Tests of Relativity Using a Cryogenic Optical Resonator

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(Received 19 June 2001; published 14 December 2001)

A 190-day comparison of the optical frequencies defined by an optical cavity and a molecular electronic transition is analyzed for the velocity independence of the speed of light (Kennedy-Thorndike test) and the universality of the gravitational redshift. The modulation of the laboratory velocity and the gravitational potential were provided by Earth's orbital motion around the Sun. We find a velocity-dependence coefficient of $(1.9 \pm 2.1) \times 10^{-5}$, 3 times lower compared to the best previous test. Alternatively, the data confirm the gravitational redshift for an electronic transition at the 4% level. Prospects for significant improvements of the tests are discussed.

DOI: 10.1103/PhysRevLett.88.010401 PACS numbers: 03.30.+p, 07.60.–j

Special relativity (SR) is one of the fundamental theories of nature. The prominent role of the theory as a basis of our physical view of nature has motivated experimenters to test its foundations and predictions with ever increasing accuracy. Added motivation for tests is provided by the theoretical efforts to unify the forces of nature. For example, approaches towards a quantum theory of gravity have been put forward that lead to modified Maxwell equations which are not necessarily Lorentz covariant [1,2].

The relationship between gravity and the other forces of nature can also be probed by measuring the gravitational frequency shift ("redshift") of clocks based on these forces. The principle of local position invariance (LPI) [3] implies that the gravitational redshift is universal, i.e., independent of the type of clock. LPI, SR, and the weak equivalence principle are the ingredients of the Einstein equivalence principle (EEP). No simple way to quantize gravity has been found; schemes aiming at quantization are found to lead to violations of the EEP at some level of accuracy. It is therefore important to improve the tests of LPI.

Thanks to worldwide developments in frequency metrology and ultrastable oscillators, especially in the optical domain, the opportunity has arisen to improve the knowledge of SR and LPI by several orders of magnitude. Here we report on a first step in this direction, based on a laboratory experiment. We remark that space experiments have also been proposed [4,5].

Test of special relativity.—According to the kinematical analysis of Robertson [6] as well as Mansouri and Sexl [7], SR follows unambiguously from experiments establishing the isotropy of space (Michelson-Morley experiments), the independence of the speed of light from the velocity of the laboratory [Kennedy-Thorndike (KT) experiments], and time dilation (Doppler spectroscopy experiments). The KT test plays a special role, since at present its accuracy is by far the lowest of the three, thus limiting the overall knowledge on the validity of SR. An improved KT experiment is, thus, the key experiment for a better verification of SR.

A violation of the constancy of the speed of light implies a dependence of $c(\mathbf{v})$ on the magnitude v of the laboratory velocity relative to a hypothetical preferred frame of reference Σ, and on the angle θ between the propagation direction of the light and the direction of $\mathbf{v}$. The natural candidate for Σ is the cosmic microwave background. From isotropy in the preferred frame Σ, it follows that $c(\mathbf{v})$ is an even function of v. According to common test theories [6,7], the dependence can be parametrized as

$$\frac{c(v,\theta)}{c_0} = 1 + A\frac{v^2}{c_0^2} + B\frac{v^2}{c_0^2}\sin^2\theta, \tag{1}$$

where c_0 is the constant speed of light in the preferred frame Σ. A and B vanish if SR is valid. In the test theory framework of [6,7], A and B can be expressed in terms of parameters entering the Lorentz transformations, $A = \alpha - \beta + 1$, and $B = \beta - \delta - 1/2$. The three parameters α, β, γ can be experimentally determined by the three types of tests described above. $|B| < 5 \times 10^{-9}$ has been determined in a Michelson-Morley experiment [8]. As this limit is over 100 times better than the current limit for A, we will assume $B = 0$ in the following.

The laboratory velocity $v(t)$ has contributions from the motion of the Sun through Σ with a constant velocity $v_s = 377$ km/s, Earth's orbital motion around the Sun (orbital velocity $v_e = 30$ km/s), and Earth's daily rotation (velocity $v_d \approx 330$ m/s at the latitude of Konstanz),

$$v(t) = v_s + v_e \sin[\Omega_y(t - t_0)]\cos\Phi_E + v_d \sin[\Omega_d(t + t_d)]\cos\Phi_A. \tag{2}$$

Here $\Phi_A \approx 8°$ is the angle between the equatorial plane and the velocity of the sun. $\Phi_E = 6°$ is the declination between the plane of Earth's orbit and the velocity of the Sun [9], $2\pi/\Omega_y = 1$ yr, $2\pi/\Omega_d = 1$ sidereal day. t_0 and t_d are determined by the phase and start date of the measurement, respectively.

 0031-9007/02/88(1)/010401(4)$15.00

Only two KT tests have been performed thus far. In the original work, Kennedy and Thorndike used an interferometer and considered both the daily and the annual variation of v [10]. Hils and Hall introduced the modern technique of comparing the frequency of a stable electromagnetic resonator with the frequency ν_{mol} of an independent (optical) frequency standard achieving $|A| < 6.6 \times 10^{-5}$. They used the daily variation of v [11]. The resonance frequency $\nu_{\mathrm{res}} = mc(v)/2l$ of an optical resonator is determined by the ratio between the speed of light, the resonator length l, and the (constant) mode number m. If $c = c(v)$, then the frequency difference $\nu_{\mathrm{res}} - \nu_{\mathrm{mol}} \sim v(t)^2/c_0^2 + \mathrm{const}$ will exhibit a characteristic time signature when measured over a sufficiently long time [12].

Conventional stable optical cavities, as the one used in [11], are made from ultralow expansion glass ceramic. Because of relaxation processes in this noncrystalline material, the cavities exhibit drifts on the order of 1 kHz/h. This limits their use to searching for the daily variation of $c(v)$. In contrast, crystalline resonators operated at liquid helium temperature (COREs) have no discernible drift [13]. Their excellent long-term stability thus allows one to access the 100-fold larger orbital modulation of v.

Figure 1 shows the experimental setup, which has been described in detail elsewhere [14]. The frequency information ν_{res} of the CORE is transferred to a Nd:YAG laser which is frequency stabilized to it using the Pound-Drever-Hall method. As a reference, an electronic transition ν_{mol} between two rovibrational levels of different electronic states of the iodine molecule I_2 is used, by means of a second Nd:YAG laser stabilized to it via its second-harmonic wave.

The 3 cm long CORE consists of two dielectric mirrors optically contacted to the spacer. All three parts were fabricated from a single piece of a highly pure sapphire crystal. The finesse is ~100 000, corresponding to a linewidth of 50 kHz at 1064 nm. The CORE was kept at 4.3 K for the whole measurement. With a measured sensitivity of less than 0.1 Hz/μW to laser induced thermal frequency shifts and using <100 μW of laser power during the measurement, the related shift is less than 10 Hz.

The iodine reference is based on modulation transfer spectroscopy and employs a calibrated, 100 mm long iodine cell from the Physikalisch-Technische Bundesanstalt (Braunschweig). Operation at low iodine pressure of 4 Pa (at 273 K temperature) reduces pressure effects on the spectral line. With a measured sensitivity of the iodine frequency to temperature changes of approximately 2 kHz/K, temperature control to ±10 mK reduced this influence to maximum 20 Hz.

The pump beam is phase modulated at 455 kHz. The pump power was 5.8 mW; the probe power 600 μW. The power-broadened iodine linewidth was 1.8 MHz, approximately 4 times the natural linewidth. No systematic drift was observed on the time scale of a few hours, the Allan standard deviation being almost constant at about 6×10^{-13} for integration times between 1 and 10^4 s. These measurements used the CORE system as a reference. With a measured sensitivity of the iodine frequency to second-harmonic generation (SHG) power level of 25 kHz/mW and typically 5% relative intensity change over a few days, the iodine frequency varies by about 7 kHz. As the measured fluctuations in Fig. 2 are of that magnitude, we conclude that residual instabilities are mainly caused by these power drifts. To minimize them, the SHG power level was readjusted every few days.

The complete data set is shown in Fig. 2. The data starts on October 10, 1997, so $t_0 = -32$ days. The least square analysis of this data for a sinusoidal violation signal of a one year period (neglecting the daily contribution) leads to a signal amplitude of 1.36 kHz with a statistical error of ±1.5 kHz. As a check only, we removed 51 points that are probably polluted by SHG power drift. This does not affect the fit result by more than 10%. We also checked if the high density of data points between $t = 75$ and $t = 100$ days could lead to an unbalance of statistical weights. However, removing 2/3 of those data points (randomly chosen) for a better balance also does not affect the fit by more than 10%.

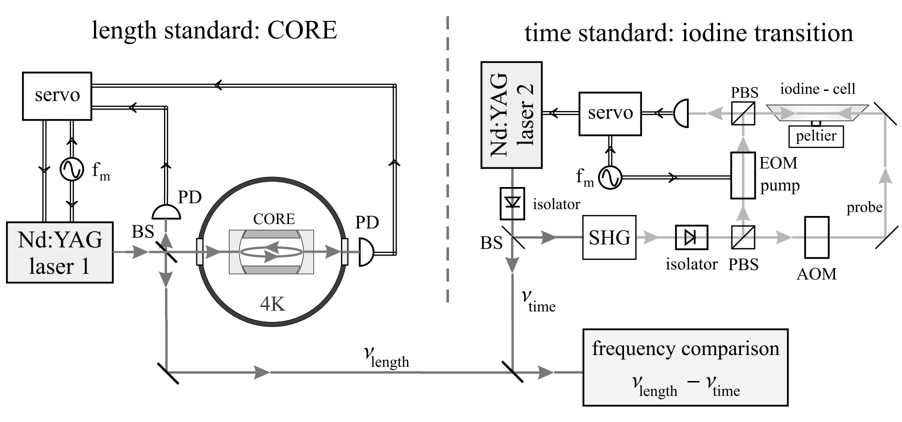

FIG. 1. Setup of the Kennedy-Thorndike experiment.

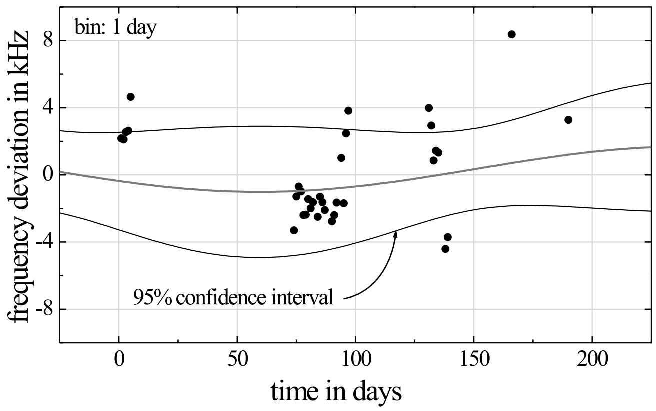

FIG. 2. Frequency difference $\nu_{res} - \nu_{mol}$ between an I_2 molecular frequency reference and a cryogenic optical resonator (CORE) over 190 days. One-day averages of the beat frequency are shown; no data points have been removed from the raw data. The central line is a least-square fit of a velocity-independence violation signal with an annual variation $a \sin[\Omega_y(t + t_d)] + b$, where a is the amplitude, and b an unknown offset. The starting time t_d is known from solar-system data. Two-sigma (95%) confidence intervals for the fit (dotted lines) are also shown. Fitting both a and b simultaneously to the data gives too high a statistical dependency ($\sim$0.9). Thus, b was determined independently as the average over the four data point groups near the zero crossing at $t = 133$ days, giving $b = (0.35 \pm 1.2)$ kHz. A subsequent fit of a yields $a = (-1.36 \pm 0.63)$ kHz. A change Δb of the offset b causes the fitted sine-wave amplitude to change linearly with $\Delta a \approx 1.125\Delta b$. The fit error of ± 0.63 kHz and the additional error caused by the unknown offset $\pm 1.125 \times 1.2$ kHz are therefore added in quadrature to give a total error of 1.5 kHz.

Given the laser frequency of $\nu_{res} = 282$ THz we have a relative signal amplitude $\Delta\nu/\nu = \Delta c/c = (4.8 \pm 5.3) \times 10^{-12}$, implying $A = (1.9 \pm 2.1) \times 10^{-5}$. This is compatible with zero within the accuracy of the experiment. This result represents a threefold improvement compared to the best previous test of Ref. [11].

Taking $\alpha = -1/2 \pm 1.7 \times 10^{-7}$ [15] and $\beta - \delta = 1/2 \pm 5 \times 10^{-9}$ [8] as established by complementary experiments, our data provides the improved limits $\beta = 1/2 \pm 2.1 \times 10^{-5}$ and $\delta = 0 \pm 2.1 \times 10^{-5}$. Our data thus improves the overall accuracy to which SR is verified by a factor of 3.

Test of local position invariance.—Assuming that SR is correct, we may interpret the measurement as a gravitational redshift universality experiment that tests LPI. A violation of the universality of the gravitational redshift effect implies a clock-type dependent magnitude of the relative frequency shift in a changing gravitational potential U. It can be parametrized as

$$\frac{\Delta\nu_C}{\nu_C} = (1 + \zeta_C)\Delta U/c^2, \tag{3}$$

where ζ_C is a clock-dependent coefficient.

As summarized in Table I limits on ζ_C have been established for a variety of clocks (i.e., oscillators), either by direct measurement of the redshift or by null experiments in which dissimilar clocks were moved together in a gravitational potential and their frequencies compared. Two previous experiments employed clocks in which a macroscopic length, arising from a solid structure, was a factor determining the frequency. In a laboratory test [16], a microwave oscillator stabilized to a superconducting cavity was compared with a Cs clock, as the laboratory moved in the gravitational field of the Sun. The measurement spanned ten days. In a space experiment, the frequency shift of an ultrastable quartz crystal oscillator in the Galileo spacecraft due to the solar gravitational potential was determined by comparison to Earth clocks [17].

Our test of LPI is a null test that compares a molecular reference frequency with an optical cavity, and thus employs a macroscopic oscillator as in the two tests mentioned above. However, it is the first test that employs an electronic transition, and the first purely optical test of LPI. The measurement yields an estimate for $\zeta_{res} - \zeta_{mol}$ from

$$\frac{\Delta\nu_{res} - \Delta\nu_{mol}}{\nu} = (\zeta_{res} - \zeta_{mol})\Delta U/c^2, \tag{4}$$

where $\nu \simeq \nu_{res} \simeq \nu_{mol}$ is the average frequency.

A test based on COREs can again make use of the orbital motion of Earth which gives rise to a modulation of the Sun's gravitational potential on Earth that is 1000 times larger than the rotation-related modulation. The time-dependent orbital contribution from the solar potential [18] is given by

$$\frac{\Delta U(t)}{c^2} = -\frac{2GM_S}{ac^2} e \cos[\Omega_y(t - t_{0,grav})], \tag{5}$$

where G is the gravitational constant, M_S the solar mass, a the semimajor axis of Earth's orbit, and e the eccentricity

TABLE I. Summary of selected LPI tests. The violation limits on $|\xi|$ refer to the clock types in the first column; to obtain the values, the results of complementary tests listed have been taken into account.

Clock type	Violation limit	Experiment type	Ref.
Nuclear transition	1×10^{-2}	Redshift measurement, ^{57}Fe-Mössbauer emitter vs ^{57}Fe	[25]
Hyperfine transition (H)	7×10^{-5}	Redshift measurement, H-maser vs H-maser	[23]
Hyperfine transition (Cs)	1×10^{-4}	Null test, Cs vs H-maser	[24]
Fine-structure transition	7×10^{-4}	Null test, Mg vs Cs	[19]
Vibrational oscillator	5×10^{-3}	Redshift measurement, quartz vs Cs	[17]
Vibrational transition	1.2×10^{-3}	Null test, OsO_4 vs Cs	[26]
Electromagnetic resonator	1.7×10^{-2}	Null test, Cs vs resonator	[16]
Electronic transition	4×10^{-2}	Null test, I_2 vs resonator	This work

[19], with $\frac{2GM_S}{ac^2}e \simeq 3.4 \times 10^{-10}$. $t_{0,\mathrm{grav}}$ is January 9, at which the attractive potential is minimum (closest approach to the Sun). The phase of an LPI violation signal thus differs by 30 or 150 days (depending on the sign of $\zeta_{\mathrm{res}} - \zeta_{\mathrm{mol}}$) from the phase of a velocity invariance violation signal. This implies that a violation signal, if it is strong enough, can, in principle, be ascribed to one of the two hypothetical effects.

To fit the data, we proceed as above and first determine the unknown offset by averaging the data points near the time of zero crossing of $\Delta U(t)$, i.e., April 10, obtaining 2.4 ± 1.2 kHz. The best-fit amplitude is (2.4 ± 0.9) kHz. The total error of the amplitude is thus ± 1.6 kHz. The universality violation limit follows as

$$|\zeta_{\mathrm{res}} - \zeta_{\mathrm{mol}}| < (2.2 \pm 1.5)\% . \tag{6}$$

In view of the previous results using macroscopic oscillators [16,17], we can interpret this as the first limit to the universality violation of the redshift of an electronic transition.

In our CORE-iodine comparison, the stability of the CORE could not be fully exploited; the experiment was clearly limited by the iodine reference. Thanks to the availability of the novel femtosecond frequency comb method [20], alternative references of much higher stability may be used. Using, e.g., a hydrogen maser stabilized on long time scales to the international time scale via the global positioning system (GPS) or a trapped ion microwave clock, we expect an improvement of the above limits by a factor of 10 in the near future. On the other hand, the near-future availability of clocks based on ultracold atoms/ions, with instability at the 10^{-16} level, gives significant impetus to develop the CORE method further to take advantage of their performance.

We also note that the optical clock frequency measurement programs under way in several laboratories (e.g., [21,22]) may also result in improved tests of LPI that are complementary to the one described here.

In summary, our experiment provides *new limits on violations of the Einstein equivalence principle*. We performed a Kennedy-Thorndike experiment by comparing a cryogenic optical resonator to an iodine frequency standard, which restricts a possible dependence of the speed of light on the laboratory velocity to $|A| = 1.9 \pm 2.1 \times 10^{-5}$. This result *improves the overall accuracy of the verification of special relativity by a factor of 3*. Alternatively, we may interpret the experiment as a null gravitational redshift experiment which tested the principle of local position invariance, as applied to length-based and electronic transition-based clocks, at the 4% level. Both tests made use of the large modulation amplitude due to the orbital motion of Earth. The potential for significant improvements of relativity tests in the near future using cryogenic optical resonators is clear.

We are indebted to Claus Lämmerzahl and Stefan Eggert for numerous discussions and thank Dr. Willy Nagl for help with the statistical evaluation of the data. This work was supported by the Gerhard Hess program of the Deutsche Forschungsgemeinschaft and by the Optik-Zentrum Konstanz.

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