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A New Multi-objective Inventory Model under Stochastic Conditions with Considering Perishable Costs

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Abstract: This paper presents a new multiple objectives model for the optimal production for an inventory control system. The stocked items may be deteriorates and the systems costs will be change over the time. In the real situation, some but not all customers will wait for backlogged items during a shortage period and therefore, the model incorporates partial backlogging. The demand rate can be a function of inflation and time value of money where the inflation and time horizon i.e., period of business, both are random in nature. The objectives of the problem are: (1) Minimization of the total expected present value of costs over time horizon (consists of the deterioration cost, production cost, inventory holding cost, backordering cost, lost sale cost and ordering cost) and (2) Decreasing the total quantity of goods in the warehouse over time horizon. We propose the ideal point approach to formulate the model. The numerical example has been provided for evaluation and validation of the theoretical results.

Key words: supply chain management; Multi-objective; Inventory; Stochastic; Optimization.

INTRODUCTION

The practical experiences reveal that the supply chain management (SCM) is under uncertain and variable conditions. One of the most important parts of SCM is inventory system management which is inherently in non-deterministic situation. The many departments of organization such as warehouse, marketing, sale, purchasing, financial, planning, production, maintenance and etc. are relevance to the inventory problem. In the past decades, the replenishment scheduling problems were typically attacked by developing proper mathematical models that consider practical factors in real world situations, such as uncertain conditions, physical characteristics of inventoried goods, effects of inflation and time value of money, partial backlogging of unsatisfied demand, etc. Inventoried goods can be broadly classified into four meta-categories based on

I. ***Obsolescence*** refers to items that lose their value through time because of rapid changes of technology or the introduction of a new product by a competitor. For example, spare parts for military aircraft are style goods, and they become obsolete when a replacement model is introduced.

II. ***Deterioration*** refers to the damage, spoilage, dryness, vaporization, etc. of the products. For example, the commonly used goods like fruits, vegetables, meat, foodstuffs, perfumes, alcohol, gasoline, radioactive substances, photographic films, electronic components, etc. where deterioration is usually observed during their normal storage period.

III. ***Amelioration*** refers to items whose value or utility or quantity increase with time. It is a practical experience the value of Persian carpet increases by age. Other examples can be wine manufacturing industry and fast growing animals like broiler, sheep, pig, etc. in farming yard.

IV. The last one refers no ***obsolescence, deterioration*** and ***amelioration***. The shelf-life of some products can be indefinite and hence they would fall under the no obsolescence/deterioration/amelioration category.

Since 1975, a series of related papers appeared that considered the effects of time value of money and inflation on the inventory system. There are a few problems in the inflationary inventory systems on obsolescence and amelioration items which have been addressed by the researchers, because, we will not use obsolesced items in the future and the amelioration products are limited in the real world. For example, Moon *et al*. (2005) considered ameliorating/deteriorating items with a time-varying demand pattern. Another research for ameliorating items has been done by Sana (2010).

The no obsolescing, deteriorating and ameliorating items have been considered in some researches on the inflationary inventory system. Misra (1979) developed a discounted cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some efforts were extended the previous works to consider more complex and realistic assumption, such as Uthayakumar and Geetha (2009), Maity (2008), Vrat and Padmanabhan (1990), Datta and Pal (1991), Hariga (1995), Hariga and Ben-Daya (1996), Chung (2003) and Chia H.H., (2011).

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The deteriorating inventory systems have been studied considerably in the recent years. For example, Chung and Tsai (2001) presented an inventory model for deteriorating items with the demand of linear trend considering the time-value of money. Wee and Law (2001) derived a deteriorating inventory model under inflationary conditions when the demand rate is a linear decreasing function of the selling price. Chen and Lin (2002) discussed an inventory model for deteriorating items with a normally distributed shelf life, continuous time-varying demand, and shortages under an inflationary and time discounting environment. Yang (2004) discussed the two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages. Chang (2004) established a deteriorating EOQ model when the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity.

Maiti *et al*. (2010) proposed an inventory model with stock-dependent demand rate and two storage facilities under inflation and time value of money. Lo *et al*. (2007) developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. A Two storage inventory problem with dynamic demand and interval valued lead-time over a finite time horizon under inflation and time-value of money considered by Dey *et al*. (2008). Other efforts on inflationary inventory systems for deteriorating items have been made by Hsieh and Dye (2010), Su *et al*. (1996), Chen (1998), Wee and Law (1999), Sarker *et al*. (2000), Yang *et al*. (2001, 2010), Liao and chen (2003), Balkhi (2004[a], 2004[b]), Hou and Lin (2004), Hou (2006), Jaggi *et al*. (2006), Chern *et al*. (2008) and Sarkar and Moon (2011). S.K. Ghosh , S. Khanra, K.S. Chaudhuri., (2011), Hern, M.S., Yang, H.L., Teng, J.T., Papachristos, S., (2008)

It can be see that in the mentioned researches, rate of inflation has been assumed completely known and certain. Yet, inflation enters the inventory picture only because it may have an impact on the future inventory costs, and the future rate of inflation is inherently uncertain and unstable. But, there are a few works in the inflationary inventory researches under stochastic conditions, especially with multiple stochastic parameters. Mirzazadeh and Sarfaraz (1997) presented multiple-items inventory system with a budget constraint and the uniform distribution function for the external inflation rate for no obsolescence, deterioration and amelioration items and Horowitz (2000) discussed an EOQ model with a normal distribution for the inflation rate. Mirzazadeh (2007) compared the average annual cost and the discounted cost methods in the inventory system's modeling with considering stochastic inflation. The results show that there is a negligible difference between two procedures for wide range values of the parameters. Furthermore, Mirzazadeh (2008) in another work, proposed an inventory model under time-varying inflationary conditions for deteriorating items. Mirzazadeh (2009) developed A Partial Backlogging Mathematical Model under Variable Inflation and Demand.

The objectives of the problem are: (1) Minimization of the total expected present value of costs and (2) Decreasing the total quantity of goods in the warehouse over the random time horizon. The second objective has seldom considered in the previous research of the inventory systems. But, decreasing in the inventory level is important for company, because: (1) decreasing in inventory level causes increasing company flexibility against changes in the market conditions, customer needs and so on, (2) the quantity of the deteriorated goods is related to inventory level so that decreasing in inventory causes decreasing destroyed good, (3) low inventory system causes faster company adaptation with technology changes, (4) decreasing in inventory causes better cash flow and rate of return.

Furthermore, the demand is a function of the inflation rate, in this paper. In the existing literature, inflationary inventory models are usually developed under the assumption of constant and well known time horizon. However, there are many real life situations where these assumptions are not valid, e.g., for a seasonal product, though time horizon is normally assumed as finite and crisp in nature, but, in every year it fluctuates depending upon the environmental effects and it is better to estimate this horizon as a stochastic parameter, which has been considered in this paper.

Additionally, the replenishment rate is finite and deteriorating items are surveyed with considering deterioration cost. In many real situations, during a shortage period, the longer the waiting time is, the smaller the backlogging rate would be. For instance, for fashionable commodities and high-tech products with the short product life cycle, the willingness for a customer to wait for backlogging is diminishing with the length of the waiting time. Therefore, the partial backlogging has been considered in this paper.

Under the mentioned situations, a new mathematical model for the optimal production for an inventory control system is formulated under stochastic environment, and the paper has been organized as follow. First, the assumptions and notations and then, the multi-objective model formulation are derived. Then, the solution procedure has been prepared with using the ideal point approach. The numerical example has been provided to clarify how the proposed model is applied. The final section has devoted to the discussion.

2. Assumptions, Notations and Description of the Model:

The developed model has been made based on the following assumptions:

1. The system brings about for a random time-horizon;

2. A constant fraction of the on-hand inventory deteriorates per unit time, as soon as the item is received into inventory;
3. Shortages are allowed and partial backlogged, except for the final cycle;
4. All of the system costs will be increase over time horizon via stochastic inflation rate;
5. The demand rate here is a linear function of the inflation rate;
6. The constant annual production (Replenishment) rate is finite. The Replenishment rate is higher than the sum of consumption and deterioration rates;
7. Lead time is negligible. Also, the initial and final inventory level is zero.

The following notations are used:

I	The stochastic inflation rate
f(i)	The pdf of inflation rate
R	The interest rate
R	The discounted rate net of inflation: R = r-i
D(i)	The demand rate per unit time is a function of inflation rate $$R(i) = a + bi \qquad a\rangle 0 , b\langle 0 \qquad (1)$$ a and b are the constant real number.
τ	The constant deterioration rate per unit time $(0 \le \tau\langle 1)$.
c_1	The ordering cost per order at time zero
c_2	The purchase cost at time zero
c_3	The inventory carrying cost per unit per unit time at time zero
c_4	The backlogging cost per unit per unit time, if the shortage is backlogged
c_5	The unit opportunity cost due to lost sale, if the shortage is lost
c_6	The deterioration cost per unit of the deteriorated item at time zero
H	The stochastic finite time horizon
f(h)	The pdf of *H*
P	The constant annual production (Replenishment) rate
$\delta(t)$	The fraction of shortages backordered that is a differentiable and decreasing function of time t, where t is the waiting time up to the next replenishment, $0 \le \delta(t) \le 1$ with $\delta(0)=1$ and $\delta(\infty)=0$. Note that if $\delta(t)=1$ (or 0) for all *t*, then shortages are completely backlogged (or lost). We assume $\delta(t)=e^{-\alpha t}$ where $\alpha \ge 0$.
T	The interval of time between replenishment
k	The proportion of time in any given inventory cycle which orders can be filled from the existing stock
n	The number of replenishments during time horizon
ETVC(n,k)	The total present value of costs over the time horizon
TI(n,k)	The total quantity of goods in warehouse over time horizon

Additional notations will be introduced later. The graphical representation of the inventory system is shown in Figure 1. The real time horizon (H) has been divided into n equal parts each of length T so that T=H/n. Initial and final inventory levels are both zero. Each inventory cycle except the last cycle can be divided into four parts. The production starts at time zero and the inventory level is gradually increasing due to production, demand and deterioration rates. This fact continues till the production stops at time α. Then the inventory level gradually decreasing mainly due to consumption and partly due to deterioration and reaches zero at time kT and shortages occur and are accumulated until time λ2. During the time interval [kT,T], we do not have any deterioration and therefore, shortages level linearly change. At time λ2 the production starts again and shortages level linearly decreases until the moment of T. The partially backordered quantity is supplied to customers during the time interval [λ2,T]. At time T, the second cycle starts and this behavior continue till the end of the (n-1)-th cycle.

In the last cycle shortages are not allowed and the inventory cycle can be divided into two parts. The production stops at time $(n-1)T+\lambda_3$ and then the inventory level decreases to lead zero at the end of the time horizon.

3. The Mathematical Modeling and Analysis:

The objectives of the problem can be explained as follows:

1. Minimization of the expected present value of costs over time horizon

$$Min \quad Z_1 = ETVC(n,k) \qquad (2)$$

2. Decreasing of the total quantity of goods in warehouse over time horizon

$$Min \quad Z_2 = TI(n,k) \tag{3}$$

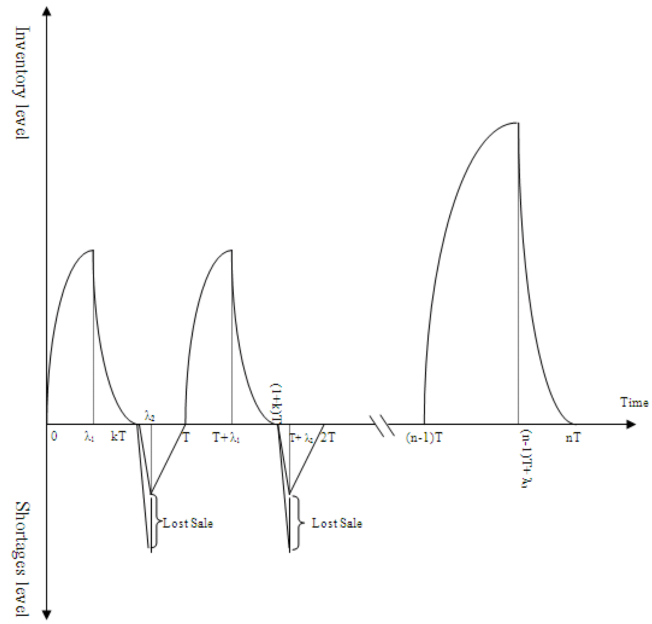

Fig. 1: Graphical representation of the inventory system.

Inventory decreasing is important, because: (1) decreasing in the inventory level causes increasing company flexibility against changes in the market conditions, customer needs and so on, (2) the quantity of the deteriorated goods is related to the inventory level so that decreasing in the inventory decreases the destroyed good, (3) the low inventory system causes faster company adaptation with the technology changes, and (4) decreasing in the inventory causes better cash flow and rate of return. The multiple objective function of the inventory system can be considered as follows

$$Min \quad Z \;= \left[MinETVC(n,k), MinTI(n,k)\right] \tag{4}$$

The inventory cycles is divided to four different parts. Let $I_i(t_i)$ denote the inventory level at any time t_i in the ith part of the first to (n-1)-th cycles ($i=1,2,3,4$). The differential equations describing the inventory level at any time in the cycle are given as

$$\frac{dI_1(t_1)}{dt_1} + \tau I_1(t_1) = P - R(i), \qquad 0 \le t_1 \le \lambda_1 \tag{5}$$

$$\frac{dI_2(t_2)}{dt_2} + \tau I_2(t_2) = -R(i), \qquad 0 \le t_2 \le kT - \lambda_1 \tag{6}$$

$$\frac{dI_3(t_3)}{dt_3} = -\delta(\lambda_2 - kT - t_3)R(i), \qquad 0 \le t_3 \le \lambda_2 - kT \tag{7}$$

$$\frac{dI_4(t_4)}{dt_4} = P - R(i), \qquad 0 \le t_4 \le T - \lambda_2 \tag{8}$$

In the last cycle shortages are not allowed and the inventory level is governed by the following differential equations ($I_i(t_i)$ denote the inventory level at any time t_i in the $(i-4)th$ part of the last cycle that $i=5,6$)

$$\frac{dI_5(t_5)}{dt_5} + \tau I_5(t_5) = P - R(i), \qquad 0 \le t_5 \le \lambda_3 \tag{9}$$

$$\frac{dI_6(t_6)}{dt_6} + \tau I_6(t_6) = -R(i), \qquad 0 \le t_6 \le T - \lambda_3 \tag{10}$$

The solution of the above differential equations along with the boundary conditions $I_1(0)=0$, $I_2(kT-\lambda_1)=0$, $I_3(0)=0$, $I_4(T-\lambda_2)=0$, $I_5(0)=0$ and $I_6(T-\lambda_3)=0$, are

$$I_1(t_1) = \frac{P - R(i)}{\tau}(1 - e^{-\tau t_1}), \qquad 0 \le t_1 \le \lambda_1 \tag{11}$$

$$I_2(t_2) = \frac{-R(i)}{\tau}(1 - e^{\tau(kT-\lambda_1-t_2)}), \qquad 0 \le t_2 \le kT - \lambda_1 \tag{12}$$

$$I_3(t_3) = \frac{R(i)e^{-(\lambda_2-kT)\alpha}}{\alpha}\left(1 - e^{\alpha t_3}\right), \qquad 0 \le t_3 \le \lambda_2 - kT \tag{13}$$

$$I_4(t_4) = (P - R(i))(t_4 - T + \lambda_2), \qquad 0 \le t_4 \le T - \lambda_2 \tag{14}$$

$$I_5(t_5) = \frac{P - R(i)}{\tau}(1 - e^{-\tau t_5}), \qquad 0 \le t_5 \le \lambda_3 \tag{15}$$

$$I_6(t_6) = \frac{-R(i)}{\tau}(1 - e^{\tau(T-\lambda_3-t_6)}), \qquad 0 \le t_6 \le T - \lambda_3 \tag{16}$$

The values of λ_1, λ_2 and λ_3 can be calculated with respect to k and T, using the above equations. Solving $I_1(\lambda_1)= I_2(0)$ for λ_1 we have

$$\lambda_1 = \frac{1}{\tau} Ln \frac{P - R(i)(1 - e^{\tau kT})}{P} \tag{17}$$

λ_2 can be calculated by solving $I_3(\lambda_2\text{-}kT)=I_4(0)$

$$\lambda_2 = \frac{[P - R(i)(1-k)]T}{P} \tag{18}$$

Finally, solving $I_5(\lambda_3)= I_6(0)$ for λ_3 we have

$$\lambda_3 = \frac{1}{\tau} Ln \frac{P - R(i)(1 - e^{\tau T})}{P} \tag{19}$$

4.1. The Expected Present Value of Costs:

Let *ECR* as the expected present value *(EPV)* of replenishment costs, *ECP* as the (*EPV*) of purchasing costs, *ECH* as the *EPV* of carrying costs, *ECS* as the *EPV* of shortages costs (backordering and lost sale) and *ECD* as the *EPV* of deterioration costs, respectively. The detailed analysis is given as follows.

4.1.1. The Expected Present Value of Ordering Cost (ECR):

Assume *CR* as the ordering cost

$$CR = c_1\left[1 + \sum_{j=0}^{n-1} e^{-R(jT+\lambda_2)}\right] \tag{20}$$

By replacing equation (15) in equation (17) and taking the expected value we have

$$ECR = c_1 E\left\{1 + e^{-\frac{T\left[(bi^2+(a-rb)i-ra)(1-k)+pR\right]}{p}}\left[\frac{1-e^{-TnR}}{1-e^{-RT}}\right]\right\} \tag{21}$$

4.1.2. The Expected Present Value of Purchasing Cost (ECP):

Let ECP_1 and ECP_2 as the *EPV* of the purchase cost in the first to (n-1)-th cycles and in the last cycle, respectively. The first purchase cost that is ordered at time zero equals to: $c_2P\lambda_1$. Then, the next purchase will occur at time λ_2 and therefore, the first cycle purchase cost is

$$c_2P\left[\lambda_1 + (T - \lambda_2)e^{-\lambda_2 R}\right] \tag{22}$$

The purchase cost for j-th cycle, (j=2, 3, …, n-1) is similar to the above equation with considering the discount factor, therefore, the *EPV* of the purchase cost in the first (n-1)-th cycles is

$$ECP_1 = c_2 PE\left\{\left[\begin{array}{l}\dfrac{Ln\left[\dfrac{P-R(i)(1-e^{\tau kT})}{P}\right]}{\tau} \\ +\left[T-\dfrac{\left(P-R(i)(1-k)\right)T}{P}\right]e^{-\left[\frac{(P-R(i)(1-k))T}{P}\right]R}\end{array}\right]\dfrac{1-e^{-T(n-1)R}}{1-e^{-TR}}\right\} \tag{23}$$

The production quantity in the last cycle will occur at time *(n-1)T* and equals to $\lambda_3 P$. Therefore, the *EPV* of the purchase cost in the last cycle will be

$$ECP_2 = c_2 PE\left[\frac{1}{\tau} Ln \frac{P-R(i)(1-e^{\tau T})}{P} e^{-(n-1)RT}\right] \tag{24}$$

The total expected purchase cost over the time horizon would be

$$ECP = ECP_1 + ECP_2 \tag{25}$$

4.1.3. Expected Present Value of Holding Cost (ECH):

Consider ECH_1 as the *EPV* of the holding cost during the first to (n-1)-th cycles. The *EPV* of the holding cost during the last cycle can be defined with ECH_2. In the first period, the holding costs for j-th cycle is

$$CH_j = c_3\left[\int_0^{\lambda_1} I_1(t_1)e^{-Rt_1}dt_1 + \int_0^{kT-\lambda_1} I_2(t_2)e^{-Rt_2}dt_2\, e^{-\lambda_1 R}\right]e^{-(j-1)RT}, \qquad j = 1,2,...,n-1 \tag{26}$$

After some complex calculations and taking the expected value we have

$$ECH_1 = c_3 E\left\{\left[\begin{array}{l}\dfrac{(P-R(i))\left\{e^{-\lambda_1 R}\left[-R(1-e^{-\tau\lambda_1})-\tau\right]+\tau\right\}}{\tau R(R+\tau)}+ \\ \dfrac{R(i)\left[-(R+\tau)e^{-\lambda_1 R}+\tau e^{-kTR}+R e^{\tau kT} e^{-\lambda_1(R+\tau)}\right]}{\tau R(R+\tau)}\end{array}\right]\dfrac{1-e^{-T(n-1)R}}{1-e^{-TR}}\right\} \tag{27}$$

For the last cycle, holding cost will be

$$CH_n = c_3\left[\int_0^{\lambda_3} I_5(t_5)e^{-Rt_5}dt_5\, e^{-R(n-1)T} + \int_0^{T-\lambda_3} I_6(t_6)e^{-Rt_6}dt_6\, e^{-R((n-1)T+\lambda_3)}\right] \tag{28}$$

After some complex calculations and taking the expected value we have

$$ECH_2 = -c_3 E\left\{e^{-R[\lambda_3+(n-1)T]}\left[\begin{array}{l}\dfrac{(P-R(i)\left[R(1-e^{-\tau\lambda_3})+\tau(1-e^{\lambda_3 R})\right]}{-\tau R(R+\tau)}+ \\ \dfrac{R(i)\left[\tau e^{-R(T-\lambda_3)}-R(1-e^{\tau(T-\lambda_3)})-\tau\right]}{-\tau R(R+\tau)}\end{array}\right]\right\} \tag{29}$$

So, the total *EPV* of the holding costs over the time horizon is

$$ECH = ECH_1 + ECH_2 \tag{30}$$

4.1.4. The Expected Present Value of Shortages Cost (ECS):

ECS shows the *EPV* of the shortages cost, including backorder and lost sales, during the first to (n-1)-th cycles. Shortages are not allowed in the last cycle. Therefore

$$ECS = \sum_{j=1}^{n-1} E\left\{\left[\begin{array}{l}\int_0^{\lambda_2-kT}\left[c_4 e^{-Rt_3}\sigma(t_3)+c_5(1-\sigma(t_3))e^{-(\lambda_2-kT)R}\right]-I_3(t_3)\right]dt_3\, e^{-kTR} \\ +\int_0^{T-\lambda_2}\left[c_4 e^{-Rt_4}\sigma(t_4)+c_5(1-\sigma(t_4))e^{-(T-\lambda_2)R}\right]-I_4(t_4)\right]dt_4\, e^{-\lambda_2 R}\end{array}\right]e^{-(j-1)RT}\right\} \tag{31}$$

Or

$$ECS = E\left\{\left[\begin{array}{l}\int_0^{\lambda_2-kT}\left[c_4 e^{-Rt_3}\sigma(t_3)+c_5(1-\sigma(t_3))e^{-(\lambda_2-kT)R}\right]-I_3(t_3)\Big]dt_3\, e^{-kTR} \\ +\int_0^{T-\lambda_2}\left[c_4 e^{-Rt_4}\sigma(t_4)+c_5(1-\sigma(t_4))e^{-(T-\lambda_2)R}\right]-I_4(t_4)\Big]dt_4\, e^{-\lambda_2 R}\end{array}\right]\frac{1-e^{-T(n-1)R}}{1-e^{-TR}}\right\} \tag{32}$$

4.1.5. The Expected Present Value of Deteriorating Cost (ECD):

Denote DI_1 the quantity of inventory items which have been deteriorated per cycle in the first to the (n-1)-th cycles

$$\begin{aligned} DI_1 &= \tau\left[\int_0^{\lambda_1} I_1(t_1)dt_1 + \int_0^{kT-\lambda_1} I_2(t_2)dt_2\right] \\ &= \frac{(P-a-bi)\left(\tau\lambda_1-1-e^{-\tau\lambda_1}\right)-(a+bi)\left(1+\tau(kT-\lambda_1)-e^{\tau(kT-\lambda_1)}\right)}{\tau} \end{aligned} \tag{33}$$

Now, assume ECD_1 as the EPV of the deterioration cost during the first to the (n-1)-th cycles. Also, ECD_2 is defined the EPV of the deterioration cost during the last cycle. ECD_1 after taking the expected value will be

$$\begin{aligned} ECD_1 &= \frac{c_6}{\tau}\sum_{j=1}^{n-1} E\left[(P-a-bi)\left(\tau\lambda_1-1-e^{-\tau\lambda_1}\right)e^{-(j-1)TR}-(a+bi)\left(1+\tau(kT-\lambda_1)-e^{\tau(kT-\lambda_1)}\right)e^{-(j-1+\lambda_1)TR}\right] \\ &= \frac{c_6}{\tau}E\left\{\left[(P-a-bi)\left(\tau\lambda_1-1-e^{-\tau\lambda_1}\right)-(a+bi)\left(1+\tau(kT-\lambda_1)-e^{\tau(kT-\lambda_1)}\right)e^{-\lambda_1 TR}\right]\frac{1-e^{-T(n-1)R}}{1-e^{-TR}}\right\} \end{aligned} \tag{34}$$

For the last cycle, deterioration cost will be

$$\begin{aligned} ECD_2 &= c_6 E\left\{\tau\left[\int_0^{\lambda_3} I_5(t_5)dt_5 e^{-(n-1)RT} + \int_0^{kT-\lambda_3} I_6(t_6)dt_6 e^{-(n+\lambda_3-1)RT}\right]\right\} \\ &= \frac{c_6}{\tau}E\left\{\left[(P-a-bi)\left(\tau\lambda_3-1+e^{-\tau\lambda_3}\right)-(a+bi)\left(1+(T-\lambda_3)\tau-e^{\tau(T-\lambda_3)}\right)e^{-\lambda_3 R}\right]e^{-(n-1)RT}\right\} \end{aligned} \tag{35}$$

Therefore, the total EPV of the deterioration cost over the time horizon is

$$ECD = ECD_1 + ECD_2 \tag{36}$$

Considering the above mentioned analysis, the EPV of the total system costs over the time horizon for a given value of H, is as follow

$$ETC(n,k) = ECR + ECP + ECH + ECS + ECD \tag{37}$$

Note that the time horizon H has a p.d.f. $f(h)$. So, the present value of expected total cost from n complete cycles, $ETVC(n,k)$, is given by

$$ETVC(n,k) = \sum_{n=0}^{\infty}\int_{nT}^{(n+1)T} ETC(n,k)f(h)dh \tag{38}$$

Therefore

$$ETVC(n,k) = c_1 E\left\{1 + e^{-\frac{T\left[(bi^2+(a-rb)i-ra)(1-k)+pR\right]}{p}}\left[\frac{1-M_H(-R)}{1-e^{-RT}}\right]\right\}$$

$$+E\left[\left\{\begin{array}{l} c_2 P\left[\begin{array}{l}\dfrac{Ln\left[\dfrac{P-R(i)(1-e^{\tau kT})}{P}\right]}{\tau} \\ +\left[T-\dfrac{(P-R(i)(1-k))T}{P}\right]e^{-\left[\frac{(P-R(i)(1-k))T}{P}\right]R}\end{array}\right] \\ +c_3\left[\begin{array}{l}\dfrac{(P-R(i))\left\{e^{-\lambda_1 R}\left[-R(1-e^{-\tau\lambda_1})-\tau\right]+\tau\right\}}{\tau R(R+\tau)}+ \\ \dfrac{R(i)\left[-(R+\tau)e^{-\lambda_1 R}+\tau e^{-kTR}+R e^{\tau kT}e^{-\lambda_1(R+\tau)}\right]}{\tau R(R+\tau)}\end{array}\right] \\ +\left[\begin{array}{l}\int_0^{\lambda_2-kT}\left[c_4 e^{-Rt_3}\sigma(t_3)+c_5(1-\sigma(t_3))e^{-(\lambda_2-kT)R}\right]\left[-I_3(t_3)\right]dt_3\, e^{-kTR} \\ +\int_0^{T-\lambda_2}\left[c_4 e^{-Rt_4}\sigma(t_4)+c_5(1-\sigma(t_4))e^{-(T-\lambda_2)R}\right]\left[-I_4(t_4)\right]dt_4\, e^{-\lambda_2 R}\end{array}\right] \\ +\dfrac{c_6}{\tau}\left[(P-a-bi)(\tau\lambda_1-1-e^{-\tau\lambda_1})-(a+bi)\left(1+\tau(kT-\lambda_1)-e^{\tau(kT-\lambda_1)}\right)e^{-\lambda_1 TR}\right]\end{array}\right\}\frac{1-e^{RT}M_{nT}(-R)}{1-e^{-RT}}\right]$$

$$+E\left[\left\{\begin{array}{l} c_2 P\left[\dfrac{1}{\tau}Ln\dfrac{P-R(i)(1-e^{\tau T})}{P}\right] \\ -c_3\left\{e^{-R\lambda_3}\left[\begin{array}{l}\dfrac{(P-R(i)\left[R(1-e^{-\tau\lambda_3})+\tau(1-e^{\lambda_3 R})\right]}{-\tau R(R+\tau)}+ \\ \dfrac{R(i)\left[\tau e^{-R(T-\lambda_3)}-R(1-e^{\tau(T-\lambda_3)})-\tau\right]}{-\tau R(R+\tau)}\end{array}\right]\right\} \\ +\dfrac{c_6}{\tau}\left[(P-a-bi)(\tau\lambda_3-1+e^{-\tau\lambda_3})-(a+bi)\left(1+(T-\lambda_3)\tau-e^{\tau(T-\lambda_3)}\right)e^{-\lambda_3 R}\right]\end{array}\right\}e^{RT}M_{nT}(-R)\right]$$

(39)

Which $M_H(-R)$ is the moment generating function of H.

4.2. The Total Quantity of Inventory (TI (n, k)):

Let TI_1 and TI_n as the total quantity of the goods held as inventory in the warehouse during the first (n-1) cycles and the last cycle respectively. TI_1 equals to quantity of the goods held as inventory during each cycle multiplied by n-1.

$$TI_1 = (n-1)\left[\int_0^{\lambda_1} I_1(t_1)dt_1 + \int_{\lambda_1}^{kT} I_2(t_2)dt_2\right] \tag{40}$$

TI_1 is calculated as follow with considering equations (11) and (12)

$$TI_1 = (n-1)\left[\frac{(P-R(i))(e^{-\tau\lambda_1}+\tau\lambda_1-1)}{\tau^2}+\frac{R(i)(\tau e^{2\tau\lambda_1}(\lambda_1-kT)-e^{\tau\lambda_1}+e^{\tau kT})}{\tau^2 e^{2\tau\lambda_1}}\right] \tag{41}$$

Note TI_1 is a decreasing function with respect to k. Referring Figure 1, it can be observed that

$$k=0 \quad \Rightarrow \quad \lambda_1 = 0 \quad \Rightarrow TI_1 = 0 \tag{42}$$

Similarly, TI_n is equal to

$$\begin{aligned} TI_n &= \int_0^{\lambda_3} I_5(t_5)dt_5 + \int_{\lambda_3}^{T} I_6(t_6)dt_6 \\ &= \frac{(P-R(i)(e^{-\tau\lambda_3}+\tau\lambda_3-1)}{\tau^2}-\frac{R(i)(\tau e^{2\tau\lambda_3}(T-\lambda_3)+e^{\tau\lambda_3}-e^{\tau T})}{\tau^2 e^{2\tau\lambda_3}} \end{aligned} \tag{43}$$

So, the total inventory over time horizon is

$$TI(n,k) = TI_1 + TI_n \tag{44}$$

The above mentioned equation includes stochastic parameters i, the inflation rate, and n=H.T, the number of replenishments during random time horizon. Therefore, the expected value approach can be used. Let ETI(n,k) as the expected total inventory over time horizon. ETI(n,k) will be as follow

$$ETI(n,k) = \sum_{n=1}^{\infty}\int_{(n-1)T}^{nT} E\left[\begin{array}{l}(n-1)\left[\begin{array}{l}\dfrac{(P-R(i))(e^{-\tau\lambda_1}+\tau\lambda_1-1)}{\tau^2}+\\ \dfrac{R(i)(\tau e^{2\tau\lambda_1}(\lambda_1-kT)-e^{\tau\lambda_1}+e^{\tau kT})}{\tau^2 e^{2\tau\lambda_1}}\end{array}\right] \\ +\left[\begin{array}{l}\dfrac{(P-R(i)(e^{-\tau\lambda_3}+\tau\lambda_3-1)}{\tau^2}-\\ \dfrac{R(i)(\tau e^{2\tau\lambda_3}(T-\lambda_3)+e^{\tau\lambda_3}-e^{\tau T})}{\tau^2 e^{2\tau\lambda_3}}\end{array}\right]\end{array}\right] f(h)dh \tag{45}$$

5. The Solution Procedure:

The problem is to determine the optimal values of n, the number of replenishments to be made during period H, and k, the proportion of time in any given inventory cycle which orders can be filled from the existing stock $(0 < k \le 1)$. The ideal point approach will be use to solve the model. Consider the following multi-objective programming problem

Min $f_j(x)$ for j=1, 2, ..., k
s.t.: $g_i(x)<0$ for i=1, 2, …, m (46)

Where x is a n-dimensional decision vector. For any $f_j(x)$, define the ideal point as $f_j(x^{*j})$ which x^{*j} minimizes $f_j(x)$. $f_j(x^{*j})$ is called the ideal point. The measure is ''closeness" and LP-metric is used. LP-metric defines the distance between two points $f_j(x)$ and $f_j(x^{*j})$ in k-dimensional space as

$$d_d = \left\{\sum_{j=1}^{k}\gamma_i\left[f_j(x^{*j})-f_j(x)\right]^d\right\}^{1/d} \qquad where \quad d \ge 1 \tag{47}$$

Where γ_j for j=1... k is relative importance (weights) of the objective function $f_j(x)$. The compromise solution for a given value of d will be minimizes the d_d-metric in (47). The measurement unit of the model objectives is not equal to each other and therefore, we need to normalize the distance family of (47) by using the reference point as follow

$$d_d = \left\{ \sum_{j=1}^{k} \gamma_i \left[\frac{f_j\left(x^{*j}\right) - f_j(x)}{f_j\left(x^{*j}\right)} \right]^d \right\}^{1/d} \qquad where \quad d \geq 1 \tag{48}$$

Therefore

$$Mind_d(n,k) = \left\{ \begin{aligned} &\gamma_1 \left[\frac{ETVC(n^*,k^*) - ETVC(n,k)}{ETVC(n^*,k^*)} \right]^d \\ &+ \gamma_2 \left[\frac{ETI(n^*,k^*) - ETI(n,k)}{ETI(n^*,k^*)} \right]^d \end{aligned} \right\}^{1/d} \qquad where \quad d \geq 1 \tag{49}$$

If k=0, the total inventory over time horizon, ETI(n,k), will be minimized for a given value of n. In this condition, the inventory level is zero over time horizon, except the last cycle, because, the shortages are not allowable. When n increases, the time interval between replenishments, especially the last cycle, will be decreases. So

$$\{n \to \infty, k = 0\} \Rightarrow ETI(n,k) \to 0 \tag{50}$$

The absolute zero inventory level is impossible for any company and therefore, the inventory system manager has to consider a minimum value none zero inventory up to internal (company) and external (market) situations. Let *ETI(n*,k*)* as the determined minimum inventory.

Since ETVC(n,k) is a function of a discrete variable n and a continuous variable k (0 < k < 1), therefore, for any given n, the necessary condition for the minimum of ETVC(n,k) is

$$\frac{dETVC(n,k)}{dk} = 0 \tag{51}$$

For a given value of n, derive k^* from Equation (51). *ETVC(n,k*)* derives by substituting (n,k^*) into equation (39). Then, n increase by the increment of one continually and *ETVC(n,k*)* calculate again. The above stages repeat until the minimum *ETVC(n,k*)* be found. The *(n*,k*)* and *ETVC(n*,k*)* values constitute the optimal solution and satisfy the following conditions

$$\Delta ETVC\left(n^* - 1, k^*\right) \rangle 0 \langle \Delta ETVC\left(n^*, k^*\right) \tag{52}$$

Where

$$\Delta ETVC\left(n^*, k^*\right) = ETVC\left(n^* + 1, k^*\right) - ETVC\left(n^*, k^*\right) \tag{53}$$

6. Numerical Example:

The following numerical example is provided to clarify how the proposed model is applied. Let the ordering, production, holding, backordering, lost sales and deterioration costs at the beginning of the time horizon as follow

c1=\$100/order; c2=\$8/unit; c3=\$2/unit/year; c4=\$3/unit/year; c5=\$10/unit and c6=\$13/unit.

The inflation rates and the time horizon are stochastic with the following p.d.f.s:

$$i \sim U(\$0.08/\$/\text{year}, \$0.15/\$/\text{year})$$
$$H \sim N\left(10, 1.5^2\right)$$

The company interest rate is 20 percent and the deterioration rate of the on–hand inventory per unit time is five percent. The constant annual production rate is 5000 units.

r=\$0.2/\$/year, *τ=0.05/unit/year*, *P=5000units/year*

The backlogging rate is $\delta(t)=e^{-0.5t}$, and the demand parametric values are *a=3000units/year* and *b=-2000*.

The problem is the optimum ordering policy for minimizing (1) the expected present value of the total inventory system costs, ETVC(n,k), and (2) the expected total quantity of the goods in warehouse over time horizon, ETI(n,k). As stated in the ideal point method, we have to first optimize objectives, separately. The ideal point of the first objective with considering the above mentioned parameters values and using the numerical methods, is calculated and the results are illustrated in Table 1. It can be seen that the minimum expected cost is *172364.82\$* for *n*=17* and *k*=0.6168* (the shortages occur after elapsing 61.68% of the cycle time).

Table 1: The optimal solution of *ETVC(n,k)* for the numerical example.

n	k	ETVC (n,k)	n	k	ETVC (n,k)
5	0.6234	183584.36	19	0.6158	173843.25
10	0.6207	178613.92	20	0.6152	174712.57
15	0.6179	174009.75	25	0.6123	179392.74
16	0.6173	173144.43	30	0.6091	184377.16
17^*	0.6168^*	172364.82^*	50	0.5986	205279.43
18	0.6163	173059.56	100	0.5765	261427.92

Let ETI*(n*,k*)*=10000.00 as the determined minimum inventory up to the internal (company) and the external (market) situations. According to d=1 and the different combinations of γ_1 and γ_2 with considering ETVC(n^*,k^*)=172364.82 and ETI(n*,k*)=10000.00, the problem is evaluated and the results are shown in Table 2. The manager can determine the optimum value of n and k with considering company policy about the importance of the goals.

Table 2: Solution of the problem.

γ_1	γ_2	n*	k*	TI*	ETVC*
0	1	31	0.0989	10000.00	204377.16
0.25	0.75	29	0.2376	11213.85	19921.74
0.5	0.5	25	0.3693	14065.35	188910.16
0.75	0.25	19	0.4929	16541.23	178852.37
1	0	17	0.6168	18157.29	172364.82

7. Discussion:

The inventory systems usually have been surveyed to minimize the total cost. In the recent decades, the companies try to maintain survival and increase their contributions in the market with considering additional objectives. Decreasing in the inventory level is one of the most important objective for the company, because: (1) decreasing in the inventory level causes increasing company flexibility against changes in the market conditions, customer needs and so on, (2) the quantity of the deteriorated goods is related to the inventory level so that decreasing in the inventory decreases the destroyed good, (3) low inventory system causes faster company adaptation with the technology changes, and (4) decreasing in the inventory causes better cash flow and rate of return. Therefore, a bi-objective inventory model has been developed in this paper.

In reality, the value or utility of goods decreases over time for deteriorating items, which in turn suggests smaller cycle length, whereas presence of inflation in cost and its impact on demand suggests larger cycle length. In this article, inventory model has been developed considering both the opposite characteristics (deterioration and inflation) of the items, with shortages over a stochastic time horizon. Shortages are partially backlogged and demand is a function of the inflation rate. The numerical example has been given to illustrate the theoretical results. The study has been conducted under the Discounted Cash Flow (DCF) approach.

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