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Interpreting Orthogonal Triple-Wire Data From Very High Turbulence Flows

For turbulence intensities of up to 30 percent, an orthogonal triple-wire probe can be used to make accurate measurements of the instantaneous velocity vector. Above this limit difficulties arise in the interpretation of the data due to the problem described as rectification. This paper presents a means by which data from an orthogonal triple-wire probe may be interpreted for single point measurements in Gaussian turbulence with intensity up to 50 percent resulting in unbiased estimates of the velocity mean vector, Reynolds stress tensor, and time correlation coefficients.

Introduction

Researchers commonly use hot wire anemometers in the study of air boundary layers, but, since a hot wire probe responds in a similar manner to either forward or backward motion in the flow, very high levels of turbulence create an uncertain relationship between the output from these anemometers and the statistics of the velocity field being measured. The studies of Tutu and Chevray[1] (1975) and Andreopoulos[2] (1983) help set limits for the domain on which instantaneous velocity measurements can be made with hot wire probes without significant errors due to the physics of the response of these probes to turbulent fluctuations. Tutu and Chevray (1975) present the problem for cross-wire probes, analyzing the response of the probe to high levels of turbulence assuming a trivariate normal distribution function for the component velocities and a simple model for the rectification of negative instantaneous velocities. Andreopoulos (1983) performs a similar analysis for triple-wire probes, concluding that mean velocities and turbulence intensities can be measured accurately at turbulence intensities of up to 30 percent.

The present study proposes to extend the domain on which an orthogonal triple wire can be reliably employed, beyond the limits set by previous investigations of the problem of rectification, by presenting a method for interpreting data from orthogonal triple-wires for turbulence intensities of up to 50 percent for situations when the underlying velocity distribution is trivariate normal or nearly so.

[1]Tuto, K. N., and Chevray, R., 1975, "Cross-wire Anemometry in High Intensity Turbulence," *Journal of Fluid Mechanics*, Vol. 71, pp. 785–801.

[2]Andreopoulos, J., 1983, "Statistical Errors Associated with Probe Geometry and Turbulence Institute in Triple Hot-Wire Anemometry," *J. Phys. E: Sci. Instrum.*, Vol. 16, pp. 1264–1271.

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The Nature of the Errors

Hot wire anemometers incur deterministic errors when subjected to highly turbulent velocity fluctuations. Tutu and Chevray (1975) identify two important physical problems in the use of an orthogonal cross-wire probe in turbulent flows of high intensity. First, the binormal (W-component for a cross-wire probe) fluctuations contributed significantly to the anemometer output and become misinterpreted in the values of the means and fluctuations of the U and V components. Second, very large turbulent fluctuations at low mean velocities may be "rectified" due to the inability of the hot wire probe to distinguish the direction of the velocity vector. Consider the following response equation for a single wire normal to the mean flow direction:

$$U_{\text{eff}}^2 = k_1 U^2 + k_2 V^2 + k_3 W^2 \tag{1}$$

where U_{eff} is the instantaneous effective cooling velocity on the wire and k_1, k_2, and k_3 are the directional sensitivity coefficients for the wire. Suppose U_{eff} is used to estimate U. For high levels of turbulent fluctuations, not only do the V and W components bias the estimate of U, but also a negative value for U cannot be distinguished from a positive value.

While both cross-wire and single-wire probes suffer both types of errors, an orthogonal triple-wire probe identifies the bi-normal component on each wire, alleviating the problem of inter-component cross-talk and leaving only the problem of rectification. Figure 1 is a sketch of a typical orthogonal triple-wire probe. Velocity components in the wire coordinate system, denoted X_j or (X, Y, Z), are related to velocity components in the laboratory coordinate system, denoted by U_i or (U, V, W), through the geometry of the probe, the pitch angle ω, and the roll angle α. The response equations for the orthogonal triple-wire relate the effective cooling velocities on the wires, $U_{\text{eff},i}$, to the component velocities in the wire coordinate system, X_j, as follows:

$$U_{\text{eff},i}^2 = K_{ij} X_j^2 \tag{2}$$

where K_{ij} is a matrix of directional sensitivities (typically as-

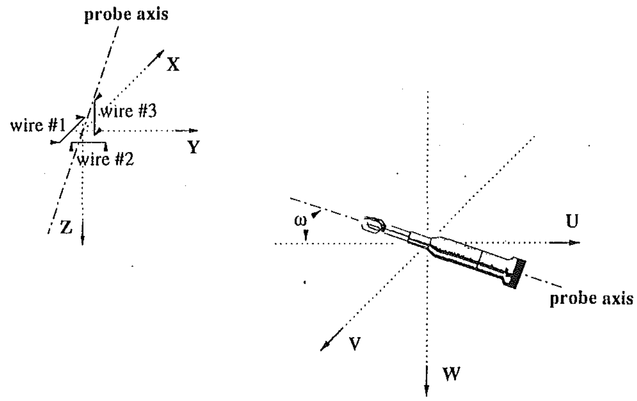

Fig. 1 The orthogonal triple-wire probe. Wire (*X*, *Y*, *Z*) and laboratory (*U*, *V*, *W*) coordinate systems

sumed constant). The effective cooling velocities, $U_{\text{eff},i}$, are related to the squares of the component velocities in wire coordinates, X_j, disregarding any directional information about X_j. A simple rotation relates the velocity vector in the laboratory coordinate system, U_j, to the velocity vector in the wire coordinate system, X_j, as follows:

$$U_i = C_{ij} X_j \tag{3}$$

where C_{ij} is a coordinate rotation matrix accounting for wire geometry including probe pitch and roll angles. In practice one would estimate $U_{\text{eff},i}$ from the component anemometer bridge output voltages, E_i, and then estimate the component velocities, X_j, via Eq. (2), assuming the direction of each component velocity is positive. Conceptually, rectification errors occur when the instantaneous velocity vector falls outside the first octant in X_j velocity space.

The Impact of Rectification Errors on Velocity Statistics

A one-dimensional, idealized illustration of the rectification process and its influence on the sample distribution of velocity vectors is presented in Fig. 2. The example presented in Fig. 2 assumes that all of the velocity fluctuations occur in the U component of velocity and that there is no prong or probe-stem interference with the velocity vector. Under these assumptions, negative velocities become interpreted as positive velocities of the original magnitude, shifting the mean of the overall distribution of velocities to the right and reducing its variance.

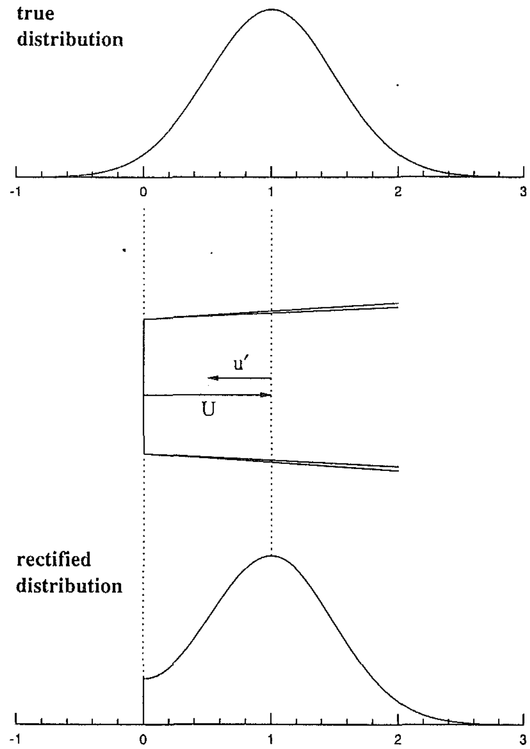

Fig. 2 Schematic of the rectification process

Although three-dimensional cases are more complicated than this simplified one-dimensional example, the influence of rectification errors on velocity statistics can generally be understood through the distortion of the distribution of velocity vectors. Rectification errors translate into incongruities between the "true" underlying distribution of velocity vectors and a measured/interpreted sample distribution. In three-dimensional cases, estimates for the moments of the velocity joint distribution function (e.g., the mean vector, Reynolds stress tensor, time correlations, etc.) are biased as a consequence of these incongruities.

A simulation of triple-wire probe response to high turbulence was used to identify which regions of the velocity joint distribution function are affected by rectification errors and to provide insight into new ways to interpret samples containing

Nomenclature

C_{ij} = coordinate rotation matrix
C_{ij}^T = transpose of the coordinate rotation matrix
CDF = cumulative distribution function
Cov = covariance
E_i = hot wire bridge voltage for ith channel
f = frequency
$I(\ )$ = indicator function
k_1, k_2, k_3 = hot wire probe directional sensitivity coefficients
K_{ij} = matrix of directional sensitivity coefficients
M = sample median
N = sample size
PSD = power spectral density function
Q_t, Z_t = random variables
R_{ij} = Reynolds stress tensor
r_τ = empirical autocorrelation coefficient for time lag τ
t = time
U, V, W = velocities in laboratory coordinate system
U_j = jth component of velocity in laboratory coordinates
$\mathrm{U_j}$ = jth component of velocity at time lag τ
$U_{\text{eff},i}$ = hot wire effective velocity for the ith channel
u', v', w' = standard deviation in U, V, and W, respectively
X, Y, Z = velocities in (triple) wire coordinate system
X_j = jth component of velocity in wire coordinates
$\mathrm{X_j}$ = jth component of velocity in wire coordinates at time lag τ
α = triple-wire probe roll angle
γ = empirical standard deviation
μ = mean
ρ = correlation coefficient
ρ_τ = autocorrelation coefficient for time lag τ
σ = standard deviation
τ = time lag between observations in a stationary time series
ω = triple-wire probe pitch angle

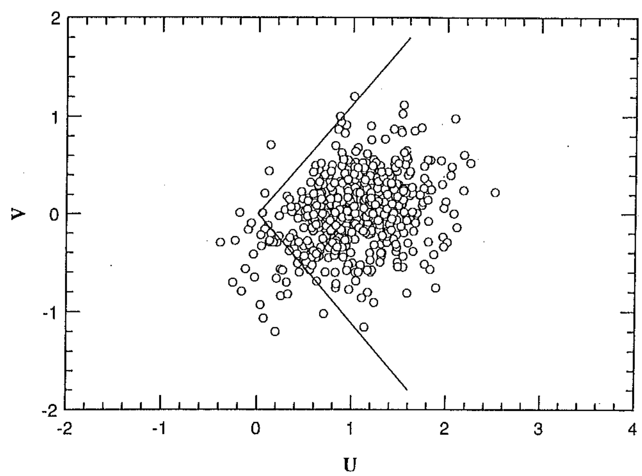

Fig. 3(*a*) Simulated "true" velocity vectors

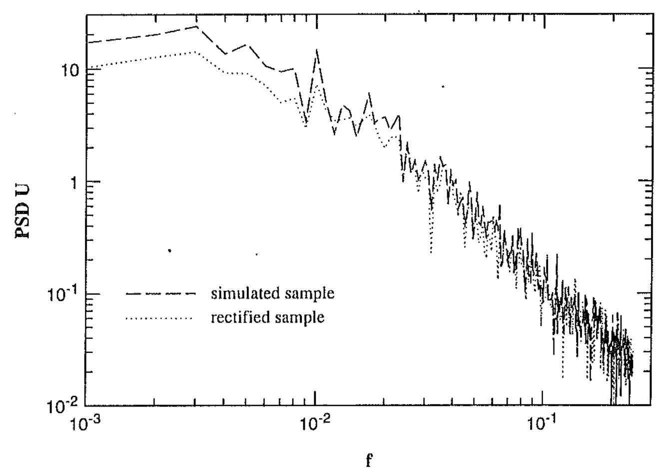

Fig. 4 Rectification affects the spectrum of component velocities

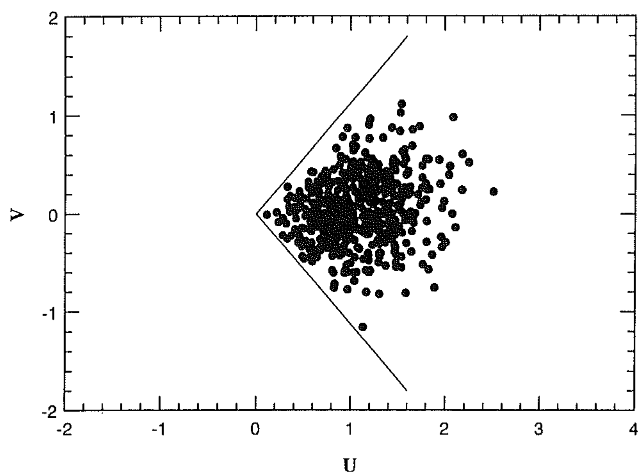

Fig. 3(*b*) Simulated response of triple-wire probe to velocity vectors plotted in Fig. 3(*a*)

observations subject to rectification errors. The simulation was performed in two parts. First, a random sample from the "true" velocity distribution was simulated by selecting vectors from a trivariate normal distribution having a specified mean vector, Reynolds stress tensor, and *U*-component autocorrelation function. Then the response of the triple-wire probe was simulated by interpreting the sample from the "true" distribution through the response equations for the triple-wire probe, Eqs. (1)–(3) above. Details of the simulation of the velocity field are given in Appendix A.

Figure 3 illustrates the simulated response of an orthogonal triple-wire probe ($\omega = 0$ deg, $\alpha = 90$ deg) to high turbulence, with the data represented in the *U*-*V* plane. For this simulation $u'/U = 0.5$, $v' = w' = 0.8u'$, and the correlation coefficient between u' and v' equals 0.3. The lines in Fig. 3 represent planes bounding the first octant of the velocity space in wire coordinates. The hollow dots in Fig. 3(*a*) correspond to "true" data points and the solid dots in Fig. 3(*b*) correspond to a mapping of the same true data points onto the *U*-*V* plane after rectification. If a velocity vector does not suffer an error due to rectification, a solid dot in Fig. 3(*b*) is plotted at the same coordinates as the hollow dot in Fig. 3(*a*) corresponding to the true velocity. If the velocity vector suffers rectification, then the true velocity plotted in Fig. 3(*a*) is mapped into a new coordinate location within the cone of interpretation in Fig. 3(*b*). It should be noted that some of the data points that fall near but still within the planes bounding the first octant of the velocity space in wire coordinates in Fig. 3(*a*) appear as rectified in Fig. 3(*b*). This occurs when a velocity vector presented in Fig. 3(*a*) has a nonzero component in the *W* direction (not shown in Fig. 3(*a*)) that is sufficiently large to place the data point outside the first octant of the velocity space in wire coordinates. The result is that the data point appears as rectified in Fig. 3(*b*).

Consider the statistics one might calculate if one had access to the true velocity readings, i.e., the hollow dots in Fig. 3(*a*), and compare them to those one would calculate using the "rectified" velocity reading, i.e., the solid dots in Fig. 3(*b*). Using the "rectified" data, one would overpredict the means of both *U* and *V*, underpredict the variance of both *U* and *V*, and underpredict the correlation between *U* and *V*. Note that if the correlation coefficient between u' and v' had been zero instead of 0.3, the mean value of *V* would be unbiased, but the variance of *V* would still be underestimated.

Figure 4 shows the power spectral density function for the *U*-component of velocity for a simulated turbulence of $u'/U = 0.5$, $v' = w' = u'$. The effects of rectification are apparent in the low frequency, energy containing portion of the spectrum, although it isn't clear from Fig. 4 how this problem arises. Reconsidering the problem in the time domain one could construct a bivariate plot of $U(t = 0)$ and $U(t = \tau)$ (where τ = lag time between observations in a stationary time series) that would display the same features as Fig. 3 did for *U* and *V*. In such a plot, the correlation between $U(t = 0)$ and $U(t = \tau)$ would then represent the autocorrelation for *U* for time lag τ. The errors in the estimates of the autocorrelations due to rectification result in the errors seen in the spectrum in Fig. 4. These errors are concentrated in the portion of the spectrum containing the largest fluctuations in the turbulence.

Figures 5 and 6 show the simulated errors in the cumulative distribution functions of the individual components of velocity for both the laboratory coordinate velocities, (*U*, *V*, *W*), and the wire coordinate velocities, (*X*, *Y*, *Z*), for $u'/U = 0.5$, $u' = v' = w'$, and with the mean velocity vector aligned with the axis of the probe stem. Figure 5 shows that in the laboratory coordinates, rectification affects the entire distribution for each component velocity. The *U* component is primarily affected from the low side of the distribution, while *V* and *W* are affected on both sides. This is due to the geometry of the probe and its orientation to the mean velocity vector. Figure 6 shows the corresponding errors in the velocity components in wire coordinates. In the wire coordinate system the errors only affect one side of the distribution for each component. Observations in the wire coordinate velocities above their respective medians are unaffected by rectification for turbulence intensities of 50 percent. This fact can be used as the basis for a technique for interpreting triple wire data in high turbulence.

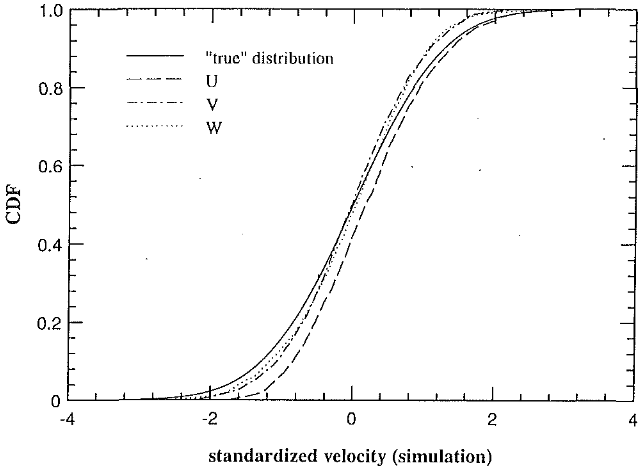

Fig. 5 In laboratory coordinates, rectification entirely distorts the velocity distributions

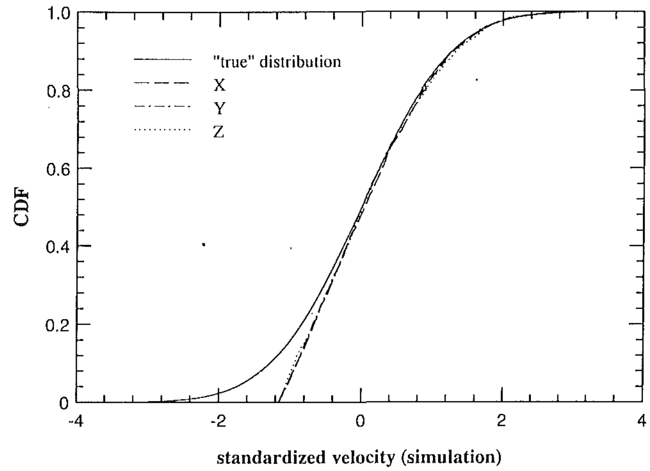

Fig. 6 In wire coordinates, rectification only distorts one side of the velocity distributions

Median Based Estimators

The rectification of the velocity signal by hot wire anemometers operating in highly turbulent flows causes significant errors in the conventional estimates of the mean velocity vector, the Reynolds stress tensor, the one-dimensional power spectrum, etc. Biased estimates of the moments of the velocity joint distribution function occur because "rectified" measurements contaminate certain regions of the distribution. Can one infer the moments of the underlying true cumulative distribution function (CDF) given the partially distorted, rectified CDF? In the wire coordinate system rectification errors contaminate the marginal distributions for X_j for low values. Are there measures of location (the center of the distributions) and scale (measure of deviations) which resist the errors in the low velocity tails of these distributions?

In the literature of statistics, Robinson[3] (1980) discusses the problem of estimating autocorrelation functions for a stationary Gaussian time series, Q_t ($t = 1, 2, \ldots, n$), that has been censored, i.e., a time series in which some of the data is selectively missing or excluded from the analysis. He specifically considers the univariate case where observations fall below the median value are censored but observations at or above the median are measured without error. Under these assumptions, Robinson (1980) proves that

$$\lim_{n\to\infty} M = \mu$$

$$\lim_{n\to\infty} \gamma = \sigma$$

$$\lim_{n\to\infty} r_\tau = \rho_\tau$$

where M is the sample median and γ and r_τ are defined as follows:

$$\gamma = \left\{ \sum_{s=1}^{n} \frac{1}{N} I(Z_s > 0) \right\}^{-1} \left\{ \sum_{t=1}^{n} Z_t^2 I(Z_t > 0) \right\}$$

$$r_\tau = 2\sqrt{2\pi}\delta_\tau - 1$$

where

$$Z_t = Q_t - M$$

$$\delta_\tau = \sum_{t=1}^{n-1} \frac{1}{N-\tau} \frac{Z_t}{\sqrt{\gamma}} I(Z_t > 0 \text{ and } Z_{t+\tau} > 0)$$

Robinson (1980) concludes that an unbiased estimate of the autocorrelation function for the true time series can be made from the censored time series using estimators based on the median of the series and observations greater than the median.

Robinson's (1980) statistical analyses of censored data can be extended to the interpretation of rectified triple-wire data. Following Robinson's (1980) logic, the correlation between two components can be estimated by

$$r_{uv} = 2\sqrt{2\pi}\delta_{uv} - 1$$

where

$$\delta_{uv} = \sum_{u=1}^{n} \frac{1}{N} \frac{Z_u}{\sqrt{\gamma}} I(Z_u > 0 \text{ and } Z_v > 0)$$

Furthermore, under the assumptions of Robinson's (1980) analysis it can be shown that

$$\lim_{n\to\infty} r_{uv} = \rho_{uv}$$

This last result can be used to estimate both the autocorrelation of U for time lag τ (from the bivariate CDF of $U(t = 0)$ and $U(t = \tau)$) and the components of the Reynolds shear stress.

For cases where the contamination of the marginal distributions of X_j due to rectification is limited to values below their respective medians and it is reasonable to assume that the underlying joint distribution is Gaussian, the following statements are true:

(1) For each component X_j, the median is an unbiased estimate of the mean.

(2) For each component X_j, γ_j is an unbiased estimate of the Reynolds normal stress for that component.

(3) For each pair of components (X_i, X_j), $r_{ij}\gamma_i\gamma_j$ is an unbiased estimate for the Reynolds shear stress between those two components (no summation on i and j).

(4) For each pair of components ($X_i(0)$, $X_j(\tau)$, r_τ is an unbiased estimate for the time correlation between components X_i and X_j for time lag τ.

The statistics for the components of velocity in wire coordinates determine the corresponding statistics in laboratory coordinates by the following relationships:

$$U_i = C_{ij} X_j \quad (4)$$

$$R_{ij,\text{lab}} = C_{ik} R_{kl,\text{wire}} C_{lj}^T \quad (5)$$

$$\text{Cov}(U_\alpha, U_\beta)_{\text{lab}} = C_{\alpha j} C_{\beta k} \text{Cov}(X_j, X_k)_{\text{wire}} \quad (6)$$

[3]Robinson, P. M., 1980, "Estimation and Forecasting for Time Series Containing Censored or Missing Observations," *Time Series Analysis*, O. D. Anderson, ed., North-Holland Publishing Co., pp. 167–182.

Experimental Validation of the Technique

The usefulness of the estimators proposed in the previous section depends on two assumptions. The first is that the true underlying distribution function is Gaussian. The second is that the contamination of the marginal distributions of X_j due to rectification is limited to values below the respective medians of X_j. The first assumption depends on the flow in which one measures and will be addressed in the next section. The second assumption depends on the response of the probe to high turbulence and will be addressed presently.

Using a simulation and a model for rectification errors, it is easy to answer the question: Will rectification errors due to a specified level of turbulence be restricted to values below the medians of X_j? It would be convenient to be able to use simulations of the response of the probe as the sole arbiter in setting limits on levels of turbulence below which the Reynolds stresses could be estimated accurately using the relations above, but there are compelling reasons to require experimental confirmations of the simulations.

There are two questions which must be answered experimentally. First, is the rectification experienced by the probe the same as the mathematical rectification used to simulate probe response? This cannot strictly be the case, since for high turbulence prong and even probe stem interference is certain to occur for some values of the instantaneous velocity vector. Nonetheless, the measured "rectified" joint CDF may be acceptably close to the mathematically rectified joint CDF. If this is the case, one could test hypotheses about the underlying distribution using the simulation. Second, even if not exact, can the simulations be used to set the limits on the domain on which the proposed estimators can be applied? In other words, is the penetration of real rectification errors into the velocity joint CDF about the same as the penetration predicted by means of the probe simulation?

A calibrated, qualified, orthogonal triple wire probe[4] was placed in a free jet at a location where the turbulence intensity was high but did not exceed 30 percent (an accepted upper limit for reasonably accurate measurements). Once the desired statistics had been measured for this location in the free jet, the probe, while remaining at the same point in the flow field, was then pitched to force the instantaneous velocity vector outside the acceptable cone for a significant proportion of the total sample in subsequent tests. The probe started at 0 degrees and was subsequently pitched to 20 and 30 degrees. (As the probe was pitched to 20 and 30 degrees, there were instances when the equation for the wire velocities in terms of the effective cooling velocities, Eq. (2), could not be solved. This was due to instantaneous interference with a prong, causing wires to "see" different velocities. When this type of error occurred components for which the equations cannot be solved were set to zero. While this interpretation would bias mean based estimators it does not affect median based estimators.) Since the results of the experiment were to be compared to the results for the probe simulation run under the same conditions as the experiment, the measurements for the probe pitched at 0 degrees were used to establish the inputs to the simulation. The simulation was then run with the probe pitched at 0, 20, and 30 degrees to the mean flow direction.

Figure 7 compares the results from the experiment to the results from the simulation of the experiment. Figures 7(*a*) and 7(*b*) show that while the actual rectification for the X component of velocity differs from the simulated rectification, both

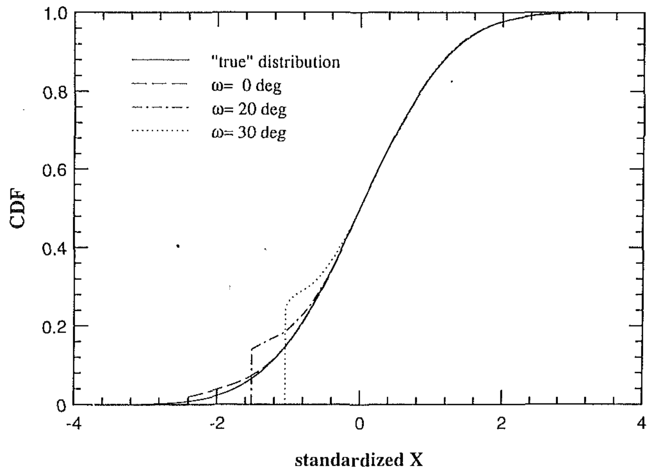

Fig. 7(*a*) Empirical distribution of X for rectification experiment

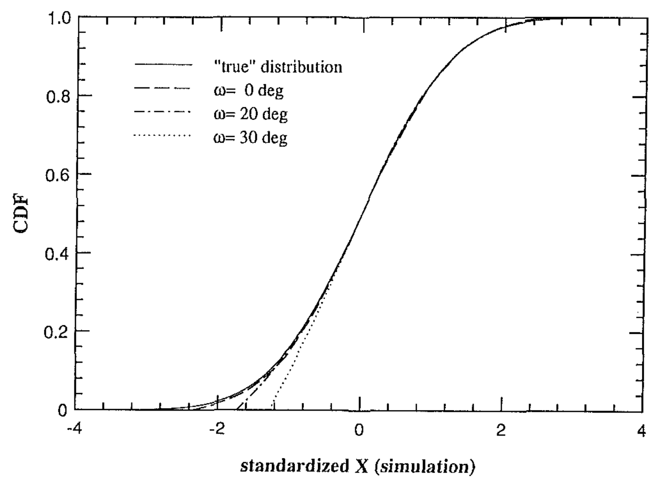

Fig. 7(*b*) Simulated distribution of X for rectification experiment

the experiment and the simulation agree on the location of the point beyond which the distribution is unaffected. Similar results have been obtained for the Y and Z components as well. While the simulation cannot be used to account for actual rectification errors, it can be used to set accurate limits for the physical conditions under which the proposed median based estimators will resist errors.

Discussion

The errors incurred by an orthogonal triple wire probe operating in high turbulence are deterministic, not random. The errors in the statistics of the velocity field are repeatedly biased. Conventional estimators (e.g., means and moments) are highly sensitive to rectification errors, but median based estimators resist these errors for turbulence intensities up to 50 percent. From a simulation at $u'/U = 0.5$, the errors in wire velocity component medians are 1 percent. At $u'/U = 0.7$, errors in these medians are only 5 percent.

The assumption that the underlying distribution is Gaussian (on which these procedures rest) can be partially tested. One could run a Kolmogorov-Smirnov test on the hypothesis that the "uncontaminated" portion of the distribution has a Gaussian shape. It should be noted that such a test cannot prove that the underlying distributions are as assumed. It only tests whether or not the available "uncontaminated" data are consistent with this assumption.

The proposed measures of turbulence come purely from the need to resist the deterministic errors incurred by hot wire probes in highly turbulent flow fields. Does it make sense from

[4]Details of the calibration and qualification procedures associated with the use of the triple wire probe are considered to be either established practices or special issues that fall outside the scope of the concerns of this paper. The details of the procedures associated with the present implementation can be found in the following: Maciejewski, P. K., and Moffat, R. J., 1989, "Heat Transfer with Very High Free-Stream Turbulence," Report No. HMT-42, Thermosciences Division of Mechanical Engineering, Stanford University.

a physical point of view to use these measures? From an empirical point of view, it is not clear which measures of turbulence will correlate with other parameters of interest. There is no a priori reason to prefer the mean to the median. The strongest argument for maintaining the conventional moments measures of turbulence is that we are familiar with how to think about them in the context of the Reynolds averaged equations. For Gaussian distributions the median based estimators proposed in this paper estimate the moments measures and can be used directly to close moments models. For underlying distributions that are non-Gaussian, one must either relate the median based measures to the expected moments measures and use existing turbulence models or develop new models which employ median based estimators directly.

Recommendations

If it is reasonable to assume the underlying velocity joint distribution function is Gaussian, the median based estimators presented above yield unbiased estimates of the mean vector, Reynolds stress tensor, and two-component time correlations. One can use these estimators on data taken with an orthogonal triple wire probe for flows with turbulence intensities in the neighborhood of 50 percent. Simulations of the probe response can be used to establish the limits of reliable use. If the underlying distribution is significantly non-Gaussian, then further precautions should be taken in interpreting triple-wire data in very high turbulence.

Acknowledgments

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APPENDIX A

Simulating High Turbulence at a Point

A random sample from a trivariate normal distribution with a specified mean vector and covariance matrix (Reynolds stress tensor) simulates a sample from the "true" population of velocity vectors, U_{ij} (ith observation, jth component). Individual components of the sample are then correlated in time using a first order autoregessive model given by

$$U'_{ij}(t) = \alpha_j U'_{ij}(t-1) + U_{ij}(t)$$

where $U'_{ij}(0) = U_{ij}(0)$, and $U_{ij} \sim N(\mu_j, \sigma_j^2)$ and α_j are specified by the user. This model simulates a time series having component autocorrelation functions given by

$$\rho_j(\tau) = \alpha_j^\tau$$

component integral scale given by

$$T_j = -1/\ln(\alpha_j)$$

and component spectra given by

$$\mathrm{PSD}_j(f) = -2\ln(\alpha_j)/\{(\ln(\alpha_j))^2 + (2\pi f)^2\}$$

This model provides a simulated sample from the population of velocity vectors with an underlying analytic joint pdf and analytic component spectra. The sample joint pdf and component spectra can be compared directly to their underlying true counterparts.