$

subject to

$x \in [M]$.

To obtain the preferred solution of the FLDM problem; we transform (13) into the following multi-objective decision-making problem:

$$\underset{x}{Max}\ f_{11}(x), \tag{14}$$

subject to

$$f_{1j}(x) \geq \delta_{1j},\ j = (2, ..., N_1), \tag{15}$$

$x \in \Omega$,

$x = (x_1, x_2)$, $x \in R^{n_1+n_2}$.

So , the algorithm steps for solving (14)-(15) are as follows:

4.1.1-The Algorithm for FLDM Problem

Step 1: (a) Use Balinski's algorithm to find all the vertices of the feasible region *M*.

(b)Select one of the non-integer vertices $x^1 = \left(x_1^1, x_2^1, ..., x_n^1\right)$ of the solution space. In the tableau of this vertex, choose the row vector where the basic variable has the largest fractional value and construct its corresponding Gomory's fractional cut in the form $a_1 x \le c_1$.

(c) Add the first cut $a_1 x \le c_1$ to the original set of the constraints *M*. This will yield a new feasible region M^1.

(d) Repeat again the steps (a) $\rightarrow$ (c) until, at some step, r, the obtained vertices of the solution space all are integers.

(e) Eliminate (drop) all the redundant constraints of the applied cuts.

(f) Add all the constraints of applied s-efficient cuts to the original set of constraints *M* to get [*M*].

Step2: Formulate the equivalent linear fractional program with the constraints [*M*].

Step3: Set the satisfactoriness. Let $s = s_0$ at the beginning, and let $s = s_1, s_2, \ldots,$ respectively.

Step 4: Set the ε-constraint problem $P(\varepsilon_{-1}(s))$, if $P(\varepsilon_{-1}(s))$ has no solution or has a non-inferior solution making $f_{11}(\bar{x}) < \delta_{11}$, then go to step 3, to adjust $s = s_{j+1} < s_j$. Otherwise, go to step 5.

Step 5: Assuming that $\bar{x}$ is a non-inferior solution of $P(\varepsilon_{-1}(s))$, judge by theorem 4 whether or not $\bar{x}$ is a non-inferior solution of (14)-(15) . If $\bar{x}$ is a non-inferior solution, turn to step 6. if $\bar{x}$ is inferior solution, there must be a $\bar{x}'$, such that $f_{1i}(\bar{x}') \ge f_{1i}(\bar{x})$, and at least one "$>$"; Repeat step 5 with $\bar{x}'$.

Step 6: If the decision-maker is satisfied with $\bar{x}$, then $\bar{x}$ is a preferred solution. Otherwise, go to step 7.

Step 7: Adjust the satisfactoriness. Let $s = s_{j+1} \succ s_j$, and go to step 4.

4.2-The Second-Level Decision-Maker (SLDM) Problem

Secondly, according to the interactive mechanism of the BLMINP problem, the FLDM variables x_1^F should be given to the SLDM; hence, the SLDM problem can be written as follows:

$$\underset{x_2}{Max}\, F_2\left(x_1^F, x_2\right) = \underset{x_2}{Max}\left(f_{21}\left(x_1^F, x_2\right), \ldots, f_{2N_2}\left(x_1^F, x_2\right)\right), \tag{16}$$

subject to

$$(x_1^F, x_2) \in [M].$$

The SLDM will convert (16) into the following single objective function as follows:

$$\underset{x}{Max}\, f_{21}\left(x_1^F, x_2\right), \tag{17}$$

subject to

$$f_{2j}\left(x_1^F, x_2\right) \ge \delta_{2j}, \quad j = 2, \ldots, N_2, \tag{18}$$

$$\left(x_1^F, x_2\right) \in \Omega.$$

Our basic though on solving (17)-(18) is to find the second-level non-inferior solution $\left(x_1^F, x_2^S\right)$ that is closest to the FLDM preferred solution $\left(x_1^F, x_2^F\right)$.

Now, we will test whether $\left(x_1^F, x_2^S\right)$ is preferred solution to the FLDM or it may be changed, by the following test:

If

$$\frac{\left\| F_1\left(x_1^F, x_2^F\right) - F_1\left(x_1^F, x_2^S\right)\right\|_2}{\left\| F_1\left(x_1^F, x_2^S\right)\right\|_2} < \delta^F \tag{19}$$

So, $\left(x_1^F, x_2^S\right)$ is a preferred solution to the FLDM, where δ^F is a small positive constant given by the FLDM which means $\left(x_1^F, x_2^S\right)$ is a preferred solution of the BLMINP problem.

(5) Interactive Algorithm for BLMINP problem

Step 1: -Set $k = 0$; solve the 1st level decision-making problem to obtain a set of preferred solutions that are acceptable to the FLDM. The FLDM puts the solutions in order in the format as follows:

Preferred solution $\left(x_1^k, x_2^k\right), \ldots, \left(x_1^{k+p}, x_2^{k+p}\right)$

Preferred ranking (satisfactory ranking)

$$\left(x_1^k, x_2^k\right) \succ (x_1^{k+1}, x_2^{k+1}) \succ \ldots \succ \left(x_1^{k+p}, x_2^{k+p}\right).$$

Step 2: -Given $x_1 = x_1^F$ to the SLDM, solve the SLDM problem to obtain x_2.

Step 3: -If $\dfrac{\left\| F_1\left(x_1^F, x_2^F\right) - F_1\left(x_1^F, x_2^S\right)\right\|_2}{\left\| F_1\left(x_1^F, x_2^S\right)\right\|_2} < \delta^F$

Where δ^F is a fairly small positive number given by the FLDM, then go to step 4. Otherwise, go to step 5.

Step 4: - $\left(x_1^F, x_2^S\right)$ is the preferred solution to the BLMINP problem.

Step 5: - Set $k = k + 1$, then go to step 1.

(6) Numerical Example

To demonstrate the solution for interactive BLMINP problem, let us consider the following example:

[1st level] $\quad \underset{x_1}{Max}\, F_1\left(x_1, x_2\right) = \underset{x_1}{Max}\left[2x_1 + 3x_2 \;,\; x_1 + x_2^2\right]$

where x_2 solves

[2nd level] $\quad \underset{x_2}{Max}\, F_2\left(x_1, x_2\right) = \underset{x_2}{Max}\left[x_1 + x_2^2 \;,\; \left(x_1 + 1\right)^2 + 2x_2^2\right]$

subject to

$$2x_1 + 2x_2 \le 7,$$

$$2x_1 + 5x_2 \le 10,$$
$$x_1, x_2 \ge 0 \text{ , and integers.}$$

First, the given bi-level integer multi-objective non-linear programming problem can be converted into its equivalent bi-level multi-objective non-linear programming problem as follows:

[1st level] $\underset{x_1}{Max}\, F_1(x_1, x_2) = \underset{x_1}{Max}\, [2x_1 + 3x_2 \;, x_1 + x_2^2]$,

[2nd level] $\underset{x_2}{Max}\, F_2(x_1, x_2) = \underset{x_2}{Max}\, [x_1 + x_2^2 \;, (x_1 + 1)^2 + 2x_2^2]$

Subject to

$$2x_1 + 2x_2 \le 7 ,$$
$$2x_1 + 5x_2 \le 10,$$
$$x_1 + 2x_2 \le 4,$$
$$x_1 + x_2 \le 3 ,$$
$$x_1, x_2 \ge 0 .$$

First, the FLDM solves his/her problem as follows:

1- Find individual optimal solution by solving (13), we get.
$(b_{11}, b_{12}) = (7, 4)$, $(a_{11}, a_{12}) = (0, 0)$.

2- Using the solution of FLDM problem, we can formulate (14)-(15) as follows:

$Max\; x_1 + x_2$

Subject to

$$2x_1 + 2x_2 \le 7 ,$$
$$2x_1 + 5x_2 \le 10,$$
$$x_1 + 2x_2 \le 4,$$
$$x_1 + x_2 \le 3 ,$$
$$x_1 + x_2^2 \ge 1.2 \;,$$
$$x_1, x_2 \ge 0 .$$

Where $\delta_{12} = (b_{12} - a_{12})\, s_1 + a_{12} = 1.2$.

So the FLDM solution is $(x_1^F, x_2^F) = (2,1\;)$ and ($s_1 = 0.3, \delta^F = 0.12$ are given by FLDM).

Secondly, the SLDM solves his/her problem as follows:

1- Find the individual optimal solutions by solving (16), we get:
$(b_{12}, b_{22}) = (4,16)$, $(a_{12}, a_{22}) = (0,1)$.

2- Using the results from SLDM problem, we can formulate (17)-(18) as follows:

$Max x_1 + x_2^2$

Subject to

$$2x_1 + 2x_2 \le 7 ,$$

$$2x_1 + 5x_2 \leq 10,$$
$$x_1 + 2x_2 \leq 4,$$
$$x_1 + x_2 \leq 3,$$
$$x_1 = 2,$$
$$(x_1 + 1)^2 + 2x_2^2 \geq 8.5$$
$$x_2 \geq 0 .$$

Where $\delta_{22} = (b_{22} - a_{22})\, s_2 + a_{22} = 8.5$.

So the SLDM solution is

$(x_1^F , x_2^S) = (2 , 1)$, and($s_2 = 0.5$).

Finally, by using (19), we will find that $(x_1^F , x_2^S) = (2 , 1)$ is a preferred solution to the FLDM from the following test:

$$\frac{\| F_1(2,1) - F_1(2,1) \|_2}{\| F_1(2,1) \|_2} = 0 \prec 0.12$$

So $(x_1^F , x_2^S) = (2 , 1)$ is the preferred solution to the BLMINP problem.

(7) Summary and Concluding Remarks

This paper has proposed an interactive algorithm for solving a bi-level multi-objective integer non-linear programming (BLMINP) problem with linear or non-linear constraints . We start by finding the convex hull of its original set of constraints using the cutting-plane algorithm to convert the BLMINP problem to an equivalent (BLMNP) problem. Then the algorithm simplifies an equivalent (BLMNP) problem by transforming it into separate multi-objective decision-making problems with hierarchical structure, and solving it by using ε-constraint method to avoid the difficulty associated with non-convex mathematical programming is introduced.

However, there are many other aspects, which should be explored and studied in the area of multi-level optimization such as:

1. Interactive bi-level and multi-level integer fractional multi-objective decision-making problems.
2. Interactive bi-level and multi-level integer stochastic non-linear multi-objective decision-making problems.
3. Interactive bi-level and multi-level integer large-scale non-linear multi-objective decision-making problems.

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