

# Different Policy Objectives of the Road Pricing Problem – a Game Theory Approach

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## Abstract

Using game theory we investigate a new approach to formulate and solve optimal tolls with a focus on different policy objectives of the road authority. The aim is to gain more insight into determining optimal tolls as well as into the behavior of users after tolls have been imposed on the network. The problem of determining optimal tolls is stated and defined using utility maximization theory, including elastic demand on the travelers' side and different objectives for the road authority. Game theory notions are adopted regarding different games and players, rules and outcomes of the games played between travelers on the one hand and the road authority on the other. Different game concepts (Cournot, Stackelberg and social planner game) are mathematically formulated and the relationship between players, their payoff functions, and rules of the games are defined. The games are solved for different scenarios and different objectives for the road authority, using the Nash equilibrium concept. Using the Stackelberg game concept as being most realistic for road pricing, a few experiments are presented illustrating the optimal toll design problem subject to different pricing policies considering different objectives of the road authority. Results show different outcomes both in terms of optimal tolls as well as in payoffs for travelers. There exist multiple optimal solutions and the objective functions may have a non-continuous shape. The main contribution is the two-level separation between the network users and the road authority in terms of their objectives and influences.

**Key words:** game theory, traffic assignment, Stackelberg game, policy objectives,

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# 1. Introduction and background

In recent years, researchers have become increasingly interested in the effects of introducing road pricing measures on transportation networks (see more in (1)). The view that pricing can be one of the strategies to achieve more efficient use of transportation capacity has led to the expectation that road pricing can relieve congestion on the roads and improve the use of the transportation system. Nevertheless, road pricing is a very controversial and complex topic making it necessary to consider road pricing from different perspectives. Which objectives the road authority would like to achieve? Who is involved in decision-making and how should decisions be made? How will the travelers change their travel behavior after introducing road pricing? How will travelers interact with each other and how can the road authority influence or even control travel behavior of travelers? To answer such questions we need a flexible framework for analyzing the behavior of travelers as well as of the road authority.

Game theory provides such a framework for modeling decision-making processes in which multiple players are involved with different objectives, rules of the game, and assumptions. Considering the problem of designing optimal tolls on the network, there is a need for better insights into the interactions between travelers and the road authority, their nature, and the consequences of these interactions. Which objective the road authority will apply will have a strong influence on how, where, when and how much toll will be levied, and on its resulting welfare. The focus of this paper is on assessing different objectives a road authority may adopt and on their influence on optimal toll design.

In this paper we analyze in a game-theoretic framework a very simple route choice problem with elastic demand where road pricing is introduced. First, the road-pricing problem is formulated using game theory notions with which different games are described. After that, a game-theoretic approach is applied to formulate the road pricing game as social planner (monopoly), Stackelberg, and Cournot games, respectively. The main purpose of the experiment reported here is to show the outcomes of different games established for the optimal toll design problem. The toll setting problem is defined using flow- dependent tolls on links in a network.

## **2. Literature review**

### **2.1 Transportation problems and game theory**

Game theory first appeared in solving transportation problems in the form of so-called *Wardropian equilibrium* of route choice, see (2), which is similar to the Nash equilibrium of an  $N$ -player game, see (3). For the definition of a Nash equilibrium see Section 5.

### **2.2 Optimal traffic control problems and game theory**

In (4) for the first time different problems in transportation systems modeling are described in which a game theory approach is proposed for solution algorithms. In that paper, relationships are drawn between two game theory models based on the Nash non-cooperative and Stackelberg games.

In (5) the dynamic mixed behavior traffic network equilibrium problem is formulated as a non-cooperative  $N$ -person, non zero-sum differential game. A simple network is considered where two types of players (called user equilibrium (UE)-players and Cournot-Nash (CN)-players respectively) interact through the congestion phenomenon. A procedure to compute system optimal routings in a dynamic traffic network is introduced by (6). Fictitious play is utilized within a game of identical interests wherein vehicles are treated as players. In the work of (7), a two-player, non-cooperative game is established between the network user seeking a path to minimize its expected trip cost on the one hand, and an “evil entity” choosing link performance scenarios to maximize the expected trip cost on the other. An application of game theory to solve risk-averse user equilibrium traffic assignment can be found in (8). Network users have to make their route choice decisions in the presence of uncertainty about route costs reason why they need to have a strategy towards risk. In (9) a preliminary model of dynamic multi-layer infrastructure networks is presented in the form of a differential game. In particular, three network layers (car, urban freight and data) are modeled as Cournot-Nash dynamic agents. In (10) the integrated traffic control and dynamic traffic assignment problem is presented as a non-cooperative game between the traffic authority and highway users. The objective of the combined control-assignment problem is to find dynamic system optimal signal settings and dynamic user-optimal traffic flows. The combined control-assignment problem is first formulated as a single-level Cournot game: the traffic authority and the users choose

their strategies simultaneously. Then, the combined problem is formulated as a bi-level Stackelberg game in which the traffic authority is the leader who determines the signal settings in anticipation of the user's responses.

### **2.3 Road pricing problems and game theory**

The problem of determining optimal tolls in transportation networks is a complex issue. In (11) the question what happens when jurisdictions have the opportunity to establish tollbooths at the frontier separating them is examined. If one jurisdiction would be able to set his policy in a vacuum it is clearly advantageous to impose as high a toll on non-residents as can be supported. However, the neighboring jurisdiction can set a policy in response. This establishes the potential for a classical prisoner's dilemma consideration: in this case to tax (cooperate) or to toll (defect). In (12) an application of game theory and queuing analysis to develop micro-formulations of congestion can be found. Only departure time is analyzed in the context of a two-player and three-player game respectively where interactions among players affect the payoffs for other players in a systematic way. In (13) route choice and elastic demand problem is considered with focus on different game concepts of the optimal toll problem. A few experiments are done showing that the Stackelberg game is the most promising game between the road authority and the travelers if only one road authority's objective is considered.

There is a lack in the literature about the importance of different policies the road authority may adopt, and outcomes that can be result of the different objectives and games played with the travelers. Therefore, different policy objectives of the road authority in the optimal toll design problem as well as different game concepts consequences will be the focus of this paper.

## **3. Problem statement (non-cooperative game theory)**

The interactions between travelers and the road authority can be seen as a non-cooperative, non-zero sum,  $(N+1)$  players game between a single traffic authority on the one side and  $N$  network users (travelers) on the other. The objective of the road-pricing problem, which is the combined optimal toll design and traffic assignment problem, is to find system-optimal tolls and user-optimal traffic flows simultaneously. This road-pricing is an example of a bi-level optimization problem. The user-equilibrium traffic assignment problem (lower level problem) can be formulated as

non-cooperative,  $N$ -person, non-zero-sum game solved as a Nash game. The upper level problem may have different objectives depending on what the road authority would like to achieve. This question will be the focus of this paper.

A conceptual framework for the optimal toll design problem in case of elastic demand addressed from different road authority's objectives is given in Figure 1.

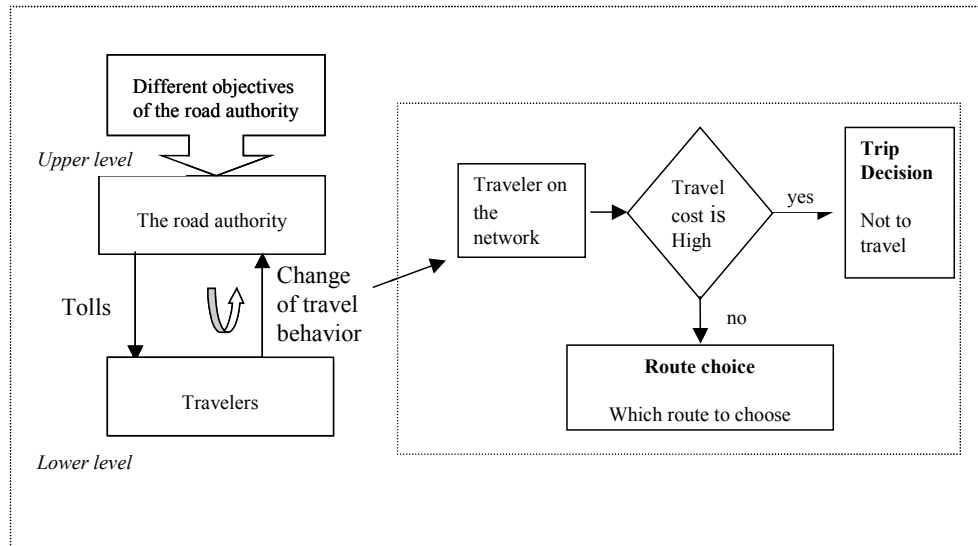


Figure 1 Conceptual framework for optimal toll design with route and trip choice

The road authority sets tolls on the network while travelers respond to tolls by changing their travel decisions. Depending on travel costs, they can decide to travel along a certain route or decide not to travel at all in the tolled network.

In the road-pricing problem, we are dealing with an  $N+1$ -player game, where there are  $N$  players (travelers) making a travel choice decision, and one player (the road manager) making a control or design decision (in this case, setting road tolls). Adding the traffic authority to the game is not as simple as extending an  $N$ -player game to an  $N+1$  player game, because the strategy space and the payoff function for this additional player differs from the rest of the  $N$  players. In fact, there are two games played in conjunction with each other. The first game is a non-cooperative game where all  $N$  travelers aim to maximize their individual utility by choosing the best travel strategy (i.e. trip choice and route choice), taking into account all other travelers' strategies. The second game is between the travelers and the road manager, where the road manager aims to maximize some network performance by choosing a control strategy, taking into account that travelers respond to the control strategy by adapting their travel strategies. The two games can be described as follows:

The outer level game, being the toll design problem, consisting of the following elements:

1. Players: the authority on the one side and  $N$  potential travelers on the other;
2. Rule 1: the authority sets the tolls taking the travelers' behavior into account as well as possible restrictions on the toll levels in order to optimize a certain objective.
3. Rule 2: travelers react on travel costs (including tolls) and change their behavior (route choice, trip choice) as to maximize their individual subjective utilities.
4. Outcomes of the game: a) optimal strategies for the authority (tolls), b) payoff for the authority (e.g. social welfare, revenues), c) optimal strategies for the travelers (trip and route decisions) and d) payoff for the travelers (utilities). The outcomes depend on the objective function for the authority used in the model.

The inner level game, being the network equilibrium problem, consisting of the following elements:

1. Players:  $N$  travelers
2. Rule: travelers make optimal trip and route choice decisions as to maximize their individual subjective utilities given a specific toll pattern.
3. Outcome of the game: a) optimal strategies for the travelers (trip and route decisions), b) payoff for the travelers (utilities)

Our main focus in this paper is to investigate the outer level game between the road authority and users, although the inner level game between travelers is part of it.

## **4. Model structure**

The objectives of the road authority and the travelers are different and sometimes even opposite. The upper level objective may be to minimize total travel time, to relieve congestion, to improve safety, to raise revenue, to improve total system utility, or anything else. The lower level objective may be the individual travel time, travel cost, or the individual travel utility. In this paper, we use the individual travel utility as the objective to maximize for travelers.

Since the purpose of this paper is to gain more insight into the structure of the optimal toll design problem under different policy objectives by using game theory, we restrict ourselves to the case of a very simple network in which only one origin-destination (OD) pair is considered. Between this OD pair, different non-overlapping route alternatives are available. The generalized route travel cost function,  $c_{pi}$  of traveler  $i$  for route  $p$  includes the travel time costs and the toll costs,

$$c_{pi} = \alpha\tau_p + \theta_p, \quad (1)$$

where  $\tau_p$  is the travel time of route  $p$ ,  $\theta_p$  is the toll costs of route  $p$ , and  $\alpha$  denotes the value-of-time (VOT) which converts the travel time into monetary costs. Let  $U_{pi}$  denote the trip utility for making a trip along route  $p$  of traveler  $i$ . This trip utility consists of a fixed net utility  $\bar{U}$  for making the trip (or arriving at the destination), and a disutility consisting of the generalized route travel costs  $c_{pi}$ :

$$U_{pi} = \bar{U} - c_{pi}. \quad (2)$$

According to utility maximization theory, a trip will be made only if the utility of doing an activity at a destination minus the utility of staying at home and the disutility of traveling is positive. In other words, if  $U_{pi} \leq 0$  then no trip will be made. By including a *fictitious* route in the route choice set representing the travelers' choice not to travel, and attaching a disutility of zero to this 'route' alternative, we combine route choice and trip choice into the model. Travelers are assumed to respond according to Wardrop's equilibrium law extended with elastic demand: *At equilibrium, no user can improve its trip utility by unilaterally making another route choice or trip choice decision.*

For the sake of simplicity we assume the deterministic utility case without a random error term.. For more elaborate definitions, see (14).

## 5. Game theory applied to road pricing

Let us consider first the  $N$ -player game of the travelers, where  $S_i$  is the set of available alternatives for traveler  $i$ ,  $i \in \{1, \dots, N\}$ . The strategy  $s_i \in S_i$  that traveler  $i$  will play depends on the control strategy set by the road manager, denoted by vector  $\theta$ , and on the strategies of all other players, denoted by  $s_{-i} \equiv (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N)$ . We

assume that each traveler decides independently seeking unilaterally the maximum utility payoff, taking into account the possible rational choices of the other travelers. Let  $J_i(s_{-i}(\theta), s_i(\theta), \theta)$  denote the utility payoff for traveler  $i$  for a given control strategy  $\theta$ . This utility payoff can include all kinds of travel utilities and travel cost. Utility payoff for traveler  $i$  can be expressed as follows:

$$J_i(s_{-i}(\theta), s_i(\theta), \theta) = U_{pi} - c_{pi} \quad (3)$$

where  $c_{pi}$  is defined in expression (1) and  $U_{pi}$  in expression (2).

If all other travelers play strategies  $s_{-i}^*$ , then traveler  $i$  will play the strategy that maximizes his payoff utility, i.e.

$$s_i^*(\theta) = \arg \max_{s_i \in S_i} J_i(s_{-i}^*(\theta), s_i^*(\theta), \theta). \quad (4)$$

If Equation (4) holds for all travelers  $i \in \{1, \dots, N\}$ , then  $s^*(\theta) \equiv (s_{-i}^*(\theta), s_i^*(\theta))$  is called a *Nash equilibrium* for the control strategy  $\theta$ . In this equilibrium, no traveler can improve his utility payoff by unilateral change of behavior. Note that this coincides with the concept of a *Wardrop user-equilibrium*.

Now consider the complete  $N+1$ -player game where the road manager faces the  $N$  travelers. The set  $\Theta$  describes the alternative strategies available to the road manager. Suppose he chooses strategy  $\theta \in \Theta$ , then, depending on this strategy and on the strategies  $s^*(\theta)$ , chosen by the travelers, his utility payoff is  $R(s^*(\theta), \theta)$ , and may represent e.g. the total system utility or the total profits made. The road manager chooses the strategy  $\theta^*$  in which he aims to maximize his utility payoff, depending on the responses of the travelers:

$$\theta^* = \arg \max_{\theta \in \Theta} R(s^*(\theta), \theta). \quad (5)$$

If Equations (4) and (5) are satisfied for all  $(N+1)$  players, where  $\theta = \theta^*$  in Equation (4), then this is a Nash equilibrium in which no player can be better off by unilaterally following another strategy. Although all equilibria use the Nash concept, a different equilibrium or game type can be defined in the  $N+1$ -player game depending on the influence each of the players has in the game. Game theory notions used in this paper are adopted from work of (15).



## 6. Different game concepts

In the following we will distinguish three different types of games between the road authority and the travelers, namely, Monopoly, Stackelberg and Cournot game, respectively.

### 6.1 Social planner game

In this case, the road manager not only sets its own control, but is also assumed able to control the strategies that the travelers will play. In other words, the road manager sets  $\theta^*$  as well as  $s^*$ . This case will lead to a so-called system optimum solution of the game. A Social planner (monopoly or solo player) game represents the best system performance and thus may serve as a ‘benchmark’ for other solutions. This game solution shows what is best for the one player (the road manager), regardless of the other players. In reality, however, a social planner solution may not be realistic since it is usually not in the users’ best interest and is it practically difficult to force travelers choosing a specific route without an incentive. From an economic point of view, in this case the road authority has complete (or full) market power. Mathematically, the problem can be formulated as follows:

$$(s^*, \theta^*) = \arg \max_{\theta \in \Theta, s \in S} R(s, \theta). \quad (6)$$

### 6.2 Stackelberg game

In this case, the road manager is the ‘leader’ by setting the control, thereby directly influencing the travelers that are considered to be ‘followers’. The travelers may only indirectly influence the road manager by making travel decisions based on the control. It is assumed that the road manager has complete knowledge of how travelers respond to control measures. The road manager sets  $\theta^*$  and the travelers follow by playing  $s^*(\theta^*)$ . From an economic point of view, in a Stackelberg game one player has more market power than others players in the game (in this case the road authority has more market power than the travelers). The Stackelberg game is a *dynamic*, multi-stage game of complete and perfect information. The order in which decisions are made is important. In a game with complete information, every player is fully informed about the rules of the game, the preferences of each player, and each player knows that every player knows. In other words, for the road pricing game we assume that apart

from the rules of the game all information about travel attributes (toll levels, available routes, travel times) are known to all travelers. A game with perfect information means that decision makers know the entire history of the game. A dynamic game has the following properties:

- the successive moves (implemented strategies) of the two players occur in sequence,
- all previous moves are observed before the next move is chosen (perfect history knowledge),
- the players' payoff from each feasible combination of moves are common knowledge (complete payoff information).

For more details about complete and imperfect games, see e.g. (16). The equilibrium is determined by *backward induction* where the traffic authority initiates the moves by setting a control strategy. Steps for the road pricing game are as follows:

1. The road authority chooses toll values from the feasible set of tolls.
2. The travelers react on the route cost (with tolls included) by adapting their route and/or trip choice.
3. Payoffs for the road authority as well as travelers are computed.
4. The optimal strategy for the road authority including the strategies of travelers is chosen.

The problem can be mathematically formulated as to find  $(s^*, \theta^*)$  such that:

$$\theta^* = \arg \max_{\theta \in \Theta} R(s^*(\theta), \theta), \quad \text{where } s_i^*(\theta) = \arg \max_{s_i \in S_i} J_i(s_i, s_{-i}^*, \theta), \quad \forall i = 1, \dots, N. \quad (7)$$

### 6.3 Cournot game

In contrast to the Stackelberg game, in this case the travelers are now assumed to have a direct influence on the road manager, having complete knowledge of the responses of the road manager to their travel decisions. The road manager sets  $\theta^*(s^*)$ , depending on the travelers' strategies  $s^*(\theta^*)$ . This type of a so-called duopoly game, in which two players choose their strategies simultaneously and therefore one's player's response is unknown in advance to others, is known as a Cournot game. Mathematically the problem can be formulated as follows. Find  $(s^*, \theta^*)$  such that

$$\theta^* = \arg \max_{\theta \in \Theta} R(s_i^*, s_{-i}^*, \theta), \text{ and } s_i^* = \arg \max_{s_i \in S_i} J_i(s_i, s_{-i}^*, \theta^*), \quad \forall i = 1, \dots, N. \quad (8)$$

The different game concepts will be illustrated in the next section. It should be pointed out that the Stackelberg game is the most realistic game approach in our pricing context. This is a dynamic game which can be solved using backward induction, see e.g. (17). Mathematical bi-level problem formulations can be used for solving more complex games, see e.g. (18).

## 7. Different objectives of the road authority

Which objective the road authority will apply will have influence on the optimal toll levels. Depending on the authority's objective, different utility payoff functions can be formulated.

Assuming the road authority's objective of maximizing total travel utility (the utility of all network users together), the objective is defined as the sum of the payoff values of all travelers:

$$\max R(s^*(\theta), \theta) = \sum_{i=1}^N J_i(s^*(\theta)). \quad (9)$$

In case the road authority aims at maximizing total toll revenues, the following objective may be used:

$$\max R(s^*(\theta), \theta) = \sum_p q_p(s^*(\theta))\theta_p, \quad (10)$$

where  $q_p(s^*)$  denotes the number of travelers using route  $p$ , which can be derived from the optimal strategies  $s^*$ . Clearly, setting tolls equal to zero does not provide any revenues, while setting very high tolls will make all travelers decide not to travel at all.

Combining these two objectives leads to the notion of social surplus maximization. The social surplus can be computed by adding the toll revenues to the total trip utilities, such that the following problem will maximize social surplus as an objective:

$$\max R(s^*(\theta), \theta) = \sum_{i=1}^N J_i(s^*(\theta)) + \sum_p q_p(s^*(\theta))\theta_p. \quad (11)$$

## 8. A few experiments

Let us now look at the following simple problem to illustrate how the road-pricing problem can be analyzed using game theory. Suppose there are two individuals wanting to travel from A to B. There are two alternative routes available to go to B. The first route is tolled (toll is equal to  $\theta$ ), the second route is untolled. Depending on the toll level, the travelers decide to take either route 1 or route 2, or not to travel at all. The latter choice is represented by a third virtual route, such that we can consider three route alternatives as available strategies to each traveler, i.e.  $S_i = \{1,2,3\}$  for traveler  $i = 1,2$ . Figure 2 illustrates the problem.

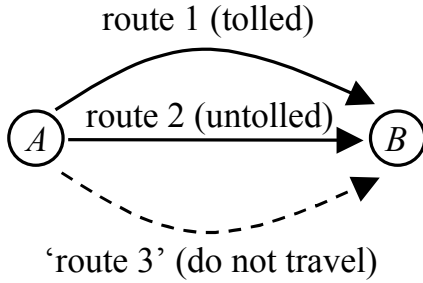


Figure 2 Network description for the road-pricing problem

Each strategy yields a different payoff, depending on the utility to make the trip, the travel time on the route (that increases whenever more travelers use it) and a possible route toll. We assume that traveler  $i$  aims to maximize its individual travel utility (payoff,) given by

$$J_i(s_1(\theta), s_2(\theta)) = \begin{cases} \bar{U} - \alpha\tau_1(s_1(\theta), s_2(\theta)) - \theta, & \text{if } s_i(\theta) = 1, \\ \bar{U} - \alpha\tau_2(s_1(\theta), s_2(\theta)), & \text{if } s_i(\theta) = 2, \\ 0, & \text{if } s_i(\theta) = 3. \end{cases} \quad (12)$$

In Equation (5),  $\bar{U}$  represents the trip utility when making the trip to destination B (in the calculations we assume  $\bar{U} = 210$ ),  $\tau_p^{rs}(\cdot)$  denotes the route travel time for route  $r$  depending on the chosen strategies, while  $\alpha$  represents the value of time (we assume  $\alpha = 6$  for all travelers). Note that negative net utilities on route 1 and 2 imply that one will not travel, i.e. if the cost (disutility) of making the trip is larger than the utility of the trip itself. The route travel times are given as a function of the chosen strategies in the sense that the more travelers use a certain route, the higher the travel time:

$$\tau_1(s_1(\theta), s_2(\theta)) = \begin{cases} 10, & \text{if } s_1(\theta) = 1 \text{ or } s_2(\theta) = 1 \text{ (e.g. flow on route 1 is 1),} \\ 18, & \text{if } s_1(\theta) = 1 \text{ and } s_2(\theta) = 1 \text{ (e.g. flow on route 1 is 2),} \end{cases} \quad (13)$$

and

$$\tau_2(s_1(\theta), s_2(\theta)) = \begin{cases} 20, & \text{if } s_1(\theta) = 2 \text{ or } s_2(\theta) = 2 \text{ (e.g. flow on route 2 is 1),} \\ 40, & \text{if } s_1(\theta) = 2 \text{ and } s_2(\theta) = 2 \text{ (e.g. flow on route 2 is 2).} \end{cases} \quad (14)$$

Solving the game between the two travelers for a Nash equilibrium corresponds to a Wardrop equilibrium with elastic demand, in which no traveler can improve his/her utility by unilaterally changing route or deciding not to travel. For the sake of clarity we will only look at pure strategies in this example, but the case may be extended to mixed strategies as well. In pure strategies, each player is assumed to adopt only one strategy, whereas in mixed strategies, the players are assumed to adopt probabilities for choosing each of the available strategies. In our example we are thus looking at discrete flows instead of continuous flows so that Wardrop's first principle according to which all travel utilities are equal for all used alternatives may no longer hold in this case. In fact, the more general equilibrium rule applies in which each traveler aims to maximize his personal trip utility. The utility payoff table, depending on the toll  $\theta$ , is given in Table 1 for the two travelers, where the values between brackets are the payoffs for travelers 1 and 2, respectively.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$(102 - \theta, 102 - \theta)$	$(150 - \theta, 90)$	$(150 - \theta, 0)$
	Route 2	$(90, 150 - \theta)$	$(-30, -30)$	$(90, 0)$
	Route 3	$(0, 150 - \theta)$	$(0, 90)$	$(0, 0)$

Table 1 Utility Payoff Table for Travellers

For example, if traveler 1 chooses route 1 and traveler 2 chooses route 2, then the travel utility for traveler 1 is  $J_1(1,2) = 210 - 6 \cdot 10 - \theta = 150 - \theta$ .

In the experiments we will consider three different road authority's objectives: total travel utility, social surplus, and generating revenues. For the first objective, three different game concepts are applied: social planner, Stackelberg and Cournot game,

respectively. Because Stackelberg game is the most realistic game and, we apply only Stackelberg game for the other two objective functions.

### 8.1 CASE STUDY 1: maximize total TRAVEL utility

Now, let us add the road manager as a player, assuming that he tries to maximize total travel utility, i.e.

$$\text{Max } R(s^*(\theta), \theta) = J_1(s^*(\theta)) + J_2(s^*(\theta)). \quad (15)$$

The strategy set of the road manager is assumed to be  $\Theta = \{\theta | \theta \geq 0\}$ . The payoffs for the road manager are presented in Table 2 depending on the strategy  $\theta \in \Theta$  that the road manager plays and depending on the strategies the travelers play.

		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$204 - 2\theta$	$240 - \theta$	$150 - \theta$
	Route 2	$240 - \theta$	-60	90
	Route 3	$150 - \theta$	90	0

Table 2 Utility Payoff Table for the Road Manager if his Objective is to Maximize the Total Travel Utility

Let us solve the previously defined payoff tables for different game concepts and different values of tolls. First, we discuss the social planner game, then the Stackelberg game and finally the Cournot game.

#### 8.1.1 Social planner game

In the social planner game, the road manager sets the toll as well as the travel decisions of the travelers such that his payoff is maximized. Note that the travel utility always decreases as  $\theta$  increases, hence  $\theta^* = 0$ . In this case, the maximum utility can be obtained if the travelers distribute themselves between routes 1 and 2, i.e.  $s^* = \{(1,2), (2,1)\}$ . Hence, in this system optimum, the total travel utility in the system is 240. Note that this optimum would not occur if travelers have free choice, since  $\theta = 0$  yields a Nash-Wardrop equilibrium for both travelers to choose route 1.

#### 8.1.2. Stackelberg game

Let us formulate the Stackelberg game in the following way. The road authority may choose one of the following strategies for the toll level on route 2:

$Q : \{0 < Q < 12; 12 < Q < 120; Q > 120\}$

After the toll is being set by the road authority, the travelers react on this toll by reconsidering their travel choices; maybe choosing different routes. The extensive form of the Stackelberg game is shown in Figure 3.

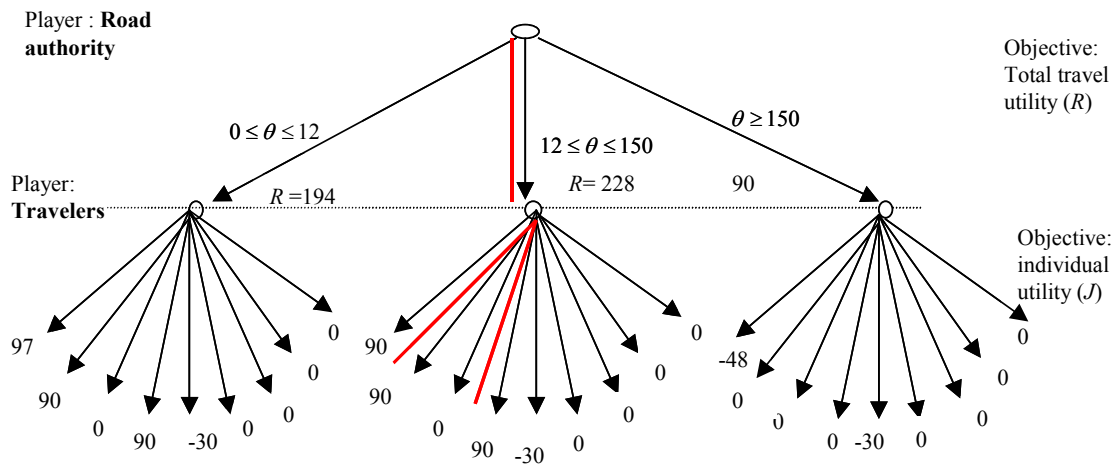
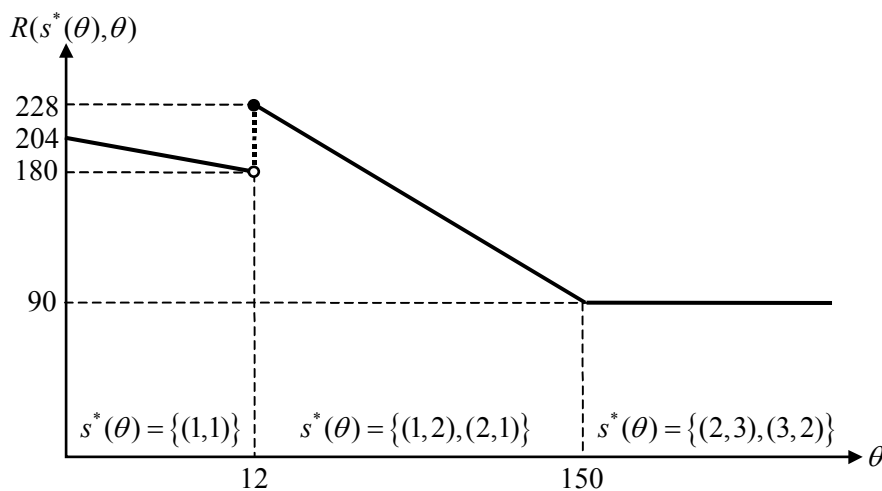


Figure 3 Outcomes of Stackelberg game applied to maximize the total travel utility

Now the travelers will maximize individually their own travel utility, depending on the toll set by the road manager. Figure 2 illustrates the total travel utility for different values of  $\theta$  with the corresponding optimal strategies played by the travellers. When  $0 \leq \theta < 12$ , travellers will both choose route 1. If  $12 \leq \theta < 150$ , travellers distribute themselves between route 1 and 2, while for  $\theta \geq 150$  one traveller will take route 2 and



another traveller will not travel at all. Clearly, the optimum for the road manager is  $\theta^* = 12$ , yielding a total travel utility of 228.

Figure 4 Total travel utilities depending on toll value for the objective of maximizing total travel utility

### 8.1.3. Cournot game

It can be shown that in case the travelers and the road manager have equal influence on each others strategies, multiple Cournot solutions exist. There is however one dominating strategy, being that the travelers both take route 1 and that the road manager sets zero tolls, yielding a total system utility of 204.

Table 3 summarizes the outcomes for the three different games.

Game	$\theta^*$	$s_i^*(\theta)$	$R$	$J_i$
Social planner	0	$\{(1,2),(2,1)\}$	240	$\{(90,150),(150,90)\}$
Stackelberg	12	$\{(1,2),(2,1)\}$	228	$\{(90,138),(138,90)\}$
Cournot	0	$\{(1,1)\}$	204	$\{(102,102)\}$

Table 3 Comparison of Outcomes of Different Games for the Objective of Maximizing Total Travel Utility

## 8.2 CASE STUDY 2: Maximize Social Surplus

Now, the road manager is assumed to maximize social surplus (see formula (11)). The strategy set of the road manager is assumed to be  $\Theta = \{\theta | \theta \geq 0\}$ . The payoffs for the road manager are presented in Table 4 depending on the strategy  $\theta \in \Theta$  that the road manager plays and depending on the strategies the travelers play.

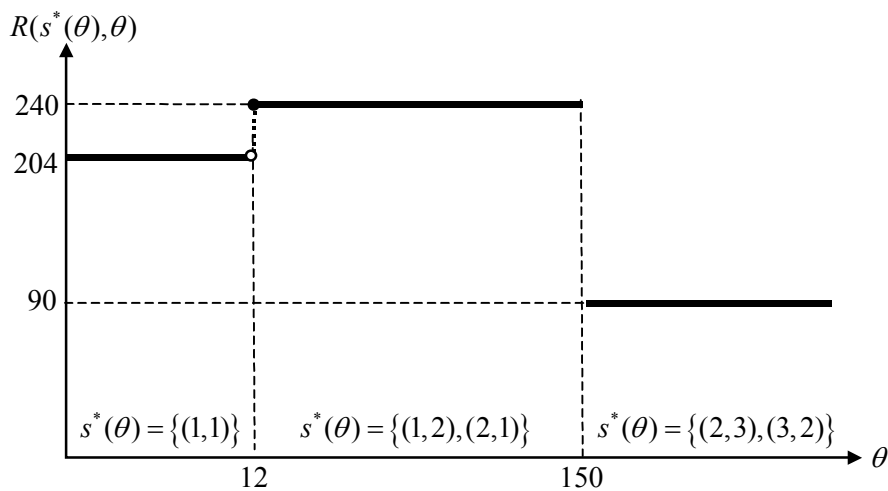
		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	204	240	150
	Route 2	240	-60	90
	Route 3	150	90	0

Table 4 Utility Payoff Table for the Road Manager if his Objective is to Maximize Social Surplus



### 8.2.1 Stackelberg game

Now the travelers will maximize individually their own travel utility (see formula (9)). Figure 5 illustrates the social surplus for different values of  $\theta$  with the corresponding optimal strategies played by the travelers. When  $0 \leq \theta < 12$ , travelers will both choose route 1. If  $12 \leq \theta < 150$ , travelers distribute themselves between route 1 and 2, while for  $\theta \geq 150$  one traveler will take route 2 and another traveler will not travel at all. Clearly, the optimum for the road manager is  $12 \leq \theta^* \leq 150$ , yielding a total



system utility of 240.

Figure 5 Total travel utilities depending on toll value for the objective of maximizing social surplus

### 8.3 CASE STUDY 3: Maximize revenues

Now, let us add the road manager as a player, assuming that he tries to maximize revenues (see formula (10)). The strategy set of the road manager is assumed to be  $\Theta = \{\theta \mid \theta \geq 0\}$ . Depending on the strategy  $\theta \in \Theta$  that the road manager plays and depending on the strategies the travelers play, the payoffs for the road manager are presented in Table 5.

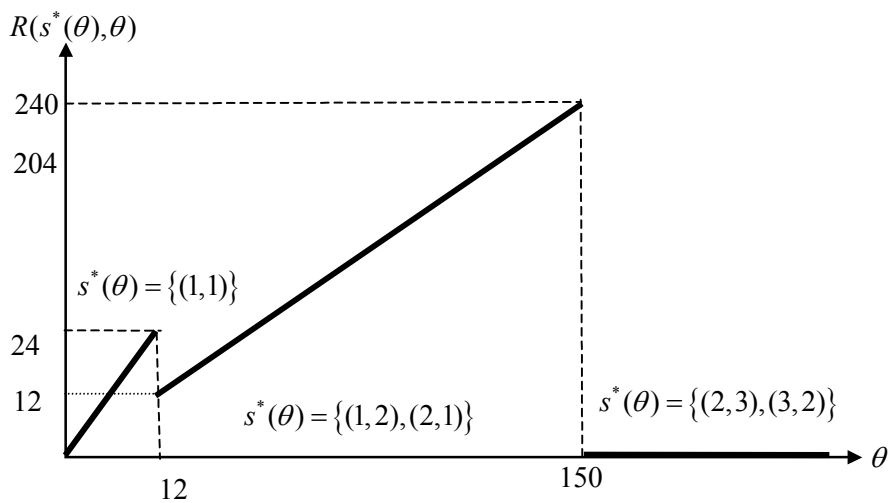
		Traveler 2		
		Route 1	Route 2	Route 3
Traveler 1	Route 1	$2\theta$	$\theta$	$\theta$
	Route 2	$\theta$	-60	90

	Route 3	$\theta$	90	0
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Table 5 Utility Payoff Table for the Road Manager if his Objective is to Maximize Revenues

### 8.3.1 Stackelberg game

Figure 6 illustrates the revenues for different values of  $\theta$  with the corresponding optimal strategies played by the travelers. When  $0 \leq \theta < 12$ , travelers will both choose route 1. If  $12 \leq \theta < 150$ , travelers distribute themselves between route 1 and 2, while for  $\theta \geq 150$  one traveler will take route 2 and another traveler will not travel at all. Clearly,



the optimum for the road manager is  $\theta^* = 150$ , yielding a total system utility of 240.

Figure 6 Total travel utilities depending on toll value for the objective of maximizing revenues

Considering all three case studies some conclusions can be drawn:

- There exist different objectives that all can be applied depending on what the road authority would like to achieve;
- Different objectives lead to different outcomes, both in terms of optimal toll system as well as in payoffs for players;
- Looking at different game types shows the span of outcomes of an optimal design and their relative worth;
- There exist multiple optimal solutions (multiple Nash equilibria)
- The objective function may have a non-continuous shape (jumps)

## 9. Conclusions and further extensions

The purpose of the paper was to gain more insight into the road-pricing problem using concepts from game theory as well as different toll designs depending on different objectives. To that end we presented the notions of game theory and presented three different game types in order to elucidate the essentials of the game theoretic approach. These game types were applied to three different toll design objectives exemplified on a simplistic demand-supply network system. This clearly revealed differences in design results in terms of toll levels and payoffs for involved actors, being the road authority and network users.

The theory presented here can be extended to include other relevant travel choices such as e.g. departure time choice as well as to include heterogeneous travellers and imperfect information on the part of the road users. An important extension is to apply the proposed game-theory framework to large cases (e.g. for large number of players or on bigger network). For practical use, the presented game-theoretic analysis should be translated into a modelling system with which tolling designs for real-size road networks become feasible. For that purpose, the bi-level optimisation framework will be used (see (18)).

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