Environmental Traffic Assignment: Developing Emission-based Models

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ABSTRACT

Vehicle tailpipe emissions are major sources of air pollution and greenhouse gases. In addition to the ongoing efforts on emissions reduction, we believe there is a need to explore an innovating approach in which drivers routing decisions are influenced to minimize emissions and fuel consumption. In order to evaluate such transportation systems, we develop environmental traffic assignment models (E-TA) based on user equilibrium (UE) and system optimal (SO) behavioral principles. Extending the traditional travel time based UE and SO principles to E-TA is not straightforward because, unlike travel time, vehicle emissions increase with the increase in vehicle speed beyond a certain point. The results of various TA models show a network-wide traffic control strategy in which vehicles are routed according to SO based E-TA, can reduce system wide emissions. However, a system in which drivers make routing decisions to minimize their own emissions, (E-UE system) results in increased individual as well as system-wide emissions.

KEY WORDS: Traffic Assignment, vehicle emissions, link cost function

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1. INTRODUCTION

Vehicle emissions are major sources of air pollution, especially in urban areas, and relatively large contributors to greenhouse gas (GHG) emissions. In 2003, the transportation sector share of GHG was 27% in the US (US Environmental Protection Agency, 2006). This share is expected to grow based on trends of increasing vehicle miles of travel. Many different factors are coalescing to motivate a need to reduce emissions and fuel consumption in the transportation system: concerns over global warming, national security of the energy supplies and fuel prices. This necessitates innovative approaches to emission reduction in addition to the stringent vehicle emission limits and fuel standards. Tailpipe emissions and fuel consumption are functions of vehicle speed, which is influenced by congestion, user decisions and network characteristics. Thus, there is a potential for reducing emissions and fuel consumptions by influencing drivers' routing decisions.

Traditionally, Wardrop's principles of equilibrium, called user equilibrium (UE) and system optimal (SO) are most commonly used to model drivers routing (Sheffi, 1984; Patriksson, 1994). According to the UE, all users are assumed to choose routes to minimize their own travel cost; while the SO assumes that all users together minimize the system-wide travel cost (Wardrop, 1952). Routing network demand or origin destination (OD) matrices in a network under certain behavioral assumption is called traffic assignment (TA). Travel time is often used to represent travel cost in TA models and is assumed to be the only performance parameter affecting users' routing decisions. However, other performance measures such as emissions, tolls, fuel consumption, safety, road conditions, and aesthetics also influence users' decisions. With existing infrastructure, it is not easy to influence driver routing decisions to minimize emissions and fuel consumption. Here, we assume that future technology and the need to reduce pollution will lead to development of emissions-conscious systems. The aim of this paper is to understand the various aspects of emissions-conscious systems and to develop models to evaluate the performance of such systems. In the initial part of this paper, we examine the emission-speed relationships for CO_2, fuel consumption, NO_x, CO, and HC and in the later part use these relationships in developing emissions-based traffic assignment, called environmental TA (E-TA).

Most of the previous efforts to reduce vehicle pollution have focused on designing exhaust after-treatment systems, developing low emission vehicle engines, and producing higher quality fuel. Although these efforts have helped on a per-vehicle basis, such efforts may not achieve the desired ambient air quality. New approaches, such as routing drivers to minimize emissions, need to be examined. Although at this point drivers do not fully understand how to achieve a minimum fuel use or emissions route, the possibility leads to consideration of environmental UE (E-UE), which is expected to exist when drivers receive perfect information on emissions on

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different routes in a network and they behave rationally to minimize their own emissions. One can also conceive of an approach to reducing vehicle emissions by controlling traffic routing in a transportation network to achieve an environmental SO (E-SO). Collecting tax or toll, proportional to the emissions and fuel consumed could influence the travelers' routing decisions.

Developing UE or SO models by changing the performance parameter from travel time to emissions and fuel consumption is not a straightforward exercise, because while higher vehicle speed decreases travel time, speed beyond a certain limit increases emissions and fuel consumption. The environmental performance functions also affect the existence and uniqueness properties of E-UE and E-SO formulations. It is possible to combine emissions and fuel consumption with travel time and develop a generalized multi-factor performance function. In this study, however, we restrict our analysis to a single parameter performance function to understand better the impact of each parameter on routing and also on the value of the other environmental parameters, including travel time. The parameters we consider in this study are: CO_2, fuel use, NO_x, CO, and HC, but the E-TA models are developed and solved for only CO_2 and fuel use. This paper aims to answer the following questions: i) What level of emissions reduction is possible by influencing travelers' routing? ii) What is the impact other parameters, including travel time, in E-TA? iii) How does a centralized traffic control system perform, with respect to various emissions and travel time, against the system in which individuals are selfishly trying to minimize their own emissions and/or fuel consumption?

This paper is organized into 7 sections. The next section discusses background and literature on traffic assignment and emission models. Analytical emissions models for various pollutants, as function of average speed, are developed in section 3, which is followed by a section describing the conventional equilibrium traffic assignment formulations. Detailed discussion on developing environmental TA models is presented in section 5. Section 6 includes the results and discussion of various TA models, while section 7 concludes the paper by discussing key findings of the study and outlining future research directions.

2. BACKGROUND AND LITERATURE REVIEW

The regulated pollutants emitted from vehicles include carbon monoxide (CO), oxides of nitrogen (NOx), hydrocarbons (HC), particles, and historically, lead. The greenhouse gas carbon dioxide (CO_2) is also emitted by vehicles. Vehicle speed and acceleration/deceleration play major roles in the amount of pollutants emitted by a vehicle (Frey, et al., 2006; Frey, et al., 2003). For example, the CO_2 versus speed plot in Figure 2 shows increasing average vehicle speed from idle reduces tailpipe emissions for lower speeds, but increasing speed beyond a certain range (around 30 to 40 mph) increases CO_2 tailpipe emissions. Thus, vehicle emissions are minimized for a certain range of speed values only.

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As explained above, travel time-based TA is not necessarily producing the minimum emissions. The emissions-minimizing routing, however, is not easy to implement in practice because drivers usually do not have information on their emissions and knowledge about factors affecting emissions. Moreover, challenges also arise from the emissions minimization TA formulations, because of the conflicting interests, evidenced in Figure 2, between travel time and emissions minimization. Nevertheless, quantifying the reduction in vehicle emissions by controlling traffic is an important and timely step. Methods of influencing traffic behavior are currently being explored. For example, in-vehicle devices which provide real-time fuel usage are becoming more commonplace. Intelligent transportation systems (ITS) such as advanced travel information systems (ATIS) have been partially successful in controlling traffic movement. Road pricing has been successful in shifting some individual drivers away from the route that minimizes their own individual travel time. Recently, researchers are proposing road pricing strategies based on emissions (Callan, et al., 1996; Yin, et al., 2006).

Only a few traffic assignment studies in the literature consider emissions as a part of travel cost. Rilett and Benedek (1994) use the CO emission model used in TRANSYT 7-F model and compare the results of travel-time based and emission-based static traffic assignment. The TRANSYT 7-F model for CO estimation was also used by Yin and Lawphongpanich (2006), who propose road pricing to internalize the emissions externality. Both studies assume a simplified flow-CO relationship. A more realistic CO model was used by Sugawara and Niemeier (2002) and emission-based traffic assignment was developed and solved using metaheuristic Simulated Annealing. All of these previous studies conclude that travel time-based TA models do not necessarily minimize emissions.

3. DEVELOPING EMISSIONS-SPEED MODELS

Vehicle emissions depend on many factors including road conditions, vehicle characteristics, driver behavior, and climate; therefore no standard analytical expressions have been developed for various pollutants. It is common practice to use mobile source emissions models like MOBILE (U.S. EPA) or EMFAC (CARB) to estimate total emissions from on-road vehicles for regulatory purposes. These aggregate models predict total fleet emission inventories at the relatively large county-scale, based on average vehicle speed. More disaggregate models such as the Comprehensive Modal Emission Model (CMEM; Barth et al., 1999) are more appropriate for estimating emissions associated with local-scale traffic improvements and "microscale" traffic simulation modeling. Modal models consider the operating mode of the vehicle (idle, steady-state cruise, acceleration and deceleration) and therefore should more accurately estimate dynamic changes in emissions over a given trip at the individual vehicle scale. EPA's latest model, MOVES, uses 14 operating bins defined by average speed and vehicle specific power to estimate emissions.

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In this study, CMEM is used to model fuel consumption and emissions of CO_2, CO, HC, NO_x based on the Highway Fuel Economy Test (HWFET) driving cycle and a single passenger car vehicle type (Category 9, Tier 1 emissions, high power-to-weight ratio with over 50,000 miles). The HWFET driving cycle vehicle speeds were adjusted to generate speed-time operation patterns with overall cycle average speeds from 2 mph ("idle or creep") to 80 mph (freeway) (see Figure 1). The CMEM grams-per-mile fuel consumption and gaseous emission rates used in this analysis are given in Table 1 for the 2-80 mph range of average speed values. Note that the values of speed in the first column are the average speed over the entire drive cycle, which include the 0 mph speed at the start and the end node of the run. The actual speed at any location in the middle of the trip may be higher than the average speed. This approach of average speed estimation is desired in static traffic assignments, because static TA does not incorporate the stops and speed fluctuation explicitly. Thus, the use of the disaggregate emissions model.

The CMEM individual pollutant emission factors (g/mi) are plotted against average speed in Figure 2. Note that NOx emissions increase with increasing average speed, but HC and CO emission factors initially decrease with increasing speed at very low speed (up to about 5 mph), but increase greatly at speeds above 50-60 mph. CO_2 and fuel consumption have similar patterns in which emission factors decrease initially for speeds up to about 35 mph, then increase slightly at higher speeds. It should be noted that the per-mile emission factor dimensions contribute somewhat to the observed patterns.
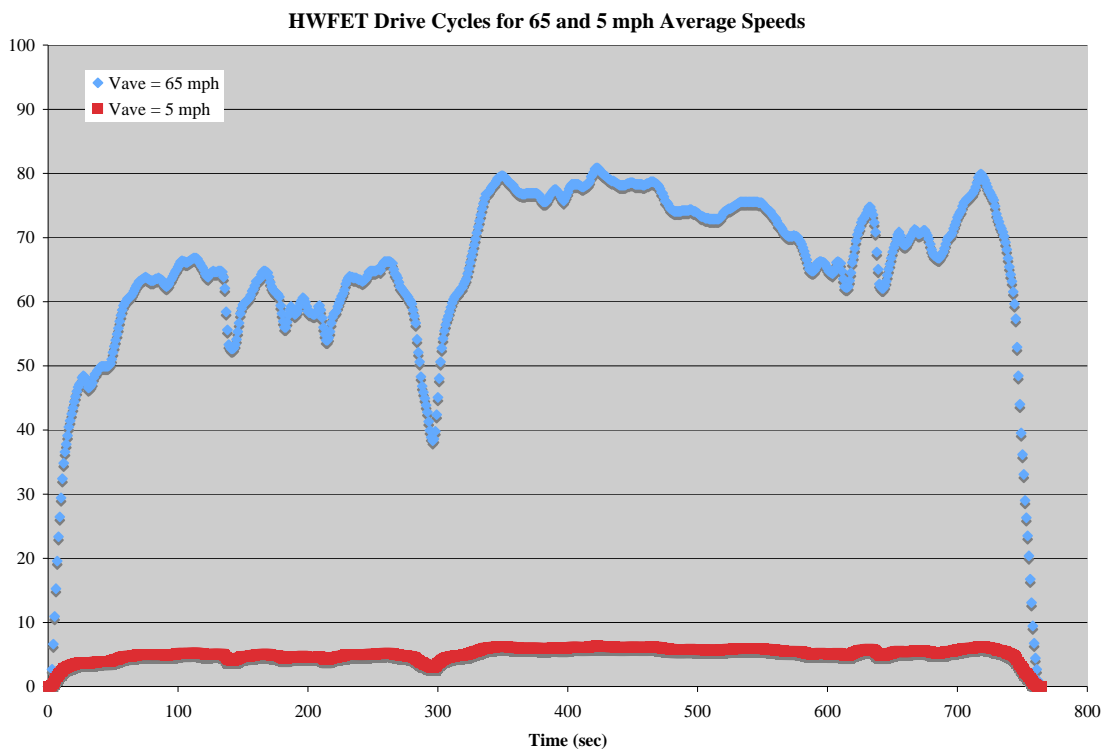
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Figure 1 Highway Fuel Economy Test (HWFET) driving cycle adjusted to average speeds of 65 and 5 mph. *Note: Second-by-second test data such as these were used to quantify vehicle emissions as a function of average vehicle speed using CMEM.*

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Table 1 Grams-per-mile Emission Factors by Average Speed of CMEM Category 9 LDV Running Various HWFET Cycles

Avg Speed (mph)	Fuel (g/mi)	CO_2(g/mi)	CO(g/mi)	HC(g/mi)	NOx(g/mi)
2	392.92	1245.50	0.539	0.036	0.000
5	163.37	517.85	0.236	0.015	0.003
10	130.36	413.03	0.296	0.016	0.014
15	101.13	320.35	0.270	0.014	0.024
20	88.40	279.94	0.275	0.014	0.032
25	73.36	232.31	0.238	0.012	0.043
30	69.22	219.14	0.257	0.013	0.054
35	68.05	215.38	0.293	0.015	0.081
40	68.69	217.32	0.343	0.017	0.116
45	67.13	212.32	0.375	0.018	0.127
50	70.56	223.05	0.463	0.022	0.151
55	74.47	235.24	0.575	0.027	0.180
60	80.69	254.23	1.020	0.036	0.223
65	88.12	276.59	1.786	0.050	0.273
70	96.51	301.21	3.00	0.07	0.33
75	107.77	328.45	8.30	0.12	0.37
80	120.83	357.22	16.19	0.19	0.42

All pollutant emission factors have nonlinear relationships with average cycle speed. We used SPSS to fit the following nonlinear models for the individual pollutants.

$$CO_2 = 129.533 + 2217.694/v + 0.027771v^2 \tag{1}$$

$$Fuel = 39.705188 + 702.856/v + 0.0096227v^2 \tag{2}$$

$$NO_x = 0.00508 - 0.000387v + 0.000069v^2 \tag{3}$$

$$HC = 0.016 + 2.30 \times 10^{-5}e^{0.122v} \tag{4}$$

$$CO = 0.279 + 6.36 \times 10^5 e^{0.155v} \tag{5}$$

Figure 2 compares the plots of the nonlinear models with the plots of CMEM data. The plots show the emissions estimation by analytical models match very well with CMEM values. This conclusion can also be verified from the high R^2 values of the analytical models. The values of R^2 for CO_2, fuel, NO_x, HC, and CO are respectively 0.988, 0.987, 0.996, 0.987, and 0.997. All five nonlinear models are convex models. The models for CO and HC underestimate the emissions at very slow speed (about 5mph and lower). The emission factor models can be categorized into two types: i) models which are non-decreasing functions of average vehicle

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speed (v) (NO_x, HC, and CO); and ii) models which are decreasing functions of v, up to an optimal average speed and then are increasing functions of v (CO_2 and fuel).
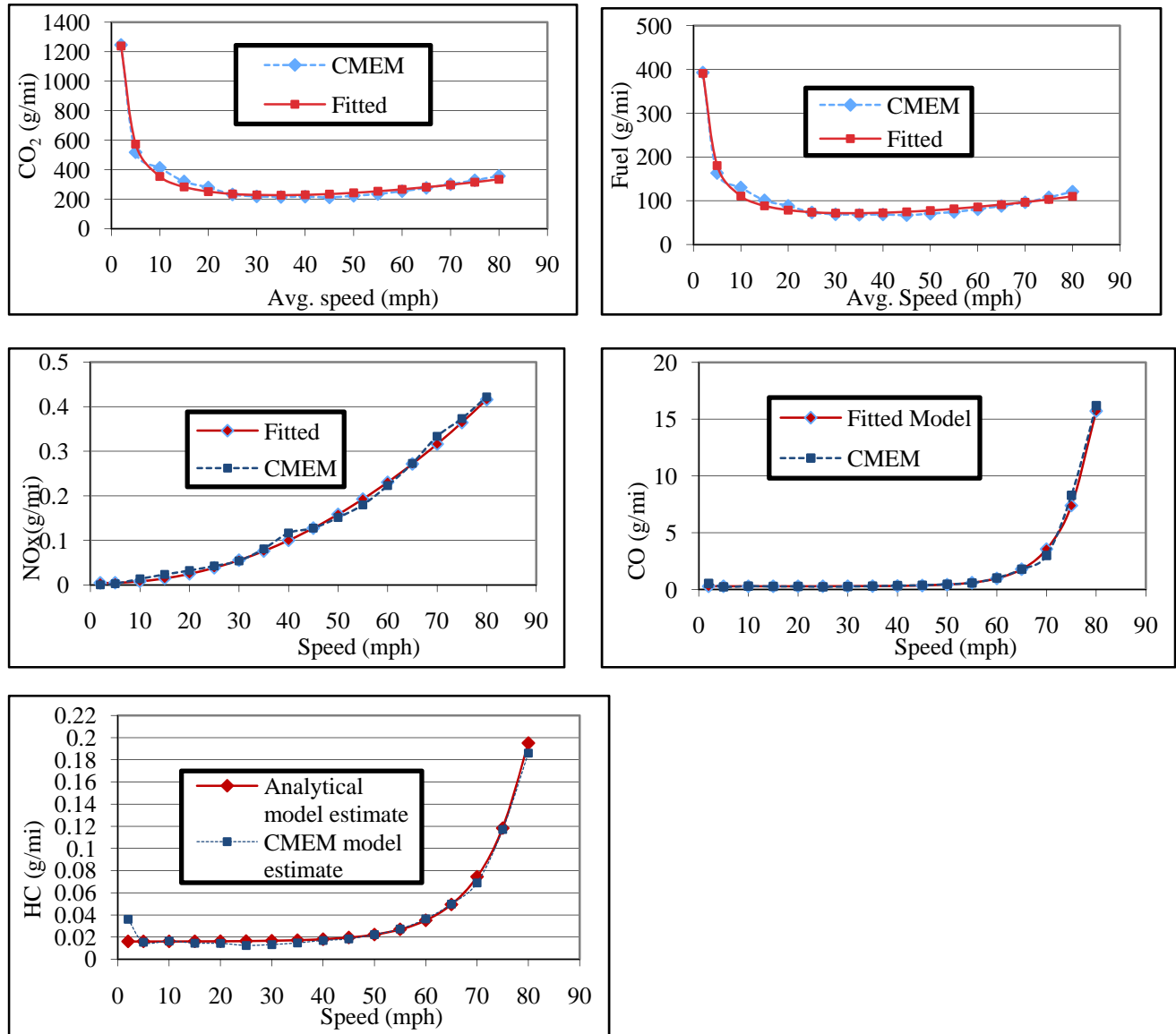

Figure 2 Comparison of the CMEM model and the analytically fitted non-linear models (Equations 1-5 in text).

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4. TRAVEL TIME BASED TRAFFIC ASSIGNMENT

We define travel time-based TA (TT-TA), to represent a TA formulation or a technique in which travel cost is replaced with travel time. Thus, under travel time-based UE, (TT-UE), all drivers are assumed to minimize their own travel time. Beckman's formulation, given by equations (6.1) through (6.4), often is used achieve UE flows.

[TT-UE]:

$$min \sum_{ij} \int_0^{x_{ij}} c_{ij}(\omega)d\omega \qquad (6.1)$$

$$\text{s. t.} \quad \sum_k f_k^{rs} = q^{rs} \qquad (6.2)$$

$$x_{ij} = \sum_r \sum_s \sum_k \delta_{ij,k}^{rs} f_k^{rs} \qquad (6.3)$$

$$f_k^{rs} \geq 0 \qquad (6.4)$$

Where,

$x_{ij}=$ flow on link (i,j)

$c_{ij}(x)=$ travel time on link (i,j)

$f_k^{rs}=$ flow on path k between OD rs

$\delta_{ij,k}^{rs}=$ a binary variable; 1 if the link (i,j) is on path k between OD rs and 0 otherwise

The above [TT-UE] problem satisfies the necessary conditions for UE: all paths carrying flow between origin r and destination s should have equal cost and the paths not carrying traffic will have equal or higher cost than the one carrying traffic. The conditions which satisfy UE can be derived from [TT-UE] problem are given by equations (7.1) – (7.4) (Sheffi, 1984):

$$f_k^{rs}(c_k^{rs} - u^{rs}) = 0 \; \forall \; k, r, s \qquad (7.1)$$

$$(c_k^{rs} - u^{rs}) \geq 0 \; \forall \; k, r, s \qquad (7.2)$$

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$$\sum_k f_k^{rs} = q^{rs} \qquad (7.3)$$

$$f_k^{rs} \geq 0 \ \forall \ k, r, s \qquad (7.4)$$

The existence and uniqueness of the [TT-UE] problem mainly depends on the properties of the travel cost function $c_{ij}(x)$. The following properties have been proven regarding the existence and uniqueness of the [TT-UE] problem:

1) The problem has an optimal solution if the link performance function is positive and continuous (Theorem 2.4, Patriksson 1994).
2) If the travel cost function $c_{ij}(x)$ is positive, continuous, and non-decreasing then the equilibrium travel costs are unique (Theorem 2.5a, Patriksson 1994). .
3) If the travel cost function is positive, continuous, and strictly increasing the equilibrium link flows are unique (Theorem 2.5c, Patriksson, 1994).

In TT-TA, the most commonly used link performance is Bureau of Public Roads (BPR) function, given by equation (8):

$$t_{ij} = T_{ij}^0 \left[1 + \beta \left(\frac{x_{ij}}{K_{ij}} \right)^\alpha \right] \qquad (8)$$

Where,

t_{ij} = travel time on link (i,j)

T_{ij}^0 = free flow travel time on link (i,j)

x_{ij} = flow on link (i,j)

K_{ij} = capacity of link (i,j)

α and β are constants; the value of α is usually 4 and that of β is 0.15. It can be easily verified that the BPR function is positive, continuous, and strictly increasing. Thus, there exists a solution to the [TT-UE] problem and the solution is unique.

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The travel time-based system optimal (TT-SO) formulation is given by equations (9.1) – (9.4). It has been shown that if the link performance function is convex then there exists a solution to [TT-SOC] problem and the solution unique with respect to link flows (Sheffi, 1984).

[TT-SO]:

$$min \sum_{ij} c_{ij}(x)x_{ij} \tag{9.1}$$

$$\text{s. t.} \quad \sum_k f_k^{rs} = q^{rs} \tag{9.2}$$

$$x_{ij} = \sum_r \sum_s \sum_k \delta_{ij,k}^{rs} f_k^{rs} \tag{9.3}$$

$$f_k^{rs} \geq 0 \tag{9.4}$$

If the BPR function (equation (8)) is used to estimate travel times, then solutions exist for both UE and SO formulations and the solutions are unique with respect to link flows and travel time. The environmental performance functions may to possess all the desired properties for the existence and uniqueness of the solution.

5. ENVIRONMENTAL TRAFFIC ASSIGNMENT MODELS

In this section, we extend the principles of UE and SO to E-TA models. For example, in CO_2 based E-SO model, vehicles are routed to minimize network-wide CO_2. Similarly in E-UE, users route themselves selfishly to minimize their own CO_2 emissions. If the performance parameter in E-UE is CO_2, then an important condition for E-UE is that the CO_2 produced by a user on all used paths between a given OD is the same and less than that on unused paths. Thus, we can define UE and SO principles for E-TA taking for CO_2:

E-UE: No user can unilaterally reduce his or her CO_2 emissions by changing paths. In other words individual CO_2 produced by all users is at a minimum.

E-SO: The total CO_2 produced in the network for all users is at a minimum.

An important assumption in both the [TT-UE] and [TT-SO] problems is that travel time is a function of flow, which is usually expressed with the BPR function (equation (8)). Similarly, for E-TA equilibrium models the emission-speed relationships in equations (1) -- (5) should be changed to equivalent emission-flow relationships.

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5.1 Emissions-flow relationships

The BPR function in equation (8) can be written as

$$t_{ij} = \frac{l_{ij}}{v_{ij}^o}\left[1 + \beta \left(\frac{x_{ij}}{K_{ij}}\right)^\alpha\right] \tag{10}$$

where,

l_{ij} = length of link (i,j)

v_{ij}^o = free flow speed on link (i,j)

All emissions models developed in Section 3 are functions of average speed. The average speed can be obtained using equation (10) as below,

$$v = \frac{l_{ij}}{t_{ij}} = \frac{v_{ij}^o}{\left[1 + \beta \left(\frac{x_{ij}}{K_{ij}}\right)^\alpha\right]} \tag{11}$$

We can substitute the average speed equation (11) into the emissions equations (1)-(5) and multiply by link length to get the emission-flow relationship for a link. For example, CO_2 on a link (i,j) as a function of link flow is given as:

$$e_{ij} = CO_2(x) = l_{ij}\left\{129.533 + 2217.694\left\{\frac{\left[1 + \beta \left(\frac{x_{ij}}{K_{ij}}\right)^\alpha\right]}{v_{ij}^o}\right\} \right. \\ \left. + 0.027771\left\{\frac{v_{ij}^o}{\left[1 + \beta \left(\frac{x_{ij}}{K_{ij}}\right)^\alpha\right]}\right\}^2\right\} \tag{12}$$

Assuming $\alpha = 4$, $\beta = 0.15$, and $l_{ij} = 1\ mile$, the relation between flow and CO_2 is plotted in Figure 3 for free flow speeds of 65 mph and 30 mph. The link flow varied from 0 to 400 veh/h and the capacity was assumed to be 150 veh/h. Unlike the BPR link performance function, the model for FF speed 65 mph is neither convex nor monotonic. However, the plot for FF speed of 30 mph is convex and non-decreasing—the CO_2 emissions on a link increase with the increase in flow.
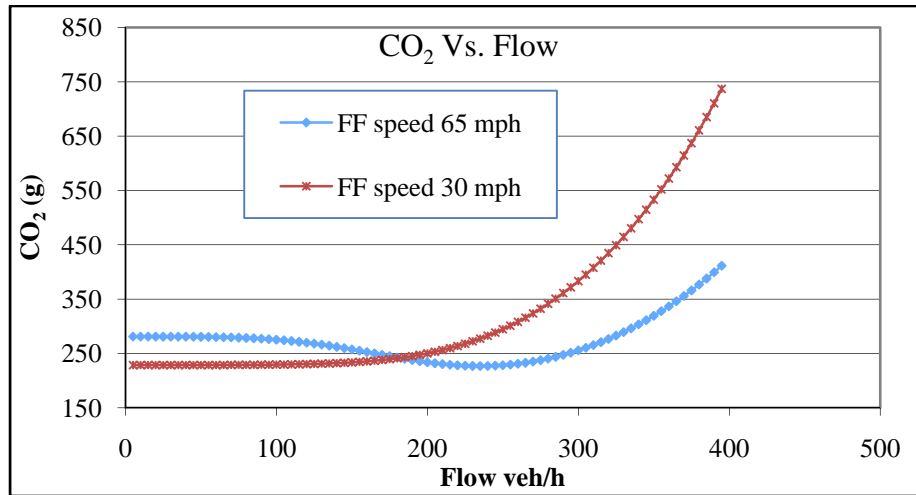
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Figure 3 CO$_2$ link emissions vs. link flow curve

5.2 Most efficient speed

An important observation from the analysis in Section 3 is that each pollutant has its minimum emission rate at a certain average operating speed. We defined the minimum emission or fuel rate speed as the *most efficient speed* (MES). The MES for CO$_2$ and fuel are between 30 to 40 mph but are very low for NOx, CO, and HC (Figure 2). If a road is not congested and if the FFS is higher than MES, travelers can drive faster than the MES, but by doing so they will be increasing their emissions.

The emissions models for CO$_2$ and fuel are convex; thus the optimal speed can be obtained by equating to zero the derivative of a model with respect to speed. Note that the emission models for both CO$_2$ and fuel are the in following form:

$$E = a + \frac{b_1}{v} + b_2 v^2 \qquad (13)$$

By taking derivative and equating to zero:

$$\frac{dE}{dv} = 2b_2 v - \frac{b_1}{v^2} = 0$$

$$\frac{2b_2 v^3 - b_1}{v^2} = 0$$

$$2b_2 v^3 - b_1 = 0$$

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$$v^* = \sqrt[3]{\frac{b_1}{2b_2}} \qquad (14)$$

Where v^* is the MES for either CO_2 or fuel. Substituting the constants from the emissions models in equation (14) , we get the MES for CO_2 and fuel as:

$$v^*_{CO_2} = 34.17 \text{ mph and } v^*_{fuel} = 33.17 \text{ mph}$$

When FFS on a link is less than the MES, the drivers cannot drive at MES. Thus we define a new speed for each link called *feasible efficient speed* (FES), $\hat{v}^*$ defined as below.

$$\hat{v}^*_{ij} = \begin{cases} v^* if \ v^o_{ij} > v^* \\ v^o_{ij} \ otherwise \end{cases} \qquad (15)$$

Because the emission factors of NOx, CO, and HC increase monotonically with increasing average speed, the FES for these pollutants is zero or very close to zero (Figure 2). Therefore, the solutions of the E-TA problems that minimize one or more of the pollutants NOx, CO, and HC will result in an undesirable network condition in which all traffic is at a standstill. However, we can combine one or more of these pollutants with travel time or any other performance parameter, in which an optimal speed is higher, to get a reasonable flow condition. We are in the process of extending this study by developing and solving traffic assignment models by combining two or more performance factors. In this study, however, we are focusing only on single parameter models, thus the TA models for NOx, CO, and HC will not be developed. We will only develop and solve E-TA models for CO_2 and fuel consumption and compare the results with the traditional travel time-based models.

5.3 Emissions-flow relationship for E-TA

Unlike TT-TA, the free flow speed is not always the desired speed for drivers in E-TA, because the emission level is higher at higher speeds (Figure 2). If the link performance parameter is CO_2, then the best strategy for the drivers is to move at an average speed of $v^*_{CO_2} = 34.17 \ mph$. When the congestion level is significantly higher, drivers may not be able to drive at MES in which case the actual speed will be lower and emissions/fuel use could be higher. The actual average speed will depend on the free flow speed and the congestion level. Two situations are discussed:

1) If $v^0_{ij} \le v^*$: When the free flow speed is less than the MES, the actual speed will always be less than optimal speed and is obtained by equation (11)

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2) If $v_{ij}^0 > v^*$: When the FFS is greater than MES, the actual speed will be either equal to or less than the optimal speed based on the level of congestion.

In order to understand the latter situation we define a term *critical flow*—the flow level on a link above which drivers will not be able to move at the feasible efficient speed. In other words, if the flow on a link is less than the critical flow, the actual speed of the drivers is equal to the MES. Similarly, we can define the critical v/c ratio. Now the actual speed if $v_{ij}^0 > v^*$:

$$v_{ij} = \begin{cases} \hat{v}_{ij}^* \ if \ x_{ij} \leq \tilde{x}_{ij} \\ eqn. (11) \ if \ x_{ij} > \tilde{x}_{ij} \end{cases} \tag{16}$$

Since, at the critical flow, the equations for critical flow and critical v/c ratio are derived by equating the equation of actual speed (equation (11)) to the optimal speed:

$$\hat{v}_{ij}^* = \frac{v_{ij}^o}{\left[1 + \beta \left(\frac{\tilde{x}_{ij}}{K_{ij}} \right)^\alpha \right]} \tag{17}$$

$$\left(\frac{\tilde{x}_{ij}}{K_{ij}} \right)^\alpha = \frac{1}{\beta} \left(\frac{v_{ij}^o}{\hat{v}_{ij}^*} - 1 \right) \tag{18}$$

Now the critical v/c ratio:

$$\frac{\tilde{x}_{ij}}{K_{ij}} = \left[\frac{1}{\beta} \left(\frac{v_{ij}^o}{\hat{v}_{ij}^*} - 1 \right) \right]^{\frac{1}{\alpha}} \tag{19}$$

And the critical flow for link (i, j)

$$\tilde{x}_{ij} = K_{ij} \left[\frac{1}{\beta} \left(\frac{v_{ij}^o}{\hat{v}_{ij}^*} - 1 \right) \right]^{\frac{1}{\alpha}} \tag{20}$$

Using the same constants used for Figure 3, the values of the critical v/c ratio and critical flow are:

1) For $v_{ij}^0 = 65 \ mph$, $\tilde{x}_{ij}= 234.9$ and $\frac{\tilde{x}_{ij}}{K_{ij}} = 1.56$

2) For $v_{ij}^0 = 30 \ mph$, $\tilde{x}_{ij}= 0$ and $\frac{\tilde{x}_{ij}}{K_{ij}} = 0$

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In this example, when the FFS is 65 mph, for the link flow of 234.9 veh/h or less, the actual speed is the FES (34.17 mph for CO_2). When flow exceeds the limit of 234.9 veh/h the congestion on the link will restrict the speed to less than the FES. However, when the FFS is limited to 30 mph the actual speed will be always less than FES (30 mph) for any positive flow. The plot of the CO_2 against link flow, with and without limiting the speed limit to the FES, when FFS is 65 mph is shown in Figure 4.
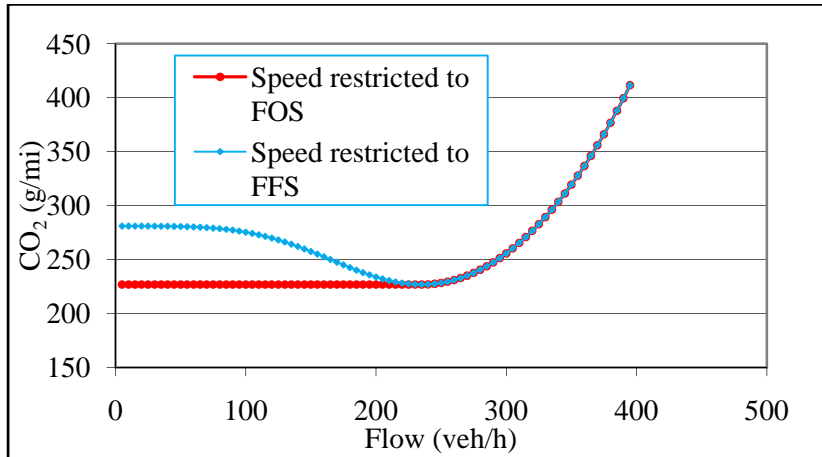

Figure 4 E-TA Flow Emission Curve

5.4 Environmental TA

As described above, in the environmental-UE, users are assumed to minimize their own vehicle emissions or fuel consumption. The formulation for E-UE is shown in equation (20).

[E-UEP]

$$min \sum_{ij} \int_0^{x_{ij}} e_{ij}(\omega)d\omega \qquad (21)$$

$$s. \; t. \quad equation \; 6.6 - 6.8$$

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Similarly, the E-SO problem can be formulated as in equation (21) :

[E-SOP]

$$min \sum_{ij} e_{ij}(x)x_{ij} \tag{22}$$

$$s.\ t.\quad equation\ 9.2 - 9.4$$

Variable $e_{ij}(x)$ in both [E-UEP] and [E-SOP] is either CO_2 emissions or fuel consumption on a link and is given by equation (22):

$$e_{ij}(x)$$
$$= \begin{cases} l_{ij}\left\{a + b_1 \times \hat{v}_{ij}^* + \dfrac{b_2}{\hat{v}_{ij}^*}\right\} & if\ x_{ij} \leq \tilde{x}_{ij} \\ l_{ij}\left[a + b_1\left(\dfrac{1 + \beta\left(\dfrac{x_{ij}}{K_{ij}}\right)^{\alpha}}{v_{ij}^o}\right) + b_2\left(\dfrac{v_{ij}^o}{1 + \beta\left(\dfrac{x_{ij}}{K_{ij}}\right)^{\alpha}}\right)^2\right] & if\ x_{ij} > \tilde{x}_{ij} \end{cases} \tag{23}$$

5.5 Nonlinear complementarity formulation for E-UE

The Frank-Wolf algorithm is commonly used to solve the Beckman's formulation of user equilibrium. The UE problem can also be formulated as a nonlinear complementarity problem (NCP) (Friesz, Tobin, Smith, & Harker, 1983; Patriksson, 1994). When UE is formulated as NCP, then one of the tools developed to solve complementarity problems can be used. KNITRO (Byrd, Nocedal, & Waltz, 2006), MILES (Rutherford, 1997) and PATH (Ferris, R, & Gay, 1999) are among the standard tools available today to solve complementarity problems. Although not guaranteed, KNITRO helps in finding a global optimal solution of a non-convex problem. In KNITRO the chances of finding the global optimal solution are increased by setting an option in which KNITRO attempts to find a local optimal with different initial solution each time. The number of the trials and the initial values can be provided in KNITRO options. To take advantage of this readily available tool, we reformulate the E-UE problem equation (20) as an equivalent NCP problem.

$$\{e_{ij}(x) + \pi_j^s - \pi_i^s\}h_{ij}^s = 0\ \forall\ (i,j), s \tag{24.1}$$

$$e_{ij}(x) + \pi_j^s - \pi_i^s \geq 0\ \forall\ (i,j), s \tag{24.2}$$

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$$\sum_{ij \in A_i^+} h_{ij}^s - \sum_{ij \in A_i^-} h_{ij}^s = q^{rs} \; \forall \, i, s \qquad (24.3)$$

$$h_{ij}^s \geq 0 \; \forall \, (i, j), s \qquad (24.4)$$

Where,
h_{ij}^s=flow on link (i, j) going to destination s
π_j^s= value of emission on a minimum emission path from node j to destination s
Equation (22) is used to calculate $e_{ij}(x)$.

6. RESULTS

In this section, we present the results for two hypothetical networks. E-TA models are solved separately for the performance parameters CO_2 and fuel. For each case the network flow is compared for SO and user UE assumptions. Here, we calculate and compare travel time, CO_2, and fuel consumptions for all models.

6.1 Network-1: 2-link network

The first network is a two-link network representing two routes from origin r to destination s. Two levels of input parameters are considered as presented below in Table 2.

Table 2 Inputs for Two-link Network

Input Scenario	q^{rs} veh/h	Link	l_{ij} (mi)	v_{ij}^0 (mph)	K_{ij} (veh/h)
SCEN-I	1000	1	3	60	600
		2	2	30	500
SCEN-II	1000	1	3	60	600
		2	3	60	500

SCEN-I:

In the first scenario, the FFS on link 1 is much higher than that on link 2. Since, the flow on one link determines the flow on the other we observed travel time and emissions levels for all combination of flows.

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The travel time with respect to link flow in TT-TA is plotted in Figure 5a), and the flow-CO_2 relationship is plotted in Figure 5b). As expected, the TT-UE has unique link flows($x_1 = 740; x_2 = 260$). For this scenario E-UE also has unique flows at the equilibrium($x_1 = 68; x_2 = 932$). The speeds on both links at different flow levels for CO_2 based TA are plotted in Figure 7. The FFS speed on link 1 is 60 mph, thus the feasible optimal speed is 34.17 mph (the optimal speed for CO_2). The traffic on link 1 will continue to flow 34.17 mph until the critical link flow limit is reached. Using equation (18), the critical link flow link 1 is equal to 899 vph. As seen in Figure 6, the speed on link 1 is constant until the flow level reaches 899 vph and then starts declining. The speed on link 2 however, is decreasing monotonically as the flow is increased, since the FFS is less than the CO_2 optimal speed. It can be noted that the TT-UE assigns most traffic to link 1. Although link 1 is longer, the higher FFS speed enables the drivers to travel quicker than that for on link 2. In case of E-UE, the drivers will move at 34.17 mph, the feasible optimal speed (FOS), until the critical flow limit. The FOS is not higher than the FOS of link 2, which is much shorter than link 1. Thus, in E-UE most of the demand is assigned to link 2 (Figure 5).
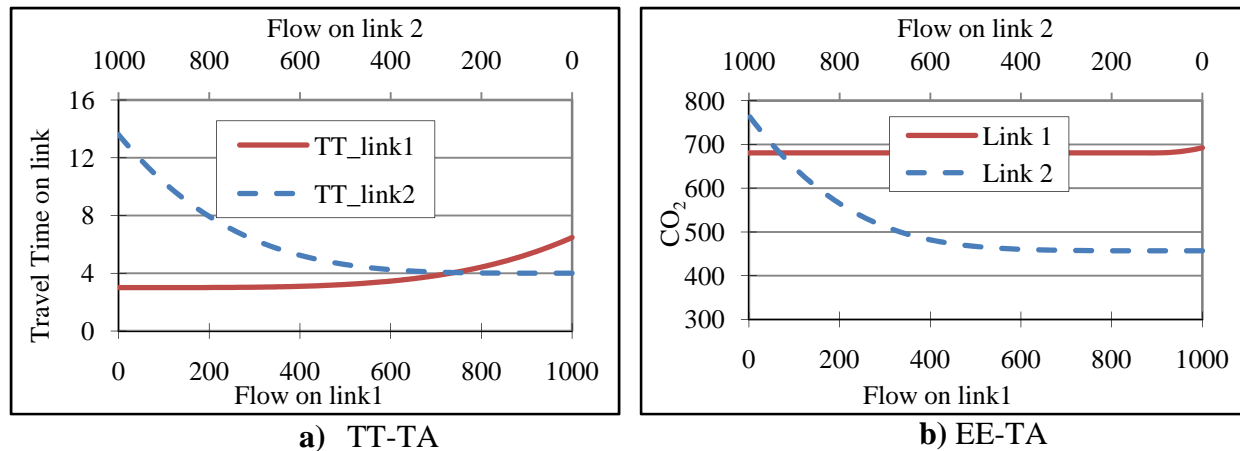

a) TT-TA   b) EE-TA

Figure 5 Link performance

The total CO_2 (in grams) at different flow combinations is plotted in Figure 7. The link flows at the minimum of this plot gives the EE-SO solution. As seen in the figure the objective function of EE-SO is a convex function and the problem has unique link flows solution. The optimal value of CO_2 in network is 559770 g and the corresponding flows are: $x_1 = 599; x_2 = 401$. The above analysis shows the inputs in SCEN-I produce unique link flow solutions to both E-UE and E-SO.
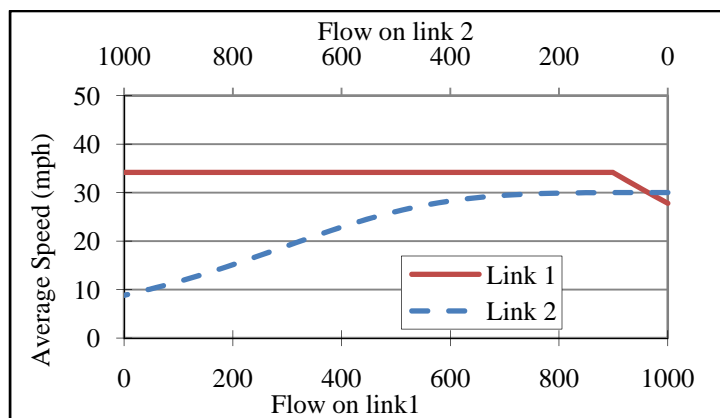
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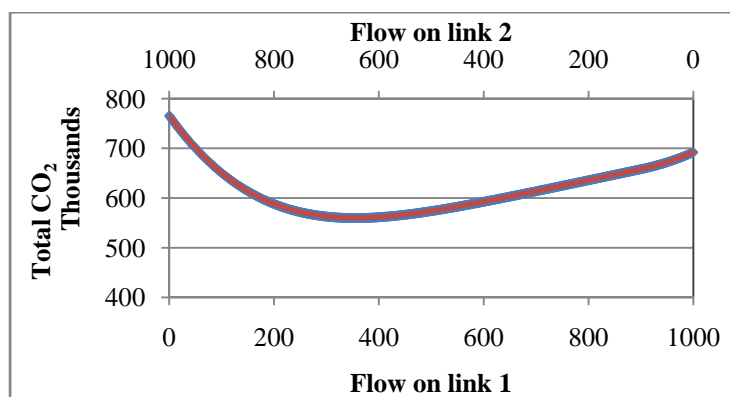

Figure 6 Average speed in CO_2 based E-TA

Figure 7 Total CO_2 in grams against link flow for SCEN-I

A summary of the results are presented in Table 3. As expected the total CO_2 (fuel) in CO_2 (fuel) based E-SO is minimum compared its values in other SO results. The CO_2 emission is reduced by 5.5% in CO_2-based SO than that by time-based SO assignment. The reduction in fuel consumption is about 5.8%. Travel time, however, increased by about 2.6% and 2.7% respectively in CO_2-based SO and fuel-based SO. One interesting observation for this network is that CO_2 and fuel consumption on link-1 are constant for all parameters under both SO and UE flow assumptions. This is because the SO and UE flow on link-1 for all parameters is less than the critical link flow, thus the flow is moving at the feasible efficient speed, emitting the minimum possible CO_2 and consuming the minimum possible fuel.

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Table 3 Results of Network -1

Flow Assumptions	Decision Parameter	Travel Time			CO$_2$			Fuel		
		Link 1	Link 2	Travel Time	Link 1	Link 2	CO$_2$	Link 1	Link 2	CO$_2$
System Optimal	Travel Time	3.45	4.25	3,768.30	680.6	460.4	592,289.2	214.5	144.5	186,460.3
	CO$_2$	3.06	5.66	4,736.27	680.6	493.3	559,770.0	214.5	154.4	175,731.4
	Fuel	3.05	5.71	4,782.12	680.6	494.8	559,791.2	214.5	154.8	175,725.4
User Equilibrium	Travel Time	4.04	4.04	4,041.89	680.6	457.4	622,564.1	214.5	143.7	196,121.4
	CO$_2$	3.00	11.24	10,682.74	680.6	681.0	680,936.0	214.5	213.3	213,392.4
	Fuel	3.00	11.34	10,795.09	680.6	684.3	684,078.4	214.5	214.4	214,381.4

Of particular interest are the results for the user equilibrium models. The results of E-UE are unexpected and paradoxical. The results indicate that when drivers behave selfishly to minimize their own emissions, they end up increasing both network wide emissions and travel time. This paradox is reasonable and is explained below.

At TT-UE the flows are $x_1 = 740; x_2 = 260$ and the CO$_2$ for each vehicle on link-1 and link-2 are respectively 680.6 and 457.4 g. These flows are unstable in CO$_2$-based UE, because a driver on shifting link-1 can reduce his/her CO$_2$ emission unilaterally by switching to link-2. When ten drivers switch to link-2, the flow levels are $x_1 = 730; x_2 = 270$, the resulting CO$_2$ level on link-2 is 457.53, which is much smaller than 680.6. Note however, since the drivers on link-1 are moving at the feasible optimal speed, the remaining drivers on that link will not experience any reduction in the CO$_2$. The drivers who switched to link-2 decreased their own CO$_2$ from 680.6 to 457.53—a significant reduction, but they increased the emissions of the original 260 drivers from 457.4 to 457.53—a tiny increase in the level. The switching reduced the system wide CO$_2$ from 622,564.1 to 620,357.5. Although better than TT-EA, this level of flow is not stable, because the drivers from link 1 can still decrease their own travel time unilaterally by changing their path. The switching will continue until the emission on link 2 reaches to 680.6. The flows at this stage are $x_1 = 68; x_2 = 932$; the total emission is 680,936, which is 9.4% higher than that for TT-UE.

The results for fuel-based UE results are also paradoxical and can be explained by similar reasoning. The E-UE increased the network wide travel time significantly in the above example. If we assume the flow is in TT-UE, and if we can control the traffic to achieve CO$_2$-based SO, a reduction in CO$_2$ of about 10% can be achieved but with increase in the travel time by 17%. The decrease in the fuel consumption in fuel-based SO is also about 10% with the increase in time by about 18%.

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SCEN-II:

In this scenario we make the length and FFS for link 2 equal to that for link 1. In SCEN-I, we found unique solutions to TT-TA and E-TA under both UE and SO flows. As discussed in Section 5, the E-TA models, with the analytical emissions expressions developed in this paper are not guaranteed to exist unique solutions. The selection of the input parameters in this scenario results in multiple solutions to both E-UE and E-SO models. Note that both links have FFS speed of 60 mph which is much higher than the feasible optimal speeds. Additionally, both links are equal in length. Since the effect of capacity level is realized only after the flow exceeds link critical flow, both links are equally attractive for a considerable range of flow combinations. As depicted in Figure 8, the CO_2 is equal on both the links for the flow range from $x_1 = 836; x_2 = 164$ to $x_1 = 303; x_2 = 697$. The total CO_2 throughout this range of flow is constant and is equal to 680,579.7; thus the EE-SO also has multiple solutions (Figure 9). The range of flows for which the problem has constant CO_2 is also the same as the range of equal costs. The multiple solutions, however, will result in different travel times. Thus we could select the solutions which produce minimum travel time. The minimum travel cost flows among the multiple solutions are $x_1 = 545; x_2 = 455$ and the corresponding network travel time is 3409.1. Coincidently, this is a TT-SO solution with global optimal time and also TT-UE solutions, thus travel time on both links are same. Thus the solution of TT-SO in the SCEN-II is also the solutions of TT-UE, E-SO, and E-UE.
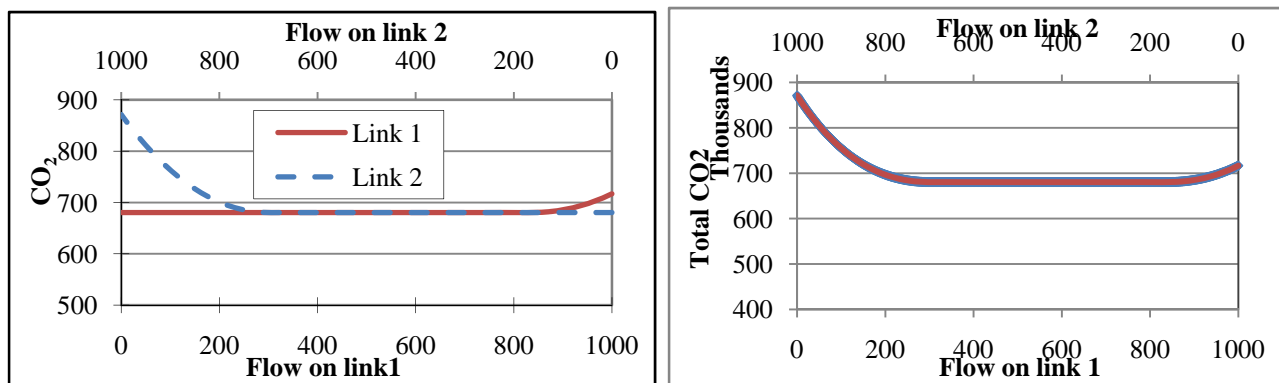

Figure 8 Total CO_2 in grams for SCEN-II

6.2 Network 2: multi-link network

The results of the network-1 provide some insights into flow-emission relations and the behavior of the objective functions. The evaluation of the link specific and network wide performance is

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conducted for all flow combinations on link-1 and link-2. This is not a feasible approach when solving larger networks. To solve the TA models for network-2 KNITRO 5.1 available on NEOS server is used. NEOS is an online optimization solver where numerous optimization tools are available to solve problems online (Czyzk, Mesnier, & Mor, 1998).

Network-2 consists of 6 nodes and 16 links with 2 OD pairs 1-6 and 6-1 (Figure 9). The input parameters are presented in Table 4; the critical flows calculated with equation (18) presented in the last two columns. Since, both E-UE and E-SO assignments can have multiple solutions, we solved the problems with 200 different initial values, but all the starting points produced the same unique solution. Thus we can reasonably conclude that for network-2 with the input levels in Table 4**,** unique solutions exist for both E-UE and E-SO.
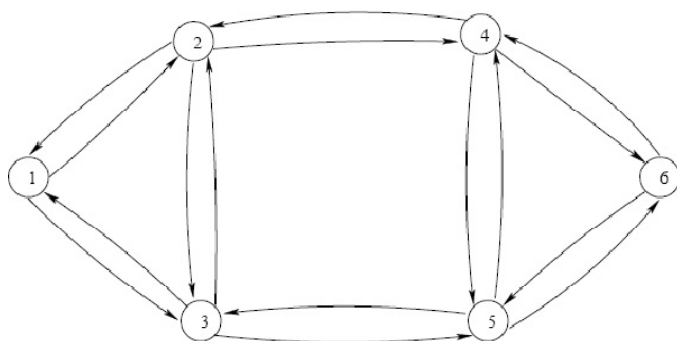

Figure 9 Network -2

To limit the length of the paper, link-wise flow, travel time, emissions, and fuel consumption are not presented, but a summary of the results of TT-TA and E-TA is presented in Table 7. The upper part of the table is for SO-TA. The network wide fuel consumption is minimum for fuel-based SO; similar results are observed for travel time and CO_2. The findings for the SO flow for this network are similar to the findings for network-1.

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Table 4 Inputs for Network-2

Link	T0 (free flow travel time in min)	Capacity (veh/h)	FFS (mph)	Length (mi)	Critical flow for CO2 (veh/hr)	Critical flow for fuel (veh/hr)
(1, 2)	1.0	3000	60.0	1.0	4494.9	4571.6
(1, 3)	2.0	7000	30.0	1.0	0.0	0.0
(2, 1)	3.0	9000	40.0	2.0	9294.5	9741.8
(2, 3)	4.0	4000	37.5	2.5	3591.2	3863.4
(2, 4)	5.0	3000	24.0	2.0	0.0	0.0
(3, 1)	2.0	2000	30.0	1.0	0.0	0.0
(3, 2)	1.0	2000	60.0	1.0	2996.6	3047.7
(3, 5)	1.0	7000	60.0	1.0	10488.1	10667.0
(4, 2)	2.0	9000	30.0	1.0	0.0	0.0
(4, 5)	3.0	3000	20.0	1.0	0.0	0.0
(4, 6)	9.0	2000	26.7	4.0	0.0	0.0
(5, 3)	4.0	6000	30.0	2.0	0.0	0.0
(5, 4)	4.0	12000	30.0	2.0	0.0	0.0
(5, 6)	2.0	9000	30.0	1.0	0.0	0.0
(6, 4)	5.0	3000	24.0	2.0	0.0	0.0
(6, 5)	6.0	4500	30.0	3.0	0.0	0.0

The results of UE assignment in the lower part of Table 5 indicate that when users attempt to minimize their own CO_2 or fuel consumption, the results are worse than the travel time minimization. The paradox, observed for the 2-link network, in which an increase in the value of the CO_2 (fuel) resulted while individual travelers were minimizing CO_2 (fuel), is also observed for network-2. The CO_2 based UE model increased the values of travel time, CO_2, and fuel by about 21%, 8%, and 8% respectively. This increase in fuel-based UE is about 18%, 3%, and 3%. Thus, we conclude that the overall network performance is worse when users are influenced to minimize their own emission levels. However, a reduction in emission level can be achieved if the drivers are routed to reach environmental SO. For this network the reduction CO_2 in the E-SO about 4% compared to corresponding pollutants in TT-UE; about same percentage of reduction is observed for fuel.

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Table 5 Results for Network-2

Flow Assumptions	Decision Parameter	Measured Parameters		
		Travel Time (min)	CO_2 (g)	Fuel (g)
System Optimal	Travel Time	170,537.6	18,158,715.6	5,718,067.3
	CO_2	173,424.8	17,632,741.7	5,534,594.8
	Fuel	173,392.1	17,634,180.4	5,534,098.1
User Equilibrium	Travel Time	184,693.6	18,370,592.9	5,783,289.7
	CO_2	223,813.3	19,919,972.1	6,260,020.3
	Fuel	226,155.7	18,927,511.7	5,945,828.8

7. CONCLUSIONS

This study focuses on developing environmental traffic assignment (E-TA) models that minimize emissions and fuel consumption. Two behavioral principles, user equilibrium (UE) and system optimal (SO), are extended to E-TA. Emissions of CO_2, fuel, NO_x, CO, and H are obtained using the CMEM modal emissions model. This data are used to fit analytical models as functions of average vehicle speed. Emissions-flow relationships of CO_2 and fuel are developed based on speed-emissions relationships. The flow-emissions relationships are used in E-TA models. Studying the properties of the flow-emission relationship indicates that any solution to the E-TA models is a global solution, but the uniqueness is not guaranteed.

The E-TA for CO_2 and fuel are solved for two hypothetical networks under both UE and SO assumptions. The results for both networks show that the system-wide emission of CO_2 (fuel) is minimum for CO_2 (fuel) based SO model. This is expected since the objective function of SO models minimizes the performance parameters (travel time, CO_2, or fuel). Interestingly, all E-UE models produced paradoxical results. When travelers make routing decisions to minimize their own CO_2 (fuel), the actual effect is to increase CO_2 (fuel) for individuals as well as for the network. In other words, CO_2 produced by an individual in CO_2-based UE model is considerably higher than the CO_2 produced in travel time-based UE model. Thus, systems in which travelers are informed of their emissions and also tolled for the amount of emissions may not result in emissions reduction. However, a system in which traffic is centrally controlled can be expected to reduce emissions and/or fuel consumption.

There is no highway transportation system in which routing decisions are based solely on minimizing emissions or fuel consumption. However, it is reasonable to expect in the future that emissions and fuel consumption will be important parameters in addition to travel time, tolls, etc. Thus, it is important to understand the impacts each parameter on travelers and network

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performance. This study using only two hypothetic networks suggests the complex interaction of factors in the system will make understanding the impacts of emissions-based user decisions harder than one would expects.

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