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A Comparison of US and Chinese Middle Grades Textbook Development of Fraction Concepts

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Abstract: This study compared US and Chinese middle grades textbooks using a theoretical framework developed by the American Association for the Advancement of Science for textbook analysis. Document analysis techniques were used to examine both textbooks using three criteria: developing mathematics ideas, promoting thinking in mathematics, and engaging students in mathematics. The US and Chinese textbooks differed significantly in all three areas. The US textbook employed more hands-on activities to develop fraction concepts than the Chinese textbook. The US textbook focused on part-whole and measure sub-constructs, while the Chinese textbook focused on part-whole and quotient sub-constructs. The US textbook contained more factual questions and fewer word problems than the Chinese textbook. The US textbook used many multiple representations, including real world connection, hands-on activities, and pictorial representations.

Key words: Chinese textbook; US textbook; Fraction; Representation; Comparison

Introduction

International comparison studies on curriculum materials reveal three levels of curriculum: the intended curriculum [national/state standards and textbooks], the enacted curriculum [classroom practices], and the attained curriculum [student achievement] (McKnight, et al., 1987; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Valverde, Bianchi, Wolfe, Schmidt, Houang, 2002). The results from the Third International Mathematics and Science Study [TIMSS] reported that East Asian eighth- graders out-performed their US counterparts (McKnight, et al., 1987; Schmidt, et al., 1997). The mathematics education community tends to agree that in the US, the textbook is the primary curriculum resource for lesson planning and classroom instruction (American Association for the Advancement of Science [AAAS], 2000; Chandler, 1995; Oliva, 2001). In addition, textbook quality was reported to be a major predictor of student achievement in studying number and algebra (Kulm and Capraro, 2008). Therefore, examination of textbooks could shed light on the reasons for the achievement gap between China and the US.

The differences in the scope of mathematics topics covered in textbooks have been the focus of textbook comparison studies (Cai, Lo, Watanabe, 2002; Li, 2002; Kulm, 2000). Criteria and strategies for evaluating textbooks have emerged since the late 1990s. For example, the AAAS (2000) developed a set of textbook analysis criteria and successfully applied them to evaluate US middle grades (Kulm, Roseman and Treistman, 1999) and algebra (Kulm, 2000) textbooks. Trafton, Reys and Wasman (2001) also proposed a six-criterion evaluation system for curriculum analysis. However, few scholars have conducted comparison studies using these criteria developed by AAAS. Number and operation has been reported as one of the topics where an achievement gap existed between students in East Asia and US (Cai, 1995; Schmidt, et al. 1997; Li, 2002). Thus, based on our research interests and experiences, we examined the differences between US and Chinese middle grades textbooks on fraction concept development. The following research questions were investigated:

1. What fraction concepts are developed in selected Chinese and USA middle grades textbooks?
2. How does each textbook promote student thinking about fractions?
3. How does each textbook engage students in learning fractions?

Review of Literature

International Comparison Studies between China and USA

TIMSS reported that US textbooks covered more mathematics topics than other countries and characterized the curriculum as “a mile wide and an inch deep” (McKnight, et al., 1987; Schmidt, et al., 1997; Valverde, et al., 2002). Several other studies have been conducted to examine the differences and similarities between Chinese and US textbooks. For example, Sun (2000) studied two sets of textbooks from the US and China, and found that US textbooks used more pictures and real world presentations than Chinese textbooks. Cai, Lo and Watanabe (2002) conducted research on how US and East Asian (including Chinese) textbooks present the concept of average. They found that US textbooks tended to present the concept of average in context of statistics, while Chinese textbooks tended to present the idea of average in the context of number and operations. They also found that the Chinese textbooks focused more on conceptual understanding than

US textbooks. Li (2002) examined the differences in integer addition and subtraction representations across US and Chinese middle grade textbooks, and pointed out that the performance requirements between the selected textbooks across countries were substantially different. Li's study stated that 36% of Chinese textbook problems focused on operations that involved rational numbers, while none of the US textbooks required students to work with fractions. Li reported somewhat different levels of cognitive demand, finding that 72% of Chinese textbook examples required the application of procedural understanding, while 63% of US textbooks required the same knowledge. The requirement for problem solving skills was low for both US and Chinese textbooks. Fan and Zhu (2007) examined problem solving strategies middle school textbooks adopted in China, Singapore, and US, and they found out that Chinese textbook basically demonstrates one problem solving stage (how to carry out the plan), while US textbook involves more stages of problem solving. Dole and Shield (2008) reported considerable amount of real world applications in a comparison study conducted in Australia.

Conceptual Understanding of Fractions

According to Van De Walle, Karp, and Bay-Williams (2010), understanding is defined as "a measure of the quality and quantity of connections that an idea has with existing ideas" (p. 23). These authors further defined conceptual understanding as "the knowledge about relationships or foundational ideas of a topic" (p. 24). They explained that if a student understands all of the sub-constructs of a concept and understands the connection between those sub-constructs, then that student will achieve the goal of understanding.

The sub-constructs of fractions have been studied for more than three decades. Scholars have proposed different views of the sub-constructs of fractions. For example, Kieren (1976) stated that fraction concepts included fractions, decimal fractions, equivalent fractions, quotient form a/b, multiplicative operations, and discrete relationship. Other researchers, including Kieren, continued to refine the sub-constructs of fractions. Later, researchers tended to agree that there are five sub-constructs of the fraction concept: part-whole, measure, division, operator, and ratio (Van De Walle, Karp, & Bay-Williams, 2010).

Misconceptions of Fractions

The introduction and development of the concept of fraction in the middle grades provides a foundation for the future study of algebra, which has been characterized as the "gate keeping" course for higher level mathematics courses (Moses & Cobb, 2002). However, fraction concepts are also reported to be one of the most difficult topics for students (Bay, 2001). Wearne and Kouba (2000) reported that more than half of children cannot solve problems involving fractions. For instance, writing mathematical symbols seemed to be difficult for students, especially when they are required to compare fractions, write a mathematics symbol to represent a fraction, or solve a word problem involving fractions (Lesh, Behr, & Post, 1987).

Some common misconceptions involving fractions are related to a fraction being interpreted as part of a whole. For example, students who are first introduced to fractions might think that 1/3 is larger than ½. They will argue that 1/3 means that a pizza was divided into 3 pieces, while ½ means a pizza is divided into 2 pieces, reasoning that 3 pieces are more than 2 pieces (Post, & Cramer, 1987). This misconception arises because students fail to make connections between the part-whole sub-construct and fraction concepts. Children likely know from real life sharing experiences that you will get more pizza if you share with fewer people. Researchers have suggested that misconceptions can be avoided or corrected through more attention to the conceptual understanding of fractions. Clarke, Roche, and Mitchell (2008) also stated that highlighting sub-constructs of fractions (i.e., part-whole, measure, division, operator, and ratio) will help students to develop conceptual understanding. Other researchers stressed the importance of making connections between different representations (Behr, Lesh, Post, & Silver, 1983).

Framework for Textbook Analysis

Researchers have developed criteria for textbook analysis. For example, AAAS (2000) developed comprehensive criteria including seven categories and 24 sub-categories for middle school mathematics textbook analysis. The categories are: 1) identifying a sense of purpose, 2) building students ideas about mathematics, 3) engaging students in mathematics, 4) developing mathematics ideas, 5) promoting student thinking in mathematics, 6) assessing student progress in mathematics, and 7) enhancing mathematics learning environment. Trafton, Reys, and Wasman (2001) proposed a six-category analysis framework including comprehensibility, coherence, development of ideas in depth, promotion of sense-making, engagement of students, and motivation for learning.

The present study adapted three of the six AAAS criteria for the following reasons. First, a close examination of the analysis AAAS and Trafton, et al. work revealed that they have three common criteria: developing mathematics ideas, promoting students thinking in mathematics, and engaging students in mathematics. Most important, in the AAAS study, the three selected criteria were the most effective in differentiating between higher and lower-rated textbooks (AAAS, 2000).

Methods

Selection of Textbooks

The following criteria were used to select a middle grades textbook from China and the US. First, the two textbooks should include the introduction to the concept of fraction. Second, the textbooks would be considered as high quality textbooks in the public schools in each country. Third, the textbooks would be widely used in each country. Fourth, the textbooks should be published about the same time. The two textbooks selected were *Mathematics* developed by the People's Education Press (PEP, 2000), and *Connected Mathematics: Bits and Pieces I* (Lappan, Fey, Fitzgerald, Friel, & Pillips, 1998). *Mathematics* has been the most popular textbook in China since its publication. It is also considered as a high quality textbook (research committee, 2002). *Bits and Pieces I* is a unit from the *Connected Mathematics,* a reform-oriented textbook developed at through a project funded by National Science Foundation [NSF]. It is considered as a high quality textbook by AAAS middle school textbook analysis project (AAAS, 2000). The analysis used the student textbook only, without including teacher guides or supplementary materials.

Analysis Criteria

The following summaries provide descriptions of the three analysis criteria.

Developing the Fraction Concept

The study used two indicators for the criteria for developing fraction ideas: Modeling and Use of Representations. For Modeling the textbook provided information or demonstrations on how to carry out a procedure. For example, textbook outlines a formula, or step by step guideline how to work out mathematics problems. Use of Representations required the textbook to provide a representation of one or more of the five fraction sub-constructs: part-whole, measure, division, operator, or ratio.

Promote Student Thinking about Fractions

The two indicators used for this criterion were Question Posing and use of Word Problems. For question posing, we examined if the textbook used questions to demonstrate or elicit mathematics thinking and reasoning. If only knowledge recall was required, we coded it as factual questions. If the questions demand students to justify their answers, we coded it as high-level questions. Word problem refers to the language context of problem.

Engaging Students in Learning Fractions
The indicator for Engaging Students was the variety of contexts the textbook used, including real-world situations, graphs, pictures, or symbols to present fraction concepts.

Analysis Procedures
The first step in the analysis was to identify the pages in each textbook that addressed the introduction of the fraction concept. If a page included one of the five fraction sub-constructs, it was included. Next, each selected page was examined to determine which of the three criteria were addressed. The first author and a second researcher, who are both fluent Chinese and English, read and coded the selected pages independently. When there was disagreement, they compared notes and discussed differences until they reached an agreement (Lincoln & Guba, 1985).

Results

The results of the analysis for each of the three research questions are provided in the following sections.

Developing Fraction Ideas
Table 1 provides a summary of the results for the criteria developing mathematics ideas.

The US textbook emphasized the measure and part-whole sub-constructs. For example, 55.6% of the content involved fraction strips as a measurement tool, while 30.5% of the content was devoted to the part-whole sub-construct. The Chinese textbook focused on the sub-construct of fraction as a division (66.7%) and fraction as a part-whole (27.8%). The US textbook tended to present part-whole and measure sub-constructs together in a real world context. The Chinese textbook connected part-whole and division with an emphasis on equal sharing.

Table 1
Percentage of Content in US and Chinese Textbooks for Developing Fraction Concepts

Criterion	Indicators		Percent of pages*	
			Chinese	USA
Develop Math Ideas	Modeling Concepts		55.6 %	66.7%
	Use of Representations	Part-whole	27.8%	30.5%
		Measure	19.4%	55.6%
		Division	66.7%	13.9%

*Percents do not total 100 since more than one criterion is present on each page.

Promote Thinking about Fractions

Table 2 presents the percentages of the types of questions each textbook devoted to promoting thinking about fraction ideas. The US textbook focused more on factual questions than the Chinese textbook (40.6% vs. 29.4%). The Chinese textbook included more word problems than the US textbook (44.1% vs. 40.6%). While both are somewhat low (26.5% vs. 18.8%), the Chinese book used more questions that required an explanation or justification, providing students with an opportunity to think about mathematics.

Table 2
Percentage of Content in US and Chinese Textbooks for Promoting Thinking about Fractions.

Criterion	Indicators		Percent of pages	
			Chinese	US
Promoting Thinking about Fractions	Question Posing	Factual	29.4%	40.6%
		Justifications Required /provided	26.5%	18.8%
	Word Problems		44.1%	40.6%

*Percents do not total to 100 since more than one criterion is present on each page.

Engaging Students in Learning Fractions

Table 3 summarizes the results for engaging students in learning fractions. The US textbook used more real world representations (51% vs. 11.1%) and pictorial representations (55.6% vs. 34.7%) than the Chinese textbook. The US textbook also used hands-on activities to engage students while Chinese textbook did not have any hands-on activities. For example, the US textbook asked children to first construct and fold fraction strips, then to use the fraction strips to understand the part-whole sub-construct. The Chinese textbook had somewhat more symbolic representations (66.7%) than the US textbook (55.6%).

Table 3

Percentage of Content in US and Chinese Textbooks for Engaging Students in Learning Fractions.

Criterion	Indicators		Percent of pages	
			Chinese	US
Engaging Students in Learning Fractions	Variety of Contexts	Real World	11.1%	40.6%
		Hands-on	0	27.8%
		Pictures	13.9%	34.7%
		Symbols	66.7%	55.6%

*Percents do not total to 100 since more than one criterion is present on each page.

Discussion

Because only one representative textbook from each country was selected for comparison, the results might not generalize to other textbooks published from these two countries. The US textbook was intended to reflect reform-based ideas and had different emphases from other textbooks that are widely used in the US (AAAS, 2000). Finally, this study examined only the introduction and early development of the concept of fractions and not other parts of the textbooks in which fractions were applied and used with other mathematics concepts. We would also like to point out that we do not aim to judge the textbooks in terms which one is better, we presented the differences of each textbook. A perfect textbook does not exist, in our opinion.

The results of the present study showed that the selected US and Chinese textbooks were quite different in terms of developing mathematics ideas about fractions, promoting thinking in math, and engaging students in mathematics.

The US textbook focused on modeling mathematics concepts through hands-on activities. This result may reflect the emphasis in the US on constructivism, building from concrete activities to more abstract understanding (Confrey, 1990). The US textbook represented the fraction concept as part of a whole and measure sub-constructs, while the Chinese textbook represented fraction concept as part of a whole and division sub-constructs. These differences might due to the fundamental differences in the mathematics standards in these two countries. For example, *Chinese National Mathematics Teaching and Learning Syllabus* emphasized whole number and fraction operations, where the division sub-construct would naturally fits the requirement. However, US *National Standards for School Mathematics* specifically stated that it is important to explore the measurement model in teaching and learning fractions (NCTM, 2000).

The US textbook focused more on factual questions than the Chinese textbook, which supports research findings from US classrooms in which the majority of questions are factual (e.g., Sahin, 2008). The US textbook also included fewer word problems than the Chinese textbook percentagewise. Since researchers suggested that word problem related to develop conceptual understandings (Van De Walle, Karp, & Bay-williams, 2010). This result is similar to Cai, Lo and Watanabe's (2002) finding that Chinese textbooks focused more on the conceptual understanding.

For the criteria of engaging students in mathematics, the US textbook provided many real world situations for students. It also included more hands-on activities and pictures in the textbook than the Chinese textbook. This finding is congruent with Sun's (2000) finding that US textbooks used more hands-on activities and real world situations to engage students than Chinese textbook. The Chinese textbook included more symbolic representations than the US textbook. These results support the results of Li (2002) who also reported that many Chinese textbook problems emphasized symbolic rational number operations.

We know that US students underperform their international counterparts in mathematics, especially those in East Asian. This study revealed the differences in fraction concept development, which might shed light on some reasons that possible led to these differences in performance. Since textbook is a predictor of students achievement Kulm,& Capraro (2008), more emphasis on different fraction sub-constructs, word problems, and variety of context could be added in the curriculum or in math teaching to compensate for the shortcomings of textbooks.

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