

## P1A.1

### Drop-Size Distribution Retrieval from Polarimetric Radar Measurements Using the Constrained-Gamma Model: Verification and Application

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#### 1. Introduction

Dual-polarization radars typically transmit horizontally and vertically polarized electromagnetic waves and receive backscattered signals that differ for the two polarizations. Changes in signal properties as the waves propagate respond to hydrometeor size, shape, and orientation. The measurements can be used to retrieve the governing parameters of assumed drop-size distribution (DSD) models. The retrieval model examined here is an adaptation of that described by Zhang et al. (2001). The theoretical basis for the method is discussed in an accompanying paper (Zhang et al. 2003). Here the method is briefly described, and retrieved DSD parameters are verified by comparison with video disdrometer observations. The model is then applied to study the spatial distribution of DSDs within storms.

#### 2. The constrained-gamma DSD model

It is assumed that raindrops can be represented by the gamma distribution (Ulbrich 1983)

$$N(D) = N_0 D^\mu \exp(-\lambda D) \quad ,$$

where  $N_0$  is a number concentration parameter,  $\mu$  is a distribution shape term, and  $\lambda$  is a slope parameter. Retrieval of the three DSD governing parameters requires three measurements or relationships. The constrained-gamma method of Zhang et al. (2001) uses the measurements and definitions of radar reflectivity at horizontal and vertical polarization

$$Z_{H,V} = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_0^{D_{\max}} N(D) |f_{a,b}(D)|^2 dD \quad ,$$

where  $f_{a,b}$  are the backscattering amplitudes along the major ( $a$ ) and minor ( $b$ ) drop axes; the differential reflectivity

$$Z_{DR} = \frac{Z_H}{Z_V} \quad ;$$

and an empirical relation between  $\mu$  and  $\lambda$  that was derived from disdrometer observations (see also Brandes et al. 2003)

$$\lambda = 1.935 + 0.735\mu + 0.0365\mu^2 \quad . \quad (1)$$

The DSD can be found by iterating between  $Z_{DR}$  and the  $\mu$ - $\lambda$  relation to find  $\mu$  and  $\lambda$  and then using  $Z_H$  to find  $N_0$ . Once the DSD is known, physical parameters, such as the total drop concentration ( $N_T$ ,  $m^{-3}$ ), drop median volume diameter ( $D_0$ , mm), rainwater content ( $W$ ,  $g\ m^{-3}$ ), and rain rate ( $R$ ,  $mm\ h^{-1}$ ), can be computed directly from integrals of the DSD. An alternative is to derive a simple set of estimators using the constrained-gamma model. Radar and rain parameters were calculated for  $\lambda$  in the range 0.5 to 13 and for a fixed value of  $N_0$ . Ratios of the parameters  $N_T$ ,  $W$ , and  $R$  with  $Z_H$  are independent of  $N_0$  and functions of  $\mu$  or  $\lambda$  determined by  $Z_{DR}$  alone for the constrained-gamma model. Taking logarithms of the ratios and fitting them with polynomial relations yields

$$N_T = 2.085 Z_H \times 10^{(0.728 Z_{DR}^2 - 2.066 Z_{DR})}$$

$$W = 5.589 \times 10^{-4} Z_H \times 10^{(0.223 Z_{DR}^2 - 1.124 Z_{DR})}$$

$$R = 0.00760 Z_H \times 10^{(0.165 Z_{DR}^2 - 0.897 Z_{DR})} \quad .$$

The units of  $Z_H$  and  $Z_{DR}$  are linear ( $mm^6\ m^{-3}$ ) and dB, respectively. Similarly, we obtain

$$D_0 = 0.171 Z_{DR}^3 - 0.725 Z_{DR}^2 + 1.479 Z_{DR} + 0.717 \quad .$$

These relations are unique for the particular axis

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ratio relation used (Brandes et al. 2002) and the constrained-gamma model [Eq. (1)]. They are valid for  $Z_{DR} < 2.5$  dB.

### 3. Retrieval comparison with disdrometer observations

Figure 1 shows radar reflectivity associated with two short lines of moderate thunderstorms observed in east central Florida during the PRECIP98 field program. The southern line passed over a video disdrometer located 38 km from NCAR's S-Pol radar. The storms moved westward (to the left).

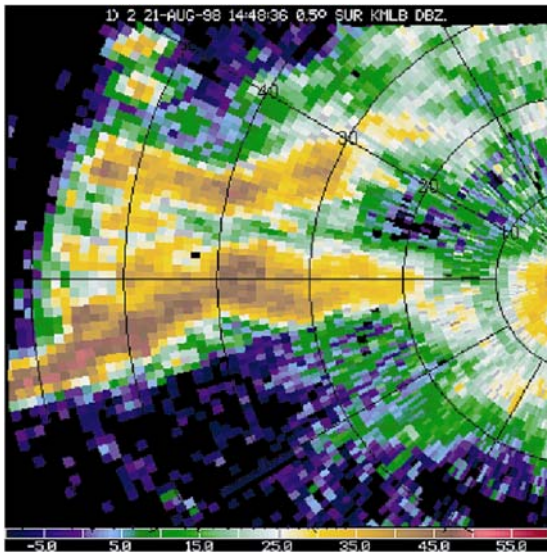


Fig. 1: Radar reflectivity measurements of moderate thunderstorms, as seen with the Melbourne WSR-88 at 1448 UTC on 21 August 1998.

A time series of  $Z_H$ ,  $Z_{DR}$ ,  $N_T$ , and  $D_0$  as measured or retrieved from radar with the above polynomial relations and computed from disdrometer observations is given in Fig. 2. Traces for  $Z_H$  and  $Z_{DR}$  are well matched. The range of values with the disdrometer is a little larger, probably due to the smaller sampling volume. Radar-derived drop concentrations are a little smaller than that with the disdrometer. No bias is evident in the  $D_0$  retrieval. Trends agree nicely for all parameters. Additional radar–disdrometer comparisons are given by Zhang et al. (2003) and Brandes et al. (2003). A DSD retrieval comparison with a wind profiler is described by Ellis et al. (2003). It is clear that the constrained-gamma method provides useful information regarding DSDs.

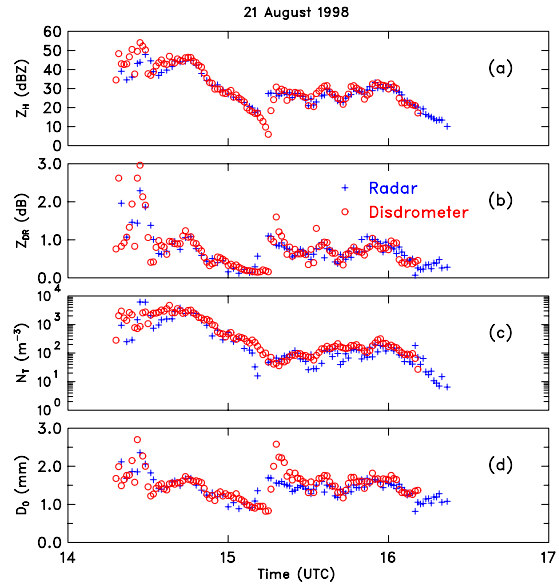


Fig. 2: Reflectivity ( $Z_H$ , panel a), differential reflectivity ( $Z_{DR}$ , panel b), total drop concentration ( $N_T$ , panel c), and drop median volume diameter ( $D_0$ , panel d) as measured or retrieved with the S-Pol radar (pluses) and computed from disdrometer observations (circles). The radar antenna elevation was  $0.5^\circ$ .

### 4. Moderate thunderstorm examples

The wind field (storm relative) for the thunderstorms depicted in Fig. 1 is presented in Fig. 3. At 0.5 km the prevailing flow is from the storm's left rear (east-southeast). The flow accelerates downwind indicating that the storms are dissipating. Winds throughout much of the southern band veer (become more southerly) with height (e.g., 2 km). Winds in the northern band become northeasterly at 2 km. The convergence sustains elevated updrafts that dominate above 2 km (not shown). Reflectivity maxima in the southern band slope southward with height, while those in the northern band slope eastward.

Retrieved DSD physical parameters are shown in Fig. 4. Inspection reveals that peak drop concentrations in the southern convective line are  $3000\text{--}5000\text{ m}^{-3}$ . Maximum drop concentrations in the northern line are about  $3000\text{ m}^{-3}$ . Drop concentrations at the edge of the storms are as small as  $30\text{--}100\text{ m}^{-3}$ . Estimated drop median volume diameters in the core of the southern storm (the region with reflectivity  $>40$  dBZ) are fairly uniform between 1.4 and 1.8 mm. The storm area with  $D_0 > 1.6$  mm is displaced slightly from the region  $>40$  dBZ. This could be a size sorting effect where the largest drops are the first to fall

21 August 1998, 1448 UTC

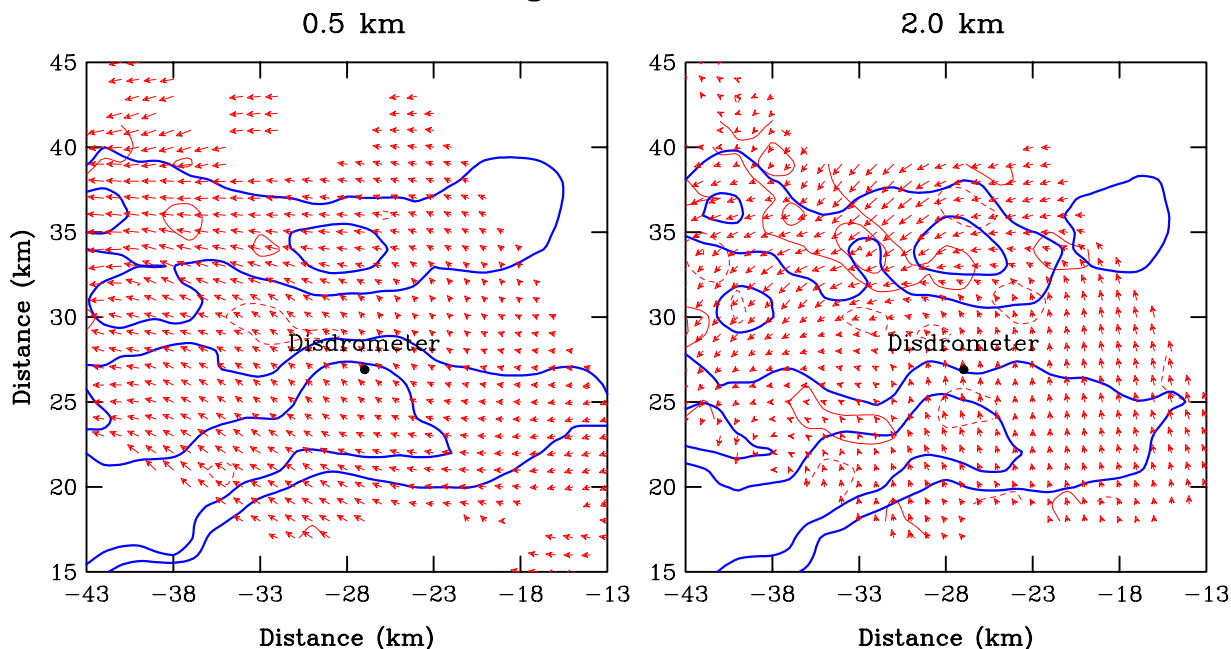


Fig. 3: Storm-relative dual-Doppler wind field analyses at 0.5 and 2 km height for storms observed on 14 August 1998. Reflectivity measurements from the Melbourne, Florida WSR-88D (30 and 40 dBZ) are shown (heavy solid contours). Thin solid (dashed) contours show updrafts (downdrafts). Contour intervals are  $0.5 \text{ m s}^{-1}$  at 0.5 km and  $1 \text{ m s}^{-1}$  at 2 km

from the elevated core.

Rainwater contents with the southern convective band are mostly  $0.5\text{--}3 \text{ g m}^{-3}$  and rainfall rates are all  $<60 \text{ mm h}^{-1}$ . An interesting feature is that the  $1 \text{ g m}^{-3}$  rainwater content contour nearly coincides with the 40-dBZ contour. This is also true for the  $20\text{-mm h}^{-1}$  rain rate contour. The relationship is altered slightly in the northern band. The correspondence suggests a reduced dependence on differential reflectivity in this case.

## 5. Summary and conclusions

An overview of the constrained-gamma method for retrieving DSD information was given, retrievals were verified with disdrometer observations, and the model was applied to an event with moderate thunderstorms. The method uses radar reflectivity, differential reflectivity, and an empirical relation between the shape and slope parameters of the gamma DSD.

Overall, good agreement was found between the retrievals and the disdrometer observations. Additional radar–disdrometer comparisons are

needed, preferably at short radar ranges to reduce sampling differences. The differential propagation phase measurement is not currently used for retrieval purposes. In theory, the measurement is sensitive to total rain content and drop shape. Perhaps the measurement can be used as an additional constraint to reduce bias by adjusting the axis ratio relation (Zhang et al. 2003). Also, retrievals at the leading edge of some convective storms often indicate drop concentrations that seem too high ( $N_T > 10^{4.5} \text{ m}^{-3}$ ). The problem is believed to arise from DSDs that are dominated by small numbers of very large drops and not well represented by the constrained-gamma model. Regardless, this study shows that useful DSD information can be deduced from polarimetric radar measurements.

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DSD Parameter Retrieval  
21 August 1998, 1448 UTC

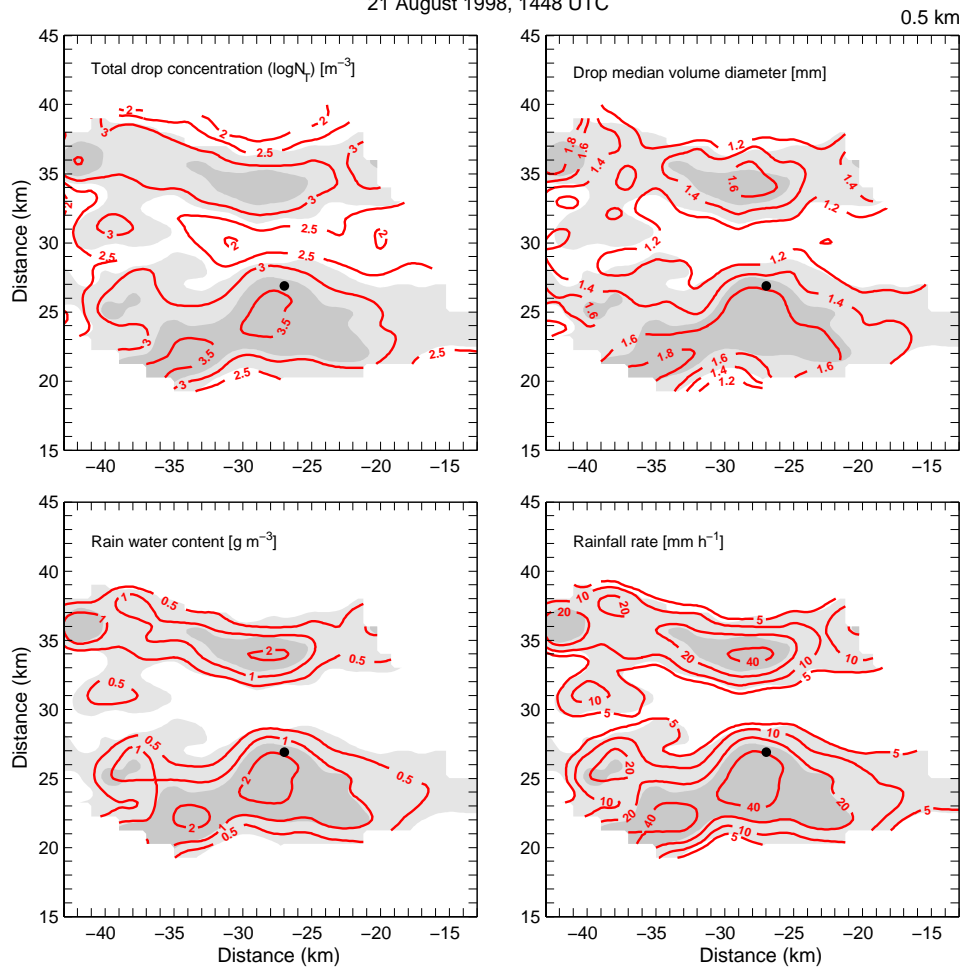


Fig. 4: Retrieved total drop concentration, drop median volume diameter, rain water content, and rain rate at 0.5 km for the storms in Fig. 3. Light (dark) shading shows radar reflectivity  $\geq 30$  dBZ ( $\geq 40$  dBZ) as measured with the S-Pol radar. The disdrometer site is shown by a dot.

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