Proactive Data Download and User Demand Shaping for Data Networks

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Abstract—In this work, we propose and study optimal proactive resource allocation and demand shaping for data networks. Motivated by the recent findings on human behavioral patterns, and the emergence of highly capable handheld devices (such as smart phones), our framework aims to smooth out the network traffic over time. Such a load balance minimizes the total cost required for data delivery.

The framework utilizes proactive data services as well as smart content recommendation schemes for shaping the demand. Proactive data services take place during the off-peak hours based on a statistical prediction *demand profile* for each user, whereas smart content recommendation assigns modified valuations to data item so as to render the users' demand less uncertain. Hence, it boosts the performance of proactive services. We conduct theoretical performance analysis that quantifies the leveraged cost reduction through the proposed framework. We show that the *cost reduction* scales with the number of users as the cost function itself does. Further, we prove that *demand shaping* through smart recommendation strictly reduces the incurred cost even below that of proactive data service only.

Index Terms—Resource allocation, wireless networks, convex optimization, predictable demand.

I. INTRODUCTION

THe vast expansion of highly capable smart wireless devices has powered a substantial transformation of the global data network into a more mobile, increasingly demanding, and socially more interconnected form. Such a wireless revolution has raised major concerns about an inevitable surge of wireless traffic load by throughput-hungry application that existing resource allocation schemes may not stand. These applications include multimedia services which constitute more than 50% of the total Internet load [1], [2]. On the other hand, there is a growing body of evidence that the available spectrum, which defines an ultimate resource for wireless communications, is suffering an inherent underutilization problem as has been reported in the recent studies by FCC [3], [4].

Consequently, there is an urged call for the development of more advanced and sophisticated techniques improving the wireless resource management and allocation. Recently, the notion of dynamic spectrum access (DSA) has been introduced as a remedy to the spectrum underutilization problem, and has been enabled through the cognitive radio technology [5], [6]. The cognitive radio approach, however, is still facing significant technological hurdles [6], [7] and, will offer only a partial solution to the problem. This limitation is tied to the main reason behind the underutilization of the spectrum;

namely *the large disparity between the average and peak traffic demand in the network*.

Actually, one can see that the traffic demand at the peak hours is much higher than that at night. Now, the cognitive radio approach assumes that the users will be able to utilize the spectrum at the off-peak times, but at those times one may expect the cognitive radio traffic characteristics to be similar to that of the primary users (e.g., at night most of the primary and cognitive radio users are expected to be idle).

Recently, we have proposed the notion of *proactive resource allocation* [8] as a remedy to the spectrum crisis. The technique aims at exploiting the *predictable* human demand as well as the powerful processing capabilities and the large memory storage offered by the smart wireless devices in smoothing out the wireless network traffic over time by *proactively serving predictable peak-hour requests during the off-peak time*. Hence, the peak-to-average demand ratio is minimized and significant utilization for the available resources is provided [9]-[11].

There exists a substantial evidence, both for general human behavior [12]-[16] and specifically for wireless data users [17]-[23], that supports the underlying premise of large-timescale (in the order of minutes to hours) user predictability which, in turn, motivates our proactive design framework. In [9], the notion of proactive resource allocation for wireless unicast networks has been introduced and analyzed under perfect predictability of users' demand, and the performance has been quantified through the diversity gain metric. The results have been extended to multicast networks in [10] where multicast alignment gain revealed a potential for a significant reduction in the resources required to attain a certain level of quality of service (QoS). In [11], proactive resource allocation has been investigated in cognitive radio networks, where the *good citizen* phenomenon is demonstrated. Such a phenomena revealed an enhanced QoS for a *non-proactive* cognitive user while primary users employ proactive resource allocation.

There are also emerging works aiming to balance the network traffic over time through *time/load-dependent pricing* (see for example [24]-[27]). The main approach is to adjust the service price depending on the total network load in a way that assigns low prices to off-peak services and higher prices to peak-hour demand. A recent study in [24] highlights the potential of smoothing-out the network traffic through time-dependent pricing. In fact, some network operators outside the USA (such as Orange, MTN and Uninor) have already started using adaptive pricing schemes to mitigate the excessive cost resulting from high bandwidth consumption [25]-[26].

On the other hand, there exists a recent work [27] that

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considers the same problem in the USA. It optimizes price allocation mechanisms that encourage network users to *delay* their demand to the off-peak hour based collected statistics about their willingness to defer to a time when the service prices are considerably reduced. While pricing can be used for demand shaping (see e.g. [28]), our proposed approach sheds light on the use of recommendation schemes as another important parameter that controls the future demand. Further, we highlight the point that our proposed approach *does not* aim at pushing users to defer their demand in time. Instead, it encourages them to be deterministic about their future demand.

In this paper, we develop a framework for optimal proactive resource allocation and demand shaping for wireless networks comprising a service providers and associated subscribers. The subscribers' demand assumes cyclostationary statistics as it repeats itself over finite time durations, whereby the service provider can track, learn and construct a user **demand profile** to each subscriber. These profiles are then used to determine proactive data downloads to the users in a way that minimizes the time average expected cost incurred by the service provider while providing reliable data delivery. Moreover, we consider a further improvement to the cost performance by applying slight modifications to the demand profiles of the users so as to render them more deterministic. We refer to this operation by **demand shaping**. We develop a smart recommendation scheme to perform demand shaping while maintaining the user satisfaction about the quality of offered data items. The demand shaping scheme is proved to strictly reduce the cost even below proactive downloads alone. The main contribution of this work are listed as follows.

- In Section II, we provide a description of the time-slotted system model, present the notion of user demand profile and its *cyclostationary* nature, and layout the time average expected cost for a traditional *non-proactive* network to serve the users' demand.

- In Section III, we formulate the general time average proactive resource allocation problem that incorporates joint data download and demand shaping. We prove the existence of a steady state solution to the infinite horizon version of it.

- In Section IV, we consider the proactive data download side of the problem. We adopt the cost reduction leveraged through proactive downloads as our metric of interest. An upper and lower bounds on the optimal cost reduction are established, and its asymptotic scaling laws when the number of users grows to infinity are characterized. We show that, successive time slots that witness a disparate levels of traffic load result in a cost reduction that scales in the same order of the non-proactive cost itself.

- In Section V, we study the joint allocation of proactive data downloads and user demand profile. we aim at further reduce the time average expected cost by pushing the user demand profile more deterministic. In Section V-A, we characterize the optimal solution to the problem under weak user satisfaction constraints where each user is flexible to follow the recommendations made by the service provider.

- In Section V-B, we take the user satisfaction into account. We use the entropy about the given user profile to quantify the uncertainty about the future demand, as well as the flexibility of each user to follow a new set of recommendations. The new set of user satisfaction constraints renders the problem computationally intractable. However, we propose an iterative scheme where an almost-surely *strictly* reduced cost below that of proactive downloads alone can be obtained.

- In Section VI we formulate and study a data-item recommendation problem that assigns new ratings to the recommended data items for each user that 1) achieve the modified profiles, and 2) stick as close as possible, in the Euclidean-distance sense, to the actual ratings made by the corresponding users.

- The work is concluded in Section VIII.

II. SYSTEM MODEL

We consider a network comprising N users with a variable demand and a service provider that responds to users' requests in a timely basis. The service provider has a total of M different data items that each user can request from in a random fashion. Each data item m is assumed to have a size of $S(m) > 0$, and

$$\check{S} := \min_{m \in \{1, \cdots, M\}} \{S(m)\} > 0, \tag{1}$$

$$\hat{S} := \max_{m \in \{1, \cdots, M\}} \{S(m)\} < \infty. \tag{2}$$

In a time-slotted system, the content of each data item is consistently updated every time slot, where such a content could be a movie (as in YouTube and Netflix), a soundtrack (as in Pandora), a social network update (as in Facebook and Twitter), a news update (as in CNN and Fox News), etc. We consider the application-layer timescale in which the duration of a time slot is the time taken for a user to completely run the requested data item, which can be in the order of minutes or possibly hours. At the beginning of each time slot, the service provider collects the demand of all users and supplies them with the requested data items, which have been updated over the previous time slot.

Users' demand profiles: We assume that the demand of each user can be tracked, learned, and predicted by the service provider over time. The service provider constructs a *demand profile* for every user n and time slot t, denoted $\mathbf{p}_{n,t} = (P_{n,t}(m))_{m=1}^{M}$, where $P_{n,t}(m)$ is the probability that user n requests item m in slot t. We assume that *each user can request at most one data item per time slot*. Then, we model the statistics of the predictable user demands as follows:

- The demand of user n at slot t is captured by a random variable $\mathbb{I}_{n,t}(m)$ where

$$\mathbb{I}_{n,t}(m) = \begin{cases} 1, & \text{with probability } P_{n,t}(m), \\ 0, & \text{with probability } 1 - P_{n,t}(m). \end{cases}$$

- For any time slot t, $\mathbb{I}_{n,t}(m)$ is independent of $\mathbb{I}_{n,t+1}(k)$ for all m, k.

- For any two users n, k such that $n \neq k$, $\mathbb{I}_{n,t}(m)$ is independent of $\mathbb{I}_{k,t}(j)$ for all m, j.

- At slot $t \geq 0$, user n requests at most one data item. Hence $\sum_{m=1}^{M} \mathbb{I}_{n,t}(m) \leq 1$.

- The probability that user n does not request any data item at slot t is $q_{n,t} := 1 - \sum_{m=1}^{M} P_{n,t}(m)$.
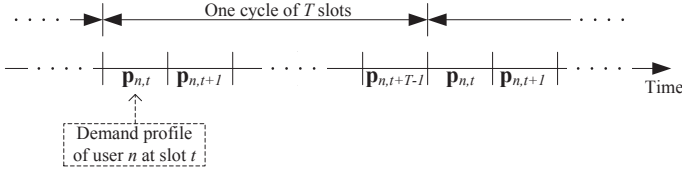
Fig. 1: Cyclostationary demand profiles over time. Every T time slots, the demand profile of user n is repeated.

Further, the demand profile of each user follows a *cyclo-stationary* pattern that repeats itself consistently in a period of T time slots as shown in Fig. 1. The T-slot period can be interpreted as a single day through which the activity of each user varies each hour, but occurs with the same statistics consistently each day. Thus, we can write $\mathbf{p}_{n,t+l} = \mathbf{p}_{n,t+kT+l}$ for any non-negative integer k, and $l = 0, \cdots, T-1$.

Data item valuations: Define $v_{n,t}(m) \in [0,1]$ as the valuation (rating) of data item m as offered by the service provider to user n at time t. The offered valuations depend on the user preferences and interests, and can be estimated through different techniques, such as collaborative filtering.

The probability that user n requests data item m at time slot t is modeled as $P_{n,t}(m) := \phi_{m,t}(\mathbf{v}_{n,t})$, where $\phi_{m,t} : [0,1]^M \rightarrow [0,1]$ is a non-negative function that maps the ratings of user n into a corresponding probability of requesting item m specifically at time slot t for any m, n, t. The function $\phi_{m,t}$ captures the relative differences between the ratings of the M data items as seen by user n.

Incurred cost: To supply requested data items, the service provider incurs a certain cost due to the resources consumed at each time slot, which is supposed to depend on the total load created by the users' demand. Suppose that L is the total network load at a given slot, then the aggregate cost incurred by the service provider is $C(L)$, where $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a smooth, strictly convex, and monotonically increasing cost function. We assume also that the total load at a given time slot is not observable by the service provider, an assumption that is justified by practical data delivery scenarios whereby users' requests arrive to a widespread content delivery network associated with the service provider (cf. [29], [30]) rendering the observability of the total load in each slot by a central entity impractical.

However, this assumption does not contradict the assumption that service provider can construct users' profiles through tracking and learning their demand since such a process is supposed to have taken place through a long period of time over which the service provider can retrieve sufficient statistics about the users' preferences and activities from the content delivery network, such a step that we assume to be performed initially.

We consider the time average expected cost incurred by a *non-proactive* network, a network whose service provider does not exploit the predictability of the user demand, as

$$C^{\mathcal{N}}(N) := \limsup_{\tau \to \infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} \mathbb{E}\left[C\left(L_t\right)\right], \qquad (3)$$
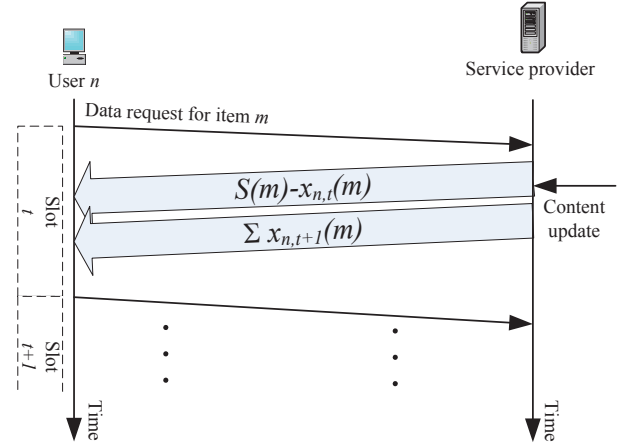

Fig. 2: Time diagram of proactive downloads for the demand of user n.

where

$$L_t := \sum_{m=1}^{M} \sum_{n=1}^{N} S(m) \mathbb{I}_{n,t}(m), \quad t \geq 0 \qquad (4)$$

is the total load at time slot t encountered by the non-proactive network. The average cost $C^{\mathcal{N}}(N)$ represents a baseline to which we compare the relative cost reduction leveraged through efficient utilization of the available users' profiles.

III. PROBLEM FORMULATION

In this section, we pose the general formulation of the proactive resource allocation problem, and highlight some of its features.

A. Proactive Data Download and Demand Shaping

Now, we propose a *proactive* network framework in which the service provider makes use of the predictable and cyclostationary nature of the users' demand in balancing the total load over time, hence minimizes the time average expected cost.

Our approach aims at producing proactive data downloads at each time slot depending on the demand statistics. It utilizes the prior knowledge about the user profile in the upcoming time slot, combines it with the statistics about the demand during the current slot, and proactively sends a portion of each potential data item to the respective users. We denote by $x_{n,t+1}(m)$ the portion of data item m sent ahead to user n at time slot t, $m = 1, \cdots, M$, $n = 1, \cdots, N$, $t = 0, 1, \cdots$. Fig. 2 provides an illustrating diagram.

Moreover, we assume that the service provider is allowed to *slightly* modify the demand profile of each user in a way that strikes a balance between enhancing the *certainty* about the future demand, and maintaining the user *satisfaction* about the quality of offered ratings to each data item. The modification of demand profiles offers a further cost reduction gain since a more deterministic user renders the proactive download process more efficient[1]. In fact, the statistical knowledge about

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[1] Addressed in more detail in Section V

the future demand might results in potential wastage in the proactive downloads as such data may not be actually requested and hence the service provider could end up incurring extra cost.

Therefore, the service provider may offer slightly different valuations from those recognized by users as follows. Suppose that $\tilde{\mathbf{p}}_{n,t} = (\tilde{P}_{n,t}(m))_{m=1}^M$ is the profile of user n at time slot t with $q_{n,t}$ being the probability that user n remains silent in slot t. Then, we model user n's flexibility to change his own profile of slot t from $\tilde{\mathbf{p}}_{n,t}$ to $\mathbf{p}_{n,t}$ by a satisfaction region $\mathcal{F}_{\tilde{\mathbf{p}}_{n,t}}$ which is a collection of probability profiles that user n is satisfied to adopt at slot t. Further, each profile $\mathbf{p}_{n,t} \in \mathcal{F}_{\tilde{\mathbf{p}}_{n,t}}$ has to always satisfy $q_{n,t} = 1 - \sum_{m=1}^M P_{n,t}(m)$, since the modification of the user preferences does not affect his activity at that slot. For simplicity of notation, let $\mathbf{v} = \{v_{n,t}\}_{n,t}$, $\mathbf{x} = \{\mathbf{x}_{n,t}\}_{n,t}$, $\mathbf{p} = \{\mathbf{p}_{n,t}\}_{n,t}$, and $\bar{C}(.) = \limsup_{\tau \to \infty} \frac{1}{\tau} \sum_{t=0}^{\tau} \mathbb{E}[C(.)]$.

Now, the proactive download and demand shaping problem for time average expected cost minimization is formulated as

$$C^{\mathcal{P}}(N) :=$$

$$\underset{\mathbf{v},\mathbf{x},\mathbf{p}}{\text{minimize}} \quad \bar{C}\left(L_t + \sum_{m=1}^M \sum_{n=1}^N x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}(m)\right)$$

subject to

$$0 \leq x_{n,t}(m) \leq S(m), \quad \forall m, n, t \geq 0,$$
$$\mathbf{p}_{n,t} \in \mathcal{F}_{\tilde{\mathbf{p}}_{n,t}}, \quad \forall n, t \geq 0,$$
$$\mathbf{p}_{n,t} = \mathbf{p}_{n,kT+t}, \quad \forall k, n, t \geq 0$$
$$P_{n,t}(m) = \phi_{m,t}(\mathbf{v}_{n,t}), \quad \forall m, n, t \geq 0,$$
$$\mathbf{v}_{n,t} \in [0,1]^M, \quad \forall n = 1, \cdots, N,$$

(5)

where the optimization is jointly done over the valuations, proactive downloads and users' profiles seeking the minimum possible expected cost under user satisfaction restrictions. We introduce the following lemma to establish the existence of the time average expected cost for both the non-proactive network (3), and the proactive network (5).

Lemma 1. *Let $\{y_n\}_{n \geq 0}$ be a bounded sequence in $\mathbb{R}_+$, then the limit $\lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} y_i$ exists and is finite.*

Proof. Please refer to Appendix A. □

Corollary 1. *For any $N \in \mathbb{N}$, the time average expected costs $C^{\mathcal{N}}(N)$ and $C^{\mathcal{P}}(N)$ exist, and $\limsup_{\tau \to \infty}$ can be replaced with $\lim_{\tau \to \infty}$ in (3), and (5).*

Proof. Follows directly from Lemma 1 while noting that $0 \leq \mathbb{E}[C(L_t)] \leq C(\hat{S} \cdot N)$ and

$$0 \leq \mathbb{E}\left[C\left(L_t + \sum_{m=1}^M \sum_{n=1}^N x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}(m)\right)\right]$$
$$\leq C(\hat{S} \cdot N), \quad \text{for all } t \geq 0.$$

□

Having established the existence of the time average expected cost, we now investigate the existence of a *one-cycle* steady-state solution.

B. One-cycle Steady-state Solution

The cyclostationary nature of the demand profiles, and the unobservability of the instantaneous load suggest the existence of a steady-state solution to (5) as follows.

Theorem 1. *For any $N \in \mathbb{N}$, the joint proactive download and demand shaping problem can be formulated as*

$$C^{\mathcal{P}}(N) =$$

$$\underset{\mathbf{v},\mathbf{x},\mathbf{p}}{\text{minimize}} \quad \overset{\circ}{C}\left(L_t + \sum_{m=1}^M \sum_{n=1}^N x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}(m)\right)$$

subject to

$$0 \leq x_{n,t}(m) \leq S(m), \quad \forall m, n, t = 0, \cdots, T-1,$$
$$\mathbf{p}_{n,t} \in \mathcal{F}_{\tilde{\mathbf{p}}_{n,t}}, \quad \forall n, t = 0, \cdots, T-1,$$
$$\mathbf{x}_{n,0} = \mathbf{x}_{n,T},$$
$$P_{n,t}(m) = \phi_{m,t}(\mathbf{v}_{n,t}), \quad \forall m, n, t = 1, \cdots, T-1,$$
$$\mathbf{v}_{n,t} \in [0,1]^M, \quad \forall n = 1, \cdots, N,$$

(6)

where $\overset{\circ}{C}(.) = \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[C(.)]$.

Proof. Please refer to Appendix B. □

Corollary 2. *For any $N \in \mathbb{N}$, the time average expected cost incurred by the non-proactive network is given by*

$$C^{\mathcal{N}}(N) = \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[C(L_t)]. \quad (7)$$

As such, only one cycle of the time average expected cost is tantamount to the infinite-horizon time average cost.

It turns out, however, that the general formulation in (6) is not convex in $(\mathbf{x}, \mathbf{p})$ because the objective function involves a collection of products of the components of both $\mathbf{x}$ and $\mathbf{p}$, where $\mathbf{x} = \{\mathbf{x}_{n,t}\}_{n,t}$, $\mathbf{p} = \{\mathbf{p}_{n,t}\}_{n,t}$ with $n = 1, \cdots, N$, and $t = 0, \cdots, T-1$. In order to tackle difficulty in obtaining a global optimal solution for the problem, we divide it into three main steps. We first address the performance of proactive downloads alone without demand shaping which characterize the the best proactive data allocation in response to a given demand profile. Then, we consider the determination of the best joint demand profile and proactive download that always improves the beyond proactive downloads alone. Finally, we investigate data item valuation approach that yields the target demand profile.

IV. PROACTIVE DATA DOWNLOAD

In this section, we quantify the performance of proactive data downloads only. We assume that the system is operating at a given demand profile $\tilde{\mathbf{p}}$ while the system provider is determining the optimal proactive downloads $\mathbf{x}^*$. As discussed in the proof of Theorem 1, the problem in this case becomes convex in $\mathbf{x}$:

$$C^{\mathcal{P}}(N, \tilde{\mathbf{p}}) := \underset{\mathbf{x}}{\text{minimize}} \quad \frac{1}{T} \sum_{t=0}^{T-1}$$

$$\mathbb{E}\left[C\left(L_t + \sum_{m=1}^M \sum_{n=1}^N x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}(m)\right)\right]$$

(8)

subject to $\quad 0 \leq x_{n,t}(m) \leq S(m), \quad \forall m, n, t,$

and $C^{\mathcal{N}}(N, \tilde{\mathbf{p}}) := \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[C(L_t)\right]$. In this section, we focus on the *cost reduction* leveraged through efficient proactive downloads only. We define the cost reduction when there are N users in the system as

$$\Delta C(N) := C^{\mathcal{N}}(N, \tilde{\mathbf{p}}) - C^{\mathcal{P}}(N, \tilde{\mathbf{p}}),$$

and consider its asymptotic performance when the number of users grows to infinity. More specifically,

$$\limsup_{N \to \infty} \frac{\Delta C(N)}{h(N)C'(\gamma \cdot N)}, \quad \text{and} \quad \liminf_{N \to \infty} \frac{\Delta C(N)}{h(N)C'(\gamma \cdot N)},$$

where C' is the first derivative of the cost function C, γ is some positive constant, and $h(N) \leq N, N \in \mathbb{N}$, is a positive non-decreasing function in N.

By the convexity of the optimization problem (8), and the compactness of the feasible set, there exists an optimal solution $\mathbf{x}^*$ with $x_{n,t+1}^*(m)$ being the **optimal** proactive download of data item m made to user n in slot t.

Definition 1 (Active users). *For each data item m and time slot t, we define a set $\mathcal{B}_t(m)$ of active users as*

$$\mathcal{B}_t(m) := \{n : \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'(L_t) - C'(L_{t-1})\right] > 0\},$$
$$t = 0, \cdots, T-1, \quad m = 1, \cdots, M,$$

with $B_t(m) := |\mathcal{B}_t(m)|$ is the cardinality of set $\mathcal{B}_t(m)$.

In the definition above, the expectation

$$\mathbb{E}\left[\mathbb{I}_{n,t}(m)C'(L_t) - C'(L_{t-1})\right]$$

captures the *marginal* contribution of of user n to the cost of time slot t, when requests item m, over the cost of the previous time slot $t-1$. The active users for any given slot have a high potential to improve the cost reduction through a proactive service of their demand.

Despite the convexity of (8), a closed form expression for the optimal value is not available for the general cost function defined above. As such, we study the asymptotic performance of $\Delta C(N)$ through upper and lower bounds that exhibit the same scaling order with N. To that end, we establish such bounds in the following subsections.

A. Upper Bound

We use the set of active users $\mathcal{B}_t(m)$ to characterize an upper bound on $\Delta C(N)$ as follows.

Lemma 2 (Upper bound on cost reduction). *Let $N \in \mathbb{N}$. For $\mathcal{B}_t(m)$ defined above,*

$$\Delta C(N) \leq \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} S(m) \times \\ \sum_{n \in \mathcal{B}_t(m)} \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'(L_t) - C'(L_{t-1})\right]. \quad (9)$$

Proof. Please refer to Appendix C. $\square$

B. Lower Bound

In order to establish a lower bound on the cost reduction, we introduce the following preliminaries.

Definition 2. *For every time slot $t \in \{0, \cdots, T-1\}$, suppose that $\mathcal{B}_t(m)$ is non-empty for some data item m. We define the quantity $\hat{x}_t$ as*

$$\hat{x}_t := \arg \min_{0 \leq x_t \leq \check{S}} \mathbb{E}\left[C\left(L_{t-1} + \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_t(m)} x_t\right) \\ + C\left(L_t - \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_t(m)} x_t \mathbb{I}_{n,t}(m)\right)\right]. \quad (10)$$

The following result about $\hat{x}_t$ holds.

Lemma 3. *If $\mathcal{B}_t(m)$ is non-empty for some time slot t and data item m, then $\hat{x}_t > 0$.*

Proof. Please refer to Appendix D. $\square$

Definition 3 (Policy A). *A proactive download allocation strategy named Policy A produces a proactive download vector $\tilde{\mathbf{x}}$ satisfying the constraints of (8) as follows. For time slot $t = 0, \cdots, T-1$, and any data item m,*

$$\tilde{x}_{n,t}(m) = \begin{cases} \tilde{x}_t & \text{if } n \in \mathcal{B}_t(m), \\ 0 & \text{if } n \notin \mathcal{B}_t(m), \end{cases} \quad (11)$$

where

$$\tilde{x}_t := \hat{x}_t - r, \quad (12)$$

for some $r > 0$ chosen such that[2] $\tilde{x}_t > 0$ for all $t \in \{0, \cdots, T-1\}$, and $\hat{x}_t$ is defined in (10).

Note that, Policy A, assigns equal proactive downloads to all *active users* in a given slot t. We utilize this policy in establishing the following lower bound on the performance of $\Delta C(N)$.

Lemma 4 (Lower bound on cost reduction). *Let $N \in \mathbb{N}$. Under Policy A and for $\mathcal{B}_t(m)$ defined above,*

$$\Delta C(N) \geq \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} \tilde{x}_t \sum_{n \in \mathcal{B}_t(m)} \\ \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(j)} \tilde{x}_t \mathbb{I}_{k,t}(j)\right) \right. \\ \left. - C'\left(L_{t-1} + \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(j)} \tilde{x}_t\right)\right] > 0. \quad (13)$$

Proof. Please refer to Appendix E. $\square$

Having established general upper and lower bounds on the potential cost reduction, we study its asymptotic performance with the number of users and present the result in the next subsection.

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[2] Such a positive r exists since T is finite, and by Lemma 3, $\hat{x}_t > 0$ for any non-empty $\mathcal{B}_t(m)$.

C. Asymptotic Analysis

When the number of users N grows to infinity, the expected time average cost also grows to infinity when $q_{n,t} < 1$, $\forall n$, and some t. The cost reduction itself will also grow to infinity with a certain scaling order with N. Such a cost reduction depends mainly on the number of elements in $\mathcal{B}_t(m)$, and how it scales with N. Throughout this subsection, we assume that $q_{n,t} < 1$, $\forall n, t$. That is, each user can request a data item at any time slot with a positive probability.

The following assumption considers the asymptotic behavior of $B_t(m)$ as $N \to \infty$.

Assumption 1. *Assume that there exists some non-decreasing function $h : \mathbb{N} \to \mathbb{N}$ such that $h(N) \leq N$ on $\mathbb{N}$, $h(N) \to \infty$ as $N \to \infty$, and for every time slot t and data item m, the limit*

$$\beta_t(m) := \lim_{N \to \infty} \frac{B_t(m)}{h(N)} \tag{14}$$

exists, and $\beta_t(m) < \infty$, $\forall m = 1, \cdots, M, t = 0, \cdots, T-1$.

Note that, under Assumption 1, $\beta_t(m) > 0$ implies the number of active users grows to infinity as the total number of users does. The following two lemmas are crucial to establish the main asymptotic scaling result.

Lemma 5. *Under Assumption 1, there exists a positive constant γ_2 such that*

$$\rho_t(m) := \limsup_{N \to \infty} \sum_{n \in \mathcal{B}_t(m)} \frac{\mathbb{E}\left[\mathbb{I}_{n,t}(m)C'(L_t) - C'(L_{t-1})\right]}{h(N)C'(\gamma_2 \cdot N)}$$
$$< \infty, \quad \forall m, t.$$

Further, if $\beta_t(m) = 0$, then $\rho_t(m) = 0$.

Proof. Please refer to Appendix F. $\qquad\square$

Lemma 6. *Under Assumption 1, suppose that the cost function C satisfies*

$$\lim_{L \to \infty} \frac{L^\delta}{C'(L)} = 0, \quad \text{for some } \delta > 0. \tag{15}$$

Suppose also that $\beta_t(m) > 0$ for some data item m and time slot t. Then,

$$\sigma_t(m) := \liminf_{N \to \infty} \frac{1}{h(N)C'(\gamma_1 \cdot N)} \times$$
$$\sum_{n \in \mathcal{B}_t(m)} \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{k \in \mathcal{B}_t(j)} \tilde{x}_t \mathbb{I}_{n,t}(j)\right)\right]$$
$$- \mathbb{E}\left[C'\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k \in \mathcal{B}_t(j)} \tilde{x}_t\right)\right]$$
$$> 0, \quad \text{for some } \gamma_1 > 0.$$

Proof. Please refer to Appendix G. $\qquad\square$

Lemmas 5, and 6 are instrumental to establish the asymptotic scaling result of the cost reduction, which is stated in the following theorem.

Theorem 2 (Asymptotic cost reduction for infinite number of active users)**.** *Under Assumption 1, suppose that the cost function C satisfies Condition (15). Then, there exist finite positive constants $\hat{\gamma}$, and $\check{\gamma}$ for which*

$$\limsup_{N \to \infty} \frac{\Delta C(N)}{h(N) \cdot C'(\hat{\gamma} \cdot N)} \leq \frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} S(m)\rho_t(m), \tag{16}$$

and

$$\liminf_{N \to \infty} \frac{\Delta C(N)}{h(N) \cdot C'(\check{\gamma} \cdot N)} \geq \frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} \chi_t \sigma_t(m), \tag{17}$$

where $\chi_t := \liminf_{N \to \infty} \tilde{x}_t$, $t = 0, \cdots, T-1$.

Proof. The proof is straightforward from Lemma 5, and Lemma 6. $\qquad\square$

Theorem 2, characterizes asymptotic upper and lower bounds on the scaling of the cost reduction leveraged through proactive data download. Moreover, we can also conclude the following.

Corollary 3. *Under Assumption 1, and Condition (15), if for some time slot t and data item m, $\beta_t(m) > 0$, then there exists a finite positive constant γ such that*

$$\Delta C(N) = \Theta\left(h(N)C'(\gamma N)\right). \tag{18}$$

Thus, if the number of active users grows to infinity as $h(N)$, the leveraged cost reduction grows unboundedly to infinity as $h(N) \cdot C'(\gamma N)$. At this point, the following remarks can be made.

Remark 1. *For a polynomial cost function with some degree $d > 1$, the leveraged cost reduction scales with the number of users as $h(N)N^{d-1}$. Further, if $h(N)$ is linear in N, then $\Delta C(N)$ grows as N^d, i.e., $\Delta C(N)$ scales with N as the cost itself does.*

Remark 2. *For an exponential cost function, the cost reduction grows exponentially with the number of users since the derivative of the cost function is also an exponential function. However, the constant γ affects the exponent of scaling.*

Remark 3. *The super-linearity of the class of cost functions satisfying Condition (15) specifies a typical form of practical types of costs, such as those measuring delays and energy consumption in communication networks.*

Remark 4. *The necessity of Condition (15) to establish the scaling result (18) is manifested by considering the following setup. Let $C(L) = L - \log(L+1)$, which is a monotonically increasing and strictly convex cost function, but does not obey Condition (15). Assuming $T = 2$, $L_0 = 0$, almost surely, and $L_1 = SN$ for a single-data item system with size S. Given this setup, the all users are active in slot 1, however, the obtained optimal cost reduction does not scale as $\Theta(\frac{\gamma N^2}{1+\gamma N})$ for any $\gamma > 0$.*

Until this point, we have addressed the scenario where the number of active users for some data items and time slots grows to infinity. Now, we investigate the potential of scaling cost reduction when the number of active users is bounded above and the total number of users grows to infinity. Such a case can essentially result from a high uncertainty about the

exact future demand for each user, especially when the user preferences are equally distributed over the set of data items. The following assumption captures this scenario.

Assumption 2. *Assume* $\liminf_{N \to \infty} B_t(m) > 0$ *for all* m, t, *and that* $B_t(m) \leq B$ *for some* $B \in [0, \infty)$, *and for all* $m = 1, \cdots, M$, $t = 0, \cdots, T-1$, *and* $N \in \mathbb{N}$.

Assumption 2 formalizes the case when there always exists a positive number of active users in the systems, but such a number is bounded above while the total number of users N grows to infinity. The scaling order of the cost function with number of users, under that assumption, is characterized in the following theorem.

Theorem 3 (Asymptotic cost reduction for bounded number of active users)**.** *Under Assumption 2, let*

$$\hat{\mathcal{B}}_t(m) := \limsup_{N \to \infty} \mathcal{B}_t(m),$$
$$\check{\mathcal{B}}_t(m) := \liminf_{N \to \infty} \mathcal{B}_t(m), \quad \forall m, t.$$

The expected time average cost $\Delta C(N)$ *satisfies*

$$\limsup_{N \to \infty} \frac{\Delta C(N)}{C'(\hat{\gamma} \cdot N)} \leq \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} S(m) \sum_{n \in \hat{\mathcal{B}}_t(m)} \rho_{n,t}(m),$$

(19)

and

$$\liminf_{N \to \infty} \frac{\Delta C(N)}{C'(\check{\gamma} \cdot N)} \geq \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} \chi_t \sum_{n \in \check{\mathcal{B}}_t(m)} \sigma_{n,t}(m), \quad (20)$$

for some finite positive constants $\hat{\gamma}, \check{\gamma}$, *where*

$$\rho_{n,t}(m) := \limsup_{N \to \infty} \frac{\mathbb{E}\left[\mathbb{I}_{n,t}(m)C'(L_t) - C'(L_{t-1})\right]}{C'(\hat{\gamma} \cdot N)},$$
$$n \in \hat{\mathcal{B}}_t(m),$$

$$\sigma_{n,t}(m) :=$$
$$\liminf_{N \to \infty} \frac{\mathbb{E}\left[\mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{n \in \mathcal{B}_t(j)}\tilde{x}_t\mathbb{I}_{n,t}(j)\right)\right]}{C'(\hat{S}N)}$$
$$- \frac{\mathbb{E}\left[C'\left(L_{t-1} + \sum_{j=1}^{M}\sum_{n \in \mathcal{B}_t(j)}\tilde{x}_t\right)\right]}{C'(\hat{S}N)}, n \in \check{\mathcal{B}}_t(m).$$

Proof. The proof follows by taking $\limsup$ and $\liminf$ of (9) and (13), respectively, as $N \to \infty$. $\square$

Note that, in Theorem 3, $\chi_t > 0$, $\forall t$, and $\sigma_{n,t}(m) > 0$, $\forall m, n, t$. This holds since the terms $\sum_{j=1}^{M}\sum_{n \in \mathcal{B}_t(j)}\tilde{x}_t\mathbb{I}_{n,t}(j)$, and $\sum_{j=1}^{M}\sum_{n \in \mathcal{B}_t(j)}\tilde{x}_t$ are bounded by Assumption 2, while the terms L_{t-1}, and L_t grow to infinity almost surely as $N \to \infty$, for all m, n, t.

Corollary 4. *Under Assumption 2, there exists a positive constant* γ *such that*

$$\Delta C(N) = \Theta(C'(\gamma N)). \quad (21)$$

The significance of (21) is boosted under the class of cost functions with $C'(N) \to \infty$ as $N \to \infty$, which yield

unbounded cost reduction for a bounded number of active users. The following remark highlights this gain.

Remark 5. *For a class of cost functions with* $C'(N) \to \infty$ *as* $N \to \infty$, *the leveraged cost reduction grows unboundedly to infinity as* $C'(\gamma N)$ *even if there is only **one** active user for only **one** data item at only **one** time slot. The reason behind such unbounded gain attributes to the coupling of the cost incurred to serve the request of this active user and the costs of serving the rest of users, which grows unboundedly with* N.

Theorem 3 and Corollary 4 have provided a worst-case scaling scenario for the cost reduction. In the asymptotic scenario of this case, the demand profiles of all but finitely many users are rather indeterministic and creating a substantial uncertainty about the expected user demand. Such a confusion forces the service provider to refrain its proactive transmission to a wide set of users. In order to tackle such a problem, we consider, in the following section, *joint* allocation of user demand profile and proactive data download.

V. DEMAND SHAPING

Our notion of demand shaping is motivated by the observation that less predictable users, whose demand profiles are divided almost equally over a subset of data items, can be expected to be more *responsive* to recommendation differences offered by the service provider. For example, a user who is indifferent to watching a documentary on the American revolution or the civil war is likely to be more responsive to the recommendation disparity between the two. We highlight the following point. Our proposed framework uses recommendations to improve the predictability of future demands, hence the accuracy of the proactive downloads. Therefore, especially less predictable users that are sensitive to data item valuation differences provide a high potential for proactive service gains.

We attack the joint design problem in two scenarios due to their structural differences. First, we consider a relaxed optimal joint allocation of proactive downloads and demand profile, a scenario whereby no restrictions are imposed on the new users' profiles other than maintaining the same probability of being inactive, $q_{n,t}$, $n = 1, \cdots, N$, $t = 0, \cdots, T-1$. Afterwards, we study a more practical scenario where users' satisfaction constraints are imposed.

A. Jointly Optimal Solution for Fully Flexible Users

In order to gain insights on the structure of the best user profile leading to minimum expected cost with proactive downloads, we consider the relaxed version of the optimization problem (6) satisfying:

Assumption 3. *Assume that user demands are fully flexible, i.e.,*

$$\mathcal{F}_{n,\tilde{\mathbf{p}}_{n,t}} = \left\{\mathbf{p}_{n,t} : \sum_{m=1}^{M} P_{n,t}(m) = 1 - q_{n,t}, P_{n,t}(m) \geq 0\right\},$$
$$\forall n, t.$$

Under Assumption 3, the joint proactive download and demand profile allocation problem is formulated as

$$C^{\mathcal{J}}(\mathbf{p}^*, \mathbf{x}^*) :=$$

$$\underset{(\mathbf{p}, \mathbf{x})}{\text{minimize}} \quad \overset{\circ}{C}\left(L_t + \sum_{m=1}^{M}\sum_{n=1}^{N} x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}(m)\right)$$

$$\text{subject to} \quad 0 \le x_{n,t}(m) \le S(m), \quad \forall m, n, t,$$

$$0 \le P_{n,t}(m), \quad \forall m, n, t,$$

$$\sum_{m=1}^{M} P_{n,t}(m) = 1 - q_{n,t}, \quad \forall n, t,$$

(22)

where $\mathbf{p} = (\mathbf{p}_n)_{n=1}^{N}$, and $\mathbf{p}_n = (\mathbf{p}_{n,t})_{t=0}^{T-1}$. Note that, the new user profile is chosen such that the probability of a user n remains silent in slot t is $q_{n,t}$ **unchanged**. Thus, the service provider may modify the preferences of the users through data item valuation techniques, but it can not change the activity of each user of whether to request a data item at all or remain silent.

Note that in this section, we focus only on *minimizing* the expected time average cost incurred by the service provider under proactive downloads instead of the expected cost reduction. The key idea is that, maximizing cost reduction through changing the user profiles affects the cost of the non-proactive network as well which renders the comparison unfair. In Section IV, however, proactive downloads only do not affect the cost of the non-proactive network, which validates the use of the cost reduction metric.

Since the feasible set of (22) is compact and the objective function is convex, then there exists a globally optimal solution to the problem [31]. We denote such a solution by $(\mathbf{p}^*, \mathbf{x}^*)$ and characterize it in the following theorem.

Theorem 4. *Under Assumption 3, define* $\mathcal{M}^* := \{m : S(m) = \check{S}\}$ *and pick some* $m^* \in \mathcal{M}^*$, *then*

$$P_{n,t}^*(m) = \begin{cases} 1 - q_{n,t}, & m = m^*, \\ 0, & m \ne m^*, \end{cases} \quad (23)$$

and

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \quad \overset{\circ}{C}\left(L_t^* + \sum_{m=1}^{M}\sum_{n=1}^{N} x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}^*(m)\right)$$

$$\text{subject to} \quad 0 \le x_{n,t}(m) \le S(m), \quad \forall m, n, t,$$

(24)

where $\mathbb{I}_{n,t}^*(m)$ *is the indicator function associate with* $P_{n,t}^*(m)$, *and* L_t^* *is the non-proactive network load under* $\mathbf{p}^*$. *Further, if* $|\mathcal{M}^*| = 1$, *then* $(\mathbf{p}^*, \mathbf{x}^*)$ *is* **unique**.

Proof. Please refer to Appendix H ☐

Theorem 4, therefore, suggests a new user profile that ensures requesting the data item with the least size if the user is to request a data item at all. Then proactive downloads under the new profile minimize the average incurred cost for service. In the next subsection, we model and study the problem under user *satisfaction* constraints.

B. Joint Design under User Satisfaction Constraints

We start this section by proposing a novel model for capturing the economic responsiveness and service flexibilities of the user demands, which will facilitate the design. Suppose that $\tilde{\mathbf{p}}_{n,t}$ is the initial (given) profile of user n for slot t, with $q_{n,t}$ being the probability that user n remains silent at slot t.

Definition 4. *We consider the term*

$$\tilde{\pi}_{n,t}(m) := \frac{\tilde{P}_{n,t}(m)}{1 - q_{n,t}}, \quad \forall m, n, t,$$

to denote the probability that user n requests data item m at time slot t given that he decided to request a data item at all. We also use $\tilde{\boldsymbol{\pi}}_{n,t} := (\tilde{\pi}_{n,t}(m))_{m=1}^{M}$.

Next, we introduce a key measure and a related constraint that captures the economic responsiveness and flexibility of users in shifting their demand profile within acceptable service quality limits.

Definition 5. *(Entropy Ball Constraint (EBC)) For user n at time slot t with initial profile $\tilde{\mathbf{p}}_{n,t}$ and probability of being silent $q_{n,t}$, we say that a new profile $\mathbf{p}_{n,t}$ satisfies the* **entropy ball constraint** *if* $\sum_m P_{n,t}(m) = 1 - q_{n,t}$ *and*

$$\frac{\|\tilde{\mathbf{p}}_{n,t} - \mathbf{p}_{n,t}\|}{1 - q_{n,t}} \le \alpha_n H(\tilde{\boldsymbol{\pi}}_{n,t}), \quad \forall n, t, \quad (25)$$

where α_n is a positive constant differentiating between user classes and normalizing the right hand side of (25), and $H(\tilde{\boldsymbol{\pi}}_{n,t}) = -\sum_m \tilde{\pi}_{n,t}(m) \log \tilde{\pi}_{n,t}(m)$ *is the entropy [33] of* $\tilde{\boldsymbol{\pi}}_{n,t}$.

The above metric utilizes the entropy of the choice of user n under the initial profile $\tilde{\mathbf{p}}_{n,t}$ to capture the radius of an M-dimensional ball centered at $\tilde{\boldsymbol{\pi}}_{n,t}$. The reason behind choosing relying on the entropy to determine size of the satisfaction region for each user is the following. The entropy of a certain user profile characterizes how deterministic such a user is. The higher the uncertainty about the user demand, the higher the entropy is, and the larger the potential that the user is willing to follow the new profile recommended by the service provider. This holds since users with indeterministic demand do not necessarily mind a specific data item, hence give more flexibility for the service provider to push their demand profiles more deterministic.

On the other hand, small entropy reflects a deterministic demand profile whereby the user is not flexible to modify his profile. Moreover, we use the parameter α_n to further control the radius of the entropy ball as user n can belong to a higher class where α_n is small resulting in a tight satisfaction region and perhaps an unchanged profile. Figure 3 depicts an illustrating shape of the EBC for $M = 3$.

Using the above EBC, the constraint set $\mathcal{F}_{n,\tilde{\mathbf{p}}_{n,t}}$ will be give as:

Assumption 4.

$$\mathcal{F}_{n,\tilde{\mathbf{p}}_{n,t}} = \{\mathbf{p}_{n,t} : \mathbf{p}_{n,t} \text{ satisfies EBC}, P_{n,t}(m) \ge 0, \forall m\},$$
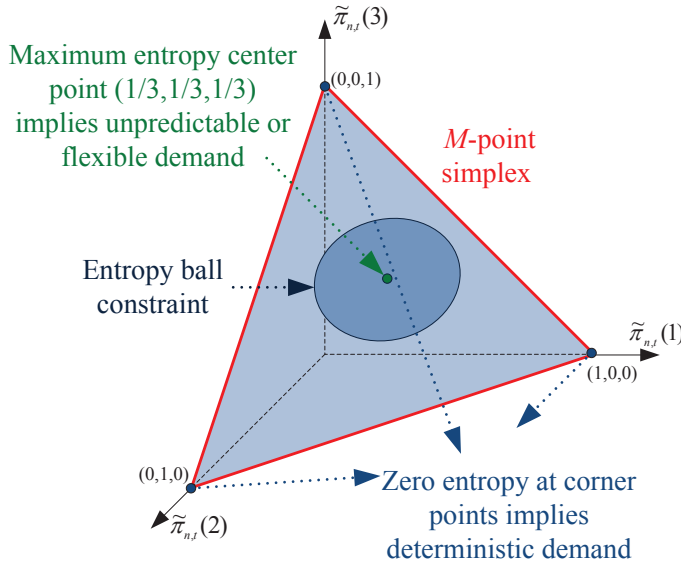
$$\forall n, t.$$

Fig. 3: An example of $M = 3$ data items that user n can choose from in slot t. Entropy ball shrinks to a zero radius at the corner points, and attains a maximum radius at the center point.

Note that EBC maintains a constant probability of remaining silent, $q_{n,t}$, for all users and time slots. Statistically, the user does not have to change his access rate.

Under Assumption 4, the *joint proactive data download and demand profile allocation problem* becomes:

$$\underset{(\mathbf{p},\mathbf{x})}{\text{minimize}} \quad \overset{\circ}{C}\left(L_t + \sum_{m=1}^{M}\sum_{n=1}^{N} x_{n,t+1}(m) - x_{n,t}(m)\mathbb{I}_{n,t}(m)\right)$$

subject to

$$\frac{\|\tilde{\mathbf{p}}_{n,t} - \mathbf{p}_{n,t}\|}{1 - q_{n,t}} \leq \alpha_n H(\tilde{\boldsymbol{\pi}}_{n,t}), \quad \forall n, t,$$

constraints of (22).

(26)

While the constraints of (26) are all convex in $(\mathbf{p}, \mathbf{x})$, the objective function, denoted as $f_0(\mathbf{p}, \mathbf{x})$, is non-convex. Thus, in contrast to the tractable structure under fully flexible demands of previous section, the characterization of a global optimal solution of (26) is computationally intractable. Nevertheless, we next show that strict performance improvement over proactive downloads can still be guaranteed.

To see this, suppose that $\tilde{\mathbf{x}}$ is the optimal proactive download allocation obtained under the initial user profile $\tilde{\mathbf{p}}$, where $(\tilde{\mathbf{p}}, \tilde{\mathbf{x}})$ does not satisfy the KKT conditions [31] of (26), then a point $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ which satisfies $f_0(\hat{\mathbf{p}}, \hat{\mathbf{x}}) < f_0(\tilde{\mathbf{p}}, \tilde{\mathbf{x}})$, as well as the KKT conditions of (26), can be obtained through iterative solution to approximate convex problems.

Lemma 7. *Let $\hat{f}^k$ be a convex function in $(\mathbf{p}, \mathbf{x})$ that replaces f_0 of (26) at iteration k. Denote by $(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})$ the optimal solution to the resulting convex optimization problem at the $k - 1^{st}$ iteration, $k = 1, 2, \cdots$. If*

1) $\hat{f}^k(\mathbf{p}, \mathbf{x}) \geq f_0(\mathbf{p}, \mathbf{x})$ *for all feasible* $(\mathbf{p}, \mathbf{x})$,
2) $\nabla \hat{f}^k(\mathbf{p}^{k-1}, \mathbf{x}^{k-1}) = \nabla f_0(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})$,

3) $\hat{f}^k(\mathbf{p}^{k-1}, \mathbf{x}^{k-1}) = f_0(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})$,

$\forall k = 1, \cdots$, *then $f_0(\mathbf{p}^{k-1}, \mathbf{x}^{k-1}) > f_0(\mathbf{p}^k, \mathbf{x}^k), \forall k$, and the sequence $\{(\mathbf{p}^k, \mathbf{x}^k)\}_k$ converges to a point $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ which is a locally optimal solution to (26).*

The above lemma is a special case of Theorem 1 in [34] which aims at providing local optimal solutions to non-convex optimization problems.

Corollary 5. *Starting from initial condition $(\mathbf{p}^0, \mathbf{x}^0) = (\tilde{\mathbf{p}}, \tilde{\mathbf{x}})$, a sequence of approximate functions $\{\hat{f}^k\}$ generated as in Lemma 7 and resulting in a KKT-satisfying point $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ leads to $f_0(\tilde{\mathbf{p}}, \tilde{\mathbf{x}}) > f_0(\hat{\mathbf{p}}, \hat{\mathbf{x}})$.*

In the following theorem, we suggest a general approximation to f_0 of (26) at each new iteration k that converges to a locally optimal solution.

Theorem 5. *For f_0 being the objective function of (26), the approximate function*

$$\hat{f}^k(\mathbf{p}, \mathbf{x}) = f_0(\mathbf{p}^{k-1}, \mathbf{x})+$$
$$\sum_{m,n,t} \left.\frac{\partial f_0(\mathbf{p}, \mathbf{x}^{k-1})}{\partial P_{n,t}(m)}\right|_{\mathbf{p}=\mathbf{p}^{k-1}} \cdot P_{n,t}(m) \quad (27)$$

at iteration $k \geq 1$ is convex in $(\mathbf{p}, \mathbf{x})$, further, the sequence of solutions to the problem resulting from replacing f_0 with $\{\hat{f}^k\}_k$ converges to a locally optimal solution of (26).

Proof. Please refer to Appendix I. □

The approximate function (27), can now be used to replace f_0 of (26) at iteration $k \geq 1$. Starting with $(\mathbf{p}^0, \mathbf{x}^0) = (\tilde{\mathbf{p}}, \tilde{\mathbf{x}})$, the successive solutions to approximate optimization problems with f_0 being replaced by $\hat{f}^k$ of (27) converges to a point $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ with $f_0(\hat{\mathbf{p}}, \hat{\mathbf{x}}) < f_0(\tilde{\mathbf{p}}, \tilde{\mathbf{x}})$. Further, the following conclusion can be made about $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$.

Theorem 6. *For a locally optimal solution $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ to (26), we have*

$$\frac{\|\tilde{\mathbf{p}}_{n,t} - \hat{\mathbf{p}}_{n,t}\|}{1 - q_{n,t}} = \alpha_n H(\tilde{\boldsymbol{\pi}}_{n,t}), \quad \forall n, t. \quad (28)$$

In other words, the locally optimal profile $\hat{\mathbf{p}}_{n,t}$ always lies on the boundary of the EBC region.

Proof. Please refer to Appendix J □

Theorem 6 motivates the design of efficient low complexity schemes for joint proactive download and demand shaping in large scale systems.

Remark 6. *The proof of Theorem 6 does not depend on the structure of the constraint region $\mathcal{F}_{n,\tilde{\mathbf{p}}_{n,t}}$ for any n, t. Consequently, we can conclude that any optimal solution to the formulation (6) always yields modified demand profiles that lie on the boundary of the constraint region $\mathcal{F}_{n,\tilde{\mathbf{p}}_{n,t}}$.*

The final step in the proposed framework is how to change the users' profiles from the initial ones obtained through tracking and learning user behavior to the new ones obtained through constrained cost minimization. Towards this end, we consider a new recommendation system that sets slightly different ratings to data item from those given by the users' themselves.

VI. Data Item Recommendation Scheme

Upon the calculation of the locally optimal demand profile $\hat{\mathbf{p}}$, the service provider has to assign new valuations, $\mathbf{v} = (\mathbf{v}_{n,t})_{n,t}$, so that the users adjust their new profiles accordingly. However, following a user-satisfaction based techniques that rely only on the user preferences and valuations (cf. [18]-[23]), the service provider has originally assigned a rating of $r_n(m)$ for data item m to user n (which is independent of the dynamics of the varying network load), but the requirements of the new demand profile necessitate a valuation of $v_{n,t}(m)$ to be assigned to data item m and user n and time slot t. A large difference between the new and the old rating values could still cause the user to feel unsatisfied with the offered valuations, especially when they do not meet his expectations. We presume that the new rating vector $\mathbf{v}_{n,t}$ must be as close as possible, in the Euclidean distance sense, to the original rating vector $\mathbf{r}_n = (r_n(m))_{m=1}^M$, while achieving the new demand profile. Figure 4 depicts a block diagram of the proposed data item valuation scheme.
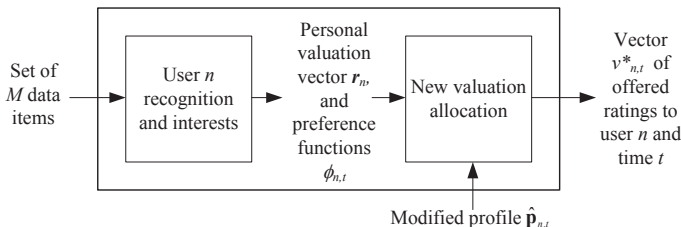

Fig. 4: Block diagram of data item valuation system for a user n at time slot t. The determination of the user interests applies through learning algorithms.

As has been hypothesized in Section II, there exists a function $\phi_{m,t} : [0,1]^M \to [0,1]$ that determines the user profile based on the offered ratings to the data items. Now, assuming that the initial profile for user n at slot t is $\tilde{\mathbf{p}}_{n,t} = (\tilde{P}_{n,t}(m))_{m=1}^M$, then according to $\phi_{m,t}$, we have $\tilde{P}_{n,t}(m) = \phi_{m,t}(\mathbf{r}_n)$, where $\phi_{m,t}$ is supposed to be monotonically increasing in $r_n(m)$ and measure the relative quality of data item m to the rest of data items. After all, the rating allocation is a user-based problem where the service provider can compute new ratings for each user separate from the others. Denoting the new profile for user n at time slot t by $\hat{\mathbf{p}}_{n,t} = (\hat{P}_{n,t}(m))_{m=1}^M$, the valuation assignment problem for user n at slot t is formulated as

$$\begin{aligned} \underset{\mathbf{v}_{n,t}}{\text{minimize}} \quad & \|\mathbf{v}_{n,t} - \mathbf{r}_n\| \\ \text{subject to} \quad & \hat{P}_{n,t}(m) = \phi_{m,t}(\mathbf{v}_{n,t}), \quad \forall m, \\ & \mathbf{v}_{n,t} \in [0,1]^M. \end{aligned} \quad (29)$$

Remark 7. *Problem* (29) *is convex if and only if $\phi_{m,t}$ is a linear fractional mapping in $\mathbf{v}_{n,t}$.*

An example case on a linear fractional mapping is

$$\phi_{m,t}(\mathbf{v}_n) = (1 - q_{n,t}) \frac{v_n(m)}{\sum_{j=1}^M v_n(j)}, \quad \forall m, n, t. \quad (30)$$

In this example, the mapping function $\phi_{m,t}$ captures the relative preference of a user to choose data item m amongst all

data items. While problem (29) is convex, a globally optimal solution can be obtained efficiently through a gradient descent algorithm [31].

In the case where $\phi_{m,t}$ is *not* a linear fractional mapping, the problem turns out to be non-convex, calling for approximate solutions. One possible way of handling such a difficulty is to replace the non-convex $\phi_{m,t}$ with an approximate linear fractional form, and iteratively solving for approximate solutions till convergence to a locally optimal rating vector, a similar approach to that used in Section V.

VII. Numerical Simulations

To validate the proposed proactive resource allocation mechanism proposed in this work, we consider the following example system for simulation. A service provider is assumed to have $M = 3$ data items with fixed sizes $\mathbf{S} = (3,2,4)$. There are $N = 2$ users that may request services on a daily basis, whereby the day is divided into $T = 2$ time slots with demand profiles following a cyclostationary distribution with period T. One time slot is supposed to represent an off-peak hour demand with $q_{n,0} = 1 - p_0$, $n = 1,2$ with $p_0 = 0.1$. The other time slot represent a peak-hour demand which has $q_{n,1} = 1 - p_1$, $n = 1,2$. The profiles of both users during the off-peak hour are $\mathbf{p}_{1,0} = p_0 \cdot (0.8, 0.1, 0.1)$, $\mathbf{p}_{2,0} = p_0 \cdot (0.3, 0.1, 0.6)$. Likewise, during the peak hour $\mathbf{p}_{1,1} = p_1 \cdot (0.8, 0.1, 0.1)$, $\mathbf{p}_{2,1} = p_1 \cdot (0.3, 0.1, 0.6)$. The parameters p_0 and p_1 represent the user activity during the off-peak and peak hours.

A. Proactive Downloads

We start by investigating the effect of the proactive downloads, being implemented during the off-peak hour, on the expected total cost and load of the whole system. We consider the two main cost functions: (1) a quadratic cost function $C(L) = L^2$, $L \in \mathbb{R}_+$ and (2) an outage-constrained cost function $C(L) = \frac{L}{\chi - L}$, where χ is the maximum load that the service provider can afford at a given time slot. We use $\chi = 9.8$ and plot the obtained results versus p_1 in Figure 5. In Figures 5a, 5b, the expected total load is plotted versus p_1 under the two different cost functions. Both figures show an expected cost reduction leveraged from the proactive resource allocation, especially when the customer activity p_1 increases.

In Figure 5c,5d, we show the total load during the off-peak and peak hours with and without proactive downloads. As can be observed, proactive downloads tend to pull the peak-hour load towards the low traffic period so as to minimize the difference between the two loads. However, because of the uncertainty about the customer choice, the allocation procedure does not exactly divide the load equally over the two traffic hours which may lead to unnecessary waste of resources and more cost. The upper histograms of both figures show the significant difference between the off-peak and peak hour loads, whereas the lower histograms reveal a considerably reduced gap between such loads, particularly in the quadratic cost function case. In the outage-constrained cost function case, the gap is not significantly reduced because of the system is fairly far from the instability point at which the load approaches the capacity.
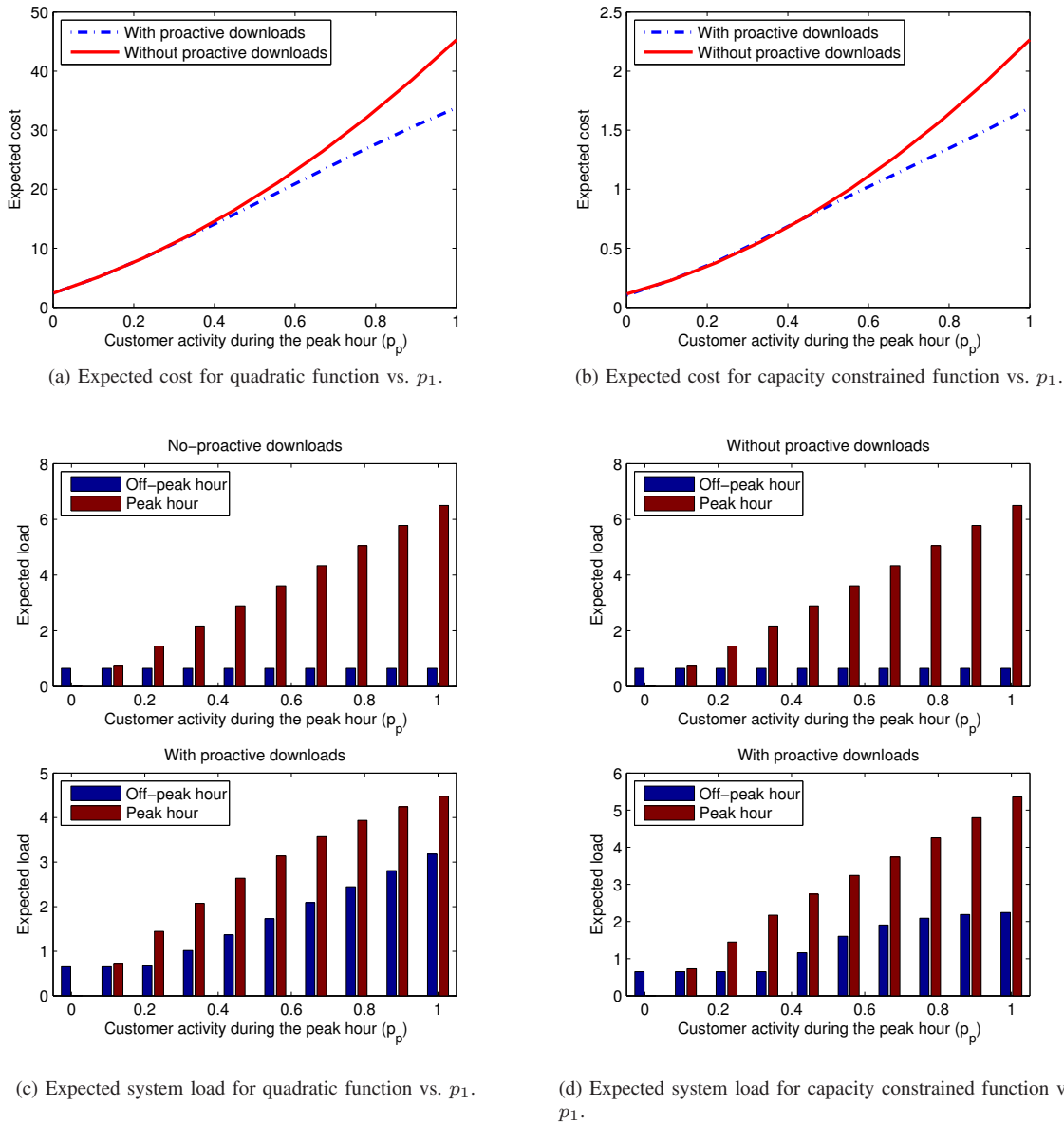
(a) Expected cost for quadratic function vs. p_1.

(b) Expected cost for capacity constrained function vs. p_1.

(c) Expected system load for quadratic function vs. p_1.

(d) Expected system load for capacity constrained function vs. p_1.

Fig. 5: Proactive downloads reduce the expected cost and tend to balance the system load.

B. Optimal Profiles and Recommendations

For the proposed joint user profile and proactive downloads scheme in Section V, we conduct a simulation which is a continuation to the above system where $p_1 = 0.9$ is fixed. For both types of cost functions, a corresponding iterative algorithm is run which takes into account the approximate convex objective function developed in (27). The convergence of results are plotted in Figure 6 where it is clear that the proposed algorithms start from the initial proactive downloads and profiles $(\tilde{\mathbf{p}}, \tilde{\mathbf{x}})$ and proceed with the iterative solutions until convergence to a local optimal solution to the original problem $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$. The resulting sequence of the objective functions (the cost functions) is strictly decreasing.

Upon obtaining the new peak-hour profiles $\hat{\mathbf{p}}_{1,1}$, and $\hat{\mathbf{p}}_{2,2}$, the recommendation process proposed in (29) is invoked to assign new ratings for the available items, where we assume

$\phi_{m,t}$ of (30). We take the original valuation vectors as $\mathbf{r}_1 = (0.8, 0.1, 0.1)$ and $\mathbf{r}_2 = (0.3, 0.1, 0.6)$. The results of the simulations are summarized in Table I, II, for the quadratic and outage-constrained cost functions respectively. It can be noted that, the modified profiles always lie on the boundary of the entropy ball as the service provider is interested in pushing the profiles in the direction of the most deterministic behavior that leads to requesting the smallest size item. Also, the new ratings are quite close to the original.

C. Scaling of Cost Reduction

In order to validate the scaling laws of the cost reduction derived in Section IV, we consider the following scenario of $M = 8$ data items, with size of $S(m) = S = 6$ units $\forall m$. The period of the cyclostationary demand profiles is We take $q_{n,0} = 0.9$ and $q_{n,1} = 0.1$, $\forall n$. We assume demand profiles

| Item index	Modified profile $\hat{\mathbf{p}}_{1,1}/p_1$	Modified profile $\hat{\mathbf{p}}_{2,1}/p_1$	New rating $\mathbf{v}_{1,1}^*$	New rating $\mathbf{v}_{2,1}^*$
1	0.8772	0.3111	0.7985	0.3381
2	0.1222	0.2211	0.1112	0.2403
3	0.0006	0.4678	0.0005	0.5084

TABLE I: Results of joint proactive downloads and user profile allocation for quadratic cost function.

Item index	Modified profile $\hat{\mathbf{p}}_{1,1}/p_1$	Modified profile $\hat{\mathbf{p}}_{2,1}/p_1$	New rating $\mathbf{v}_{1,1}^*$	New rating $\mathbf{v}_{2,1}^*$
1	0.8298	0.3546	0.8005	0.2594
2	0.1222	0.4507	0.1179	0.3297
3	0.0480	0.1947	0.0463	0.1424

TABLE II: Results of joint proactive downloads and user profile allocation for a outage-constrained cost function.
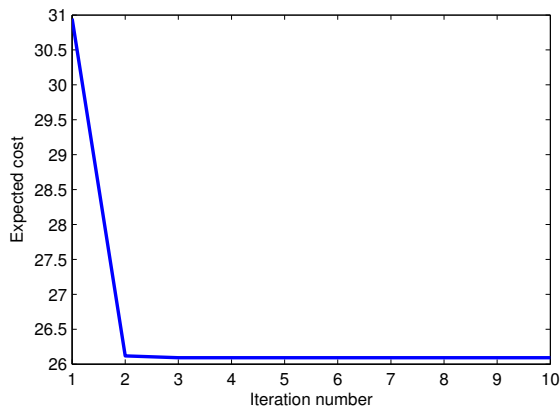
(a) Quadratic function
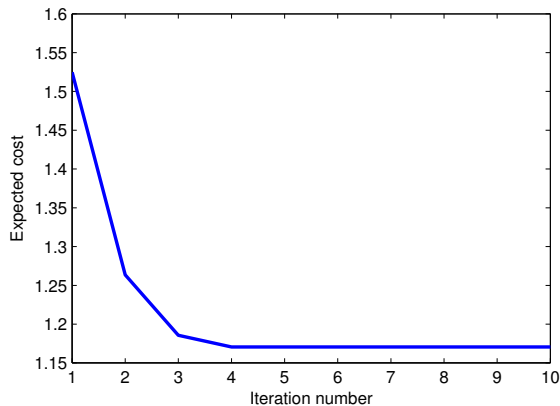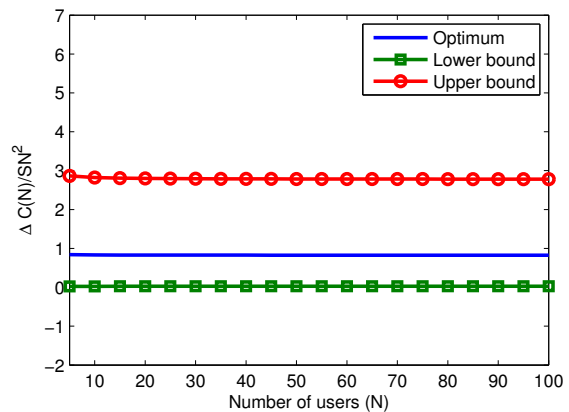

(b) Capacity-constrained function

Fig. 6: Convergence of the joint user profile and proactive downloads allocation.

Fig. 7: Asymptotic cost reduction $\Delta C(N)$ scales as $\Theta(N^2)$ under a quadratic cost function.

of the users to follow a Zipf distribution with

$$P_{n,t}(m) = G_{n,t}\frac{1-q_{n,t}}{m^2}, \quad \forall n, t,$$

where $G_{n,t}$ is a normalizing factor. While considering a quadratic cost function $C(L) = L^2$, the parameter $\beta_1(1) > 0$ under $h(N) = N$ implying an optimal cost reduction that scales with the number of users as $\Theta(N^2)$.

In Figure 7, we plot the optimal ratio of $\Delta C(N)/(SN^2)$ versus the number of users N, where it is clear that, as N increases, the ratio approaches a constant value that assures a cost reduction scaling as $\Theta(N^2)$. Further, we evaluate the upper and lower bounds established in (9), (13) respectively, divide them by SN^2 and plot the result on the same figure for the sake of validation. Obviously, both the upper and lower bounds have the same scaling order as the optimal cost reduction.

VIII. CONCLUSION

In this work, we have proposed and studied a proactive resource allocation and demand shaping framework for data networks. The framework aims to utilize the predictability of future demand in creating more opportunities for a balanced load over time, hence a considerable resource utilization. We have considered the resource allocation problem from the perspective of a service provider (SP) which incurs excessive costs due to the peak-hour demand to sustain the service of the users' requests. Inspired by the recent findings on the predictability of human behavior, we have proposed the notion of users' *demand profiles* to capture the statistical information about the future demand for each user. Such profiles are harnessed in *proactive content downloads* where a portion of highly likely future demand is downloaded to the respective users during the off-peak hour, in a way that smooths out the network load over time, and minimizes the *time average expected cost* incurred by the SPs.

We have analyzed the asymptotic scaling lows of the cost reduction leveraged through such proactive downloads with the

total number of users. We have proved the cost reduction to scale as the same order as the cost of *non-proactive* networks does. Even in a worst case scenario, where the network users are *indeterministic*, the cost reduction scales with the first derivative of the cost function. In order to improve the certainty about the users' demand, we have proposed and studied the notion of *demand shaping*, which is proved to strictly reduce the cost reduction under user satisfaction constraints. We have developed a data item *smart recommendation* scheme that enhances the certainty about the demand of each user, hence the quality of proactive downloads. We have validated the theoretical results with numerical examples to quantify the potential gains of the proposed framework.

APPENDIX A
PROOF OF LEMMA 1

Let $\bar{y}_n = \frac{1}{n} \sum_{i=0}^{n-1} y_i$. It suffices to show that $\{\bar{y}_n\}_{n \geq 0}$ is a Cauchy sequence.

Fix $\epsilon > 0$, and pick two positive integers n_1, n_2 such that $n_1 < n_2$. Then

$$
\begin{aligned}
|\bar{y}_{n_1} - \bar{y}_{n_2}| &= \left| \frac{1}{n_1} \sum_{i=0}^{n_1-1} y_i - \frac{1}{n_2} \sum_{i=0}^{n_2-1} y_i \right| \\
&= \left| \frac{n_2 \sum_{i=0}^{n_1-1} y_i - n_1 \sum_{i=0}^{n_2-1} y_i}{n_1 n_2} \right| \\
&\overset{(a)}{\leq} \frac{(n_2 - n_1) \sum_{i=0}^{n_1-1} |y_i|}{n_1 n_2} + \frac{\sum_{i=n_1}^{n_2-1} |y_i|}{n_2} \\
&\overset{(b)}{\leq} 2K \left(1 - \frac{n^* + k_1}{n^* + k_2} \right) \overset{(c)}{<} \epsilon,
\end{aligned}
$$

where (a) follows by triangular inequality, (b) follows because the boundedness of $\{y_n\}_n$ implies $y_n \leq K$, $\forall n \geq 0$ and for some $K \in \mathbb{R}_+$, and (c) follows since we replace n_1 and n_2 respectively with $n^* + k_1$ and $n^* + k_2$ and choose n^* large enough. Hence $\{y_n\}_{n \geq 0}$ is a real Cauchy sequence, and therefore converges to a finite limit smaller than K.

APPENDIX B
PROOF OF THEOREM 1

We note that the cyclostationarity of the demand profiles implies that $\{\mathbb{I}_{m,kT+t}(m)\}_{k \geq 0}$ is an independent and identically distributed (i.i.d.) sequence of random variables for every $t \geq 0$. Further, we have $P_{n,t}(m)$ if a function of $\mathbf{v}_{n,t}$ for all m, n, t, by hypothesis. Hence, it suffices to show that for any choice of $\{\mathbf{p}_{n,t}\}_{n,t}$, we have the optimal choice of $\{\mathbf{x}_{n,t}\}_{n,t}$, call it $\{\mathbf{x}_{n,t}^*\}_{n,t}$, satisfies

$$
\mathbf{x}_{n,t}^* = \mathbf{x}_{n,kT+t}, \quad \forall k \in \mathbb{Z}_+, n = 1, \cdots, N, t = 0, \cdots, T-1. \tag{31}
$$

Let us use the notation

$$
g_t(\mathbf{x}_t, \mathbf{x}_{t+1}) :=
$$
$$
\mathbb{E}\left[C \left(L_t + \sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t+1}(m) - x_{n,t}(m) \mathbb{I}_{n,t}(m) \right) \right],
$$

where $\mathbf{x}_t = (\mathbf{x}_{n,t})_{n=1}^{N}$, to denote the expected cost at time slot t under some choice of demand profile $\{\mathbf{p}_{n,t}\}_{n,t}$. We have

by the cyclostationarity of demand profiles $g_t(\mathbf{x}_t, \mathbf{x}_{t+1}) = g_{kT+t}(\mathbf{x}_t, \mathbf{x}_{t+1})$, $\forall k, t$. Also, since C is a strictly convex function, and the expectation operator preserves convexity, it follows that $g_t(\mathbf{x}_t, \mathbf{x}_{t+1})$ is strictly convex in $(\mathbf{x}_t, \mathbf{x}_{t+1})$ [31]. Now, suppose towards a contradiction that $\{\mathbf{x}_{n,t}^*\}_{n,t}$ does not satisfy (31). Then

$$
\lim_{\tau \to \infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} g_t(\mathbf{x}_t^*, \mathbf{x}_{t+1}^*) =
$$
$$
\lim_{K \to \infty} \frac{1}{KT} \sum_{k=0}^{K-1} \sum_{t=0}^{T-1} g_t(\mathbf{x}_{kT+t}^*, \mathbf{x}_{kT+t+1}^*) \overset{(a)}{\geq}
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} g_t \left(\lim_{K \to \infty} \frac{\sum_{k=0}^{K-1} (\mathbf{x}_{kT+t}^*, \mathbf{x}_{kT+t+1}^*)}{K} \right) \overset{(b)}{=}
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} g_t(\bar{\mathbf{x}}_t, \bar{\mathbf{x}}_t),
$$

where

$$
\bar{\mathbf{x}}_t = \lim_{K \to \infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{x}_{kT+t}^*. \tag{32}
$$

Inequality (a) follows by Jensen's inequality since g_t is convex, and $\sum_{k=0}^{K-1} \frac{1}{K} = 1$ for all $K \geq 0$. Equality (b) holds because the limit (32) exists by Lemma 1, and $|\mathbf{x}_{kT+t}^*| \leq \hat{S}$ for all k, t.

Now, it follows that $\{\bar{\mathbf{x}}_t\}_t$ with $\bar{\mathbf{x}}_t = \bar{\mathbf{x}}_{kT+t}, \forall k, t$ is the unique globally optimal choice of proactive downloads that minimizes the time average expected cost under $\{\mathbf{v}_n\}_n$, $\{\mathbf{p}_{n,t}\}_{n,t}$. Thus we have a contradiction as $\{\mathbf{x}_t^*\}_t$ is no longer the optimal choice of proactive downloads.

APPENDIX C
PROOF OF LEMMA 2

The cost reduction can be written as

$$
\Delta C(N) =
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[C(L_t) - C\left(L_t + \sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t+1}^*(m) \right.\right.
$$
$$
\left.\left. - \sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t}^*(m) \mathbb{I}_{n,t}(m) \right) \right] \overset{(a)}{\leq}
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} \left[C'(L_t) \left(\sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t}^*(m) \mathbb{I}_{n,t}(m) \right.\right.
$$
$$
\left.\left. - \sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t+1}^*(m) \right) \right] \overset{(b)}{=}
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[\sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t}^*(m)(\mathbb{I}_{n,t}(m) C'(L_t) - C'(L_{t-1})) \right] =
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} \sum_{n=1}^{N} x_{n,t}^*(m) \mathbb{E}\left[\mathbb{I}_{n,t}(m) C'(L_t) - C'(L_{t-1}) \right] \overset{(c)}{\leq}
$$
$$
\frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} S(m) \sum_{n=1}^{N} \mathbb{E}\left[\mathbb{I}_{n,t}(m) C'(L_t) - C'(L_{t-1}) \right].
$$

Inequality (a) follows by the mean value theorem, as for any two non-negative random variables X, Y, there exists a random variable X_0 such that

$$
\begin{aligned}
\mathbb{E}\left[C(X) - C(Y)\right] &= \mathbb{E}\left[C'(X_0)(X-Y)\right] \\
&= \mathbb{E}\left[C'(X_0)(X-Y); X > Y\right] + \\
&\quad \mathbb{E}\left[C'(X_0)(X-Y); X \leq Y\right] \\
&\leq \mathbb{E}\left[C'(X)(X-Y); X > Y\right] + \\
&\quad \mathbb{E}\left[C'(X)(X-Y); X \leq Y\right] \\
&= \mathbb{E}\left[C'(X)(X-Y)\right].
\end{aligned}
$$

as C' is non-negative and monotonically increasing function. Equality (b) follows by rearranging the terms of the right hand side (RHS) of Inequality (a). Finally, Inequality (c) holds since we consider the summation over $\mathcal{B}_t(m)$ instead of all users which can only add a positive value, by the definition of $\mathcal{B}_t(m)$ (see Definition 1). Then, we replace $x^*_{n,t}(m)$ with $S(m)$ as $x^*_{n,t}(m) \leq S(m)$ for all m, n, t.

APPENDIX D
PROOF OF LEMMA 3

By applying the optimality condition on $\hat{x}_t$, we have either $\hat{x}_t = \check{S} > 0$ or $\hat{x}_t$ is the unique solution to

$$
\begin{aligned}
&\sum_{i=1}^{M} \sum_{n \in \mathcal{B}_t(i)} \mathbb{E}\left[\mathbb{I}_{n,t}(i) C'\left(L_t - \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(j)} \hat{x}_t \mathbb{I}_{k,t}(j)\right)\right. \\
&\left. - C'\left(L_{t-1} + \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(j)} \hat{x}_t\right)\right] = 0.
\end{aligned}
\tag{33}
$$

The proof follows since $\mathbb{E}\left[\mathbb{I}_{n,t}(m) C'(L_t)\right] > \mathbb{E}\left[C'(L_{t-1})\right]$ for all $n \in \mathcal{B}_t(m)$, and C' is monotonically increasing.

APPENDIX E
PROOF OF LEMMA 4

We have the cost reduction satisfies

$$
\Delta C(N) \overset{(a)}{\geq}
$$

$$
\begin{aligned}
&\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[C(L_t) - C\left(L_t - \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_t(m)} \tilde{x}_t \mathbb{I}_{n,t}(m)\right.\right. \\
&\left.\left. + \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_{t+1}(m)} \tilde{x}_{t+1}\right)\right] \overset{(b)}{\geq}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[C'\left(L_t - \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_t(m)} \tilde{x}_t \mathbb{I}_{n,t}(m)\right.\right. \\
&\left. + \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_{t+1}(m)} \tilde{x}_{t+1}\right) \times \left(\sum_{m=1}^{M} \sum_{n \in \mathcal{B}_t(m)} \tilde{x}_t \mathbb{I}_{n,t}(m)\right.
\end{aligned}
$$

$$
\left.\left. - \sum_{m=1}^{M} \sum_{n \in \mathcal{B}_{t+1}(m)} \tilde{x}_{t+1}\right)\right] \overset{(c)}{\geq}
$$

$$
\frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} \tilde{x}_t \sum_{n \in \mathcal{B}_t(m)}
$$

$$
\begin{aligned}
&\mathbb{E}\left[\mathbb{I}_{n,t}(m) C'\left(L_t - \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(k)} \tilde{x}_t \mathbb{I}_{k,t}(j)\right) - \right. \\
&\left. C'\left(L_{t-1} + \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(j)} \tilde{x}_t\right)\right] > 0.
\end{aligned}
$$

Inequality (a) follows since Policy A does not necessarily solve (8) optimally. Inequality (b) holds by the first order condition on the convexity of the cost function C. Inequality (c) follows by rearranging the terms of the RHS of Inequality (b) and replacing the terms $\sum_{j=1}^{M} \sum_{k \in \mathcal{B}_{t+1}(j)} \tilde{x}_{t+1}$, $-\sum_{j=1}^{M} \sum_{k \in \mathcal{B}_{t-1}(k)} \tilde{x}_{t-1} \mathbb{I}_{k,t-1}(j)$ with zeros while noting that C' is monotonically increasing on its domain. Finally, the last strict inequality holds since $\tilde{x}_t < \hat{x}_t$ which, combined with the monotonicity and non-negativity of C', and the optimality condition (33), yields an always positive sum as long as $\mathcal{B}_t(m)$ is non-empty for some t and m.

APPENDIX F
PROOF OF LEMMA 5

Set $\gamma_2 = \hat{S}$. We have by the positivity of C',

$$
\rho_t(m) \leq
$$

$$
\limsup_{N \to \infty} \sum_{n \in \mathcal{B}_t(m)} \frac{\mathbb{E}[\mathbb{I}_{n,t}(m) C'(L_t)]}{h(N) C'(\hat{S} \cdot N)} \leq
$$

$$
\limsup_{N \to \infty} \sum_{n \in \mathcal{B}_t(m)} \frac{\mathbb{E}[\mathbb{I}_{n,t}(m) C'(\hat{S} \cdot N)]}{h(N) C'(\hat{S} \cdot N)}
$$

since $L_t \leq \hat{S} \cdot N$ almost surely. It follows therefore that

$$
\begin{aligned}
\rho_t(m) &\leq P_{n,t}(m) \limsup_{N \to \infty} \frac{B_t(m)}{h(N)} \\
&= \beta_t(m) < \infty,
\end{aligned}
$$

by Assumption 1.

APPENDIX G
PROOF OF LEMMA 6

The proof follows in two steps. In one step, we show that if $\liminf_{N \to \infty} \tilde{x}_t > 0$ then

$$
\liminf_{N \to \infty} \frac{1}{h(N) C'(\gamma_1 \cdot N)} \times
$$

$$
\sum_{n \in \mathcal{B}_t(m)} \mathbb{E}\left[\mathbb{I}_{n,t}(m) C'\left(L_t - \sum_{j=1}^{M} \sum_{k \in \mathcal{B}_t(j)} \tilde{x}_t \mathbb{I}_{n,t}(j)\right)\right] > 0,
$$

for some $\gamma_1 > 0$. In the other step, we prove that $\liminf_{N\to\infty} \tilde{x}_t > 0$, and

$$\liminf_{N\to\infty} \frac{\mathbb{E}\left[\sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \tilde{x}_t\mathbb{I}_{n,t}(j)\right)\right]}{\sum_{n\in\mathcal{B}_t(m)} \mathbb{E}\left[C'\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \tilde{x}_t\right)\right]} > 1.$$

Step 1: Suppose that $\liminf_{N\to\infty} \tilde{x}_t > 0$. By Fobini's theorem, we can move the summation inside the expectation, since all the summands are non-negative. Also, by Fatou's lemma, we have

$$\liminf_{N\to\infty} \frac{1}{h(N)C'(\gamma_1 \cdot N)} \times$$

$$\sum_{n\in\mathcal{B}_t(m)} \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \tilde{x}_t\mathbb{I}_{n,t}(j)\right)\right] \geq$$

$$\mathbb{E}\left[\liminf_{N\to\infty} \frac{\sum_{n\in\mathcal{B}_t(m)} P_{n,t}(m)}{h(N)} \times \right.$$

$$\left. \liminf_{N\to\infty} \frac{C'\left(-\hat{S} + \sum_{j=1}^{M}\sum_{k=1}^{N}(S(j)-\tilde{x}_t)\mathbb{I}_{k,t}(j)\right)}{C'(\gamma_1 N)}\right] =$$

$$\mathbb{E}\left[\liminf_{N\to\infty} \frac{\sum_{n\in\mathcal{B}_t(m)} P_{n,t}(m)}{h(N)} \times \right.$$

$$\left. \liminf_{N\to\infty} \frac{C'\left(-\hat{S} + N\cdot\sum_{j=1}^{M}\frac{\sum_{k=1}^{N}(S(j)-\tilde{x}_t)\mathbb{I}_{k,t}(j)}{N}\right)}{C'(\gamma_1 N)}\right].$$

But for any $n \in \mathcal{B}_t(m)$, $P_{n,t}(m) > 0$ for otherwise $n \notin \mathcal{B}_t(m)$. Further, $\beta_t(m) > 0$ by hypothesis. Therefore,

$$\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{P_{n,t}(m)}{h(N)} = \beta_t(m) \lim_{B_t(m)\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{P_{n,t}(m)}{B_t(m)}$$

$$> 0.$$

On the other hand, Kolmogorov's strong law of large numbers implies

$$\gamma_1 := \lim_{N\to\infty} \sum_{j=1}^{M} \frac{\sum_{k=1}^{N}(S(j)-\tilde{x}_t)\mathbb{I}_{k,t}(j)}{N}$$

$$= \lim_{N\to\infty} \sum_{j=1}^{M}(S(j)-\tilde{x}_t)\frac{P_{k,t}(j)}{N} \quad \text{almost surely}$$

$$> 0 \quad \text{almost surely},$$

since $q_{n,t} < 1, \forall n,t$.

Hence, we have

$$\mathbb{E}\left[\liminf_{N\to\infty} \frac{\sum_{n\in\mathcal{B}_t(m)} P_{n,t}(m)}{h(N)} \times \right.$$

$$\left. \liminf_{N\to\infty} \frac{C'\left(-\hat{S} + N\cdot\sum_{j=1}^{M}\frac{\sum_{k=1}^{N}(S(j)-\tilde{x}_t)\mathbb{I}_{k,t}(j)}{N}\right)}{C'(\gamma_1 N)}\right] \geq$$

$$\mathbb{E}\left[\beta_t(m)\cdot\lim_{B_t(m)\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{P_{n,t}(m)}{B_t(m)}\right] > 0.$$

Step 2: In this step, we prove that there exists $\chi > 0$, independent of N, for which if $\tilde{x}_t = \chi$, then

$$\liminf_{N\to\infty} \frac{\sum_{n\in\mathcal{B}_t(m)} \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\mathbb{I}_{n,t}(j)\right)\right]}{\sum_{n\in\mathcal{B}_t(m)} \mathbb{E}\left[C'\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\right)\right]} > 1.$$

We set $\tilde{x}_t = \chi$, independent of N, and we will prove that $0 < \chi < \check{S}$.

By Fubini's theorem and Fatou's lemma, as in ***Step 1***, it suffices to prove that

$$\frac{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\mathbb{I}_{n,t}(j)\right)\right]}{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} C'\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\right)\right]} > 1,$$

for some $0 < \chi < \check{S}$.

We have from Condition (15),

$$\frac{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)C'\left(L_t - \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\mathbb{I}_{n,t}(j)\right)\right]}{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} C'\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\right)\right]} \geq$$

$$\stackrel{(a)}{\geq} \frac{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)\left(L_t - \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\mathbb{I}_{n,t}(j)\right)^{\delta}\right]}{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)}\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\right)^{\delta}\right]}$$

$$\stackrel{(b)}{=} \frac{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)\left(\sum_{j=1}^{M}\sum_{k=1}^{N}(S(j)-\chi)\mathbb{I}_{n,t}(j)\right)^{\delta}\right]}{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)}\left(L_{t-1} + \sum_{j=1}^{M}\sum_{k=1}^{N}\chi\right)^{\delta}\right]}$$

$$\frac{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{\mathbb{I}_{n,t}(m)}{B_t(m)}\left(\sum_{j=1}^{M}\sum_{k=1}^{N}\frac{(S(j)-\chi)\mathbb{I}_{n,t}(j)}{N}\right)^{\delta}\right]}{\mathbb{E}\left[\liminf_{N\to\infty}\left(\sum_{j=1}^{M}\sum_{k=1}^{N}\frac{S(j)\mathbb{I}_{k,t-1}(j)}{N} + M\chi\right)^{\delta}\right]},$$

where Inequality (a) follows since we extend the negative sum $-\sum_{j=1}^{M}\sum_{k\in\mathcal{B}_t(j)} \chi\mathbb{I}_{n,t}(j)$ to include the terms from outside set $\mathcal{B}_t(j)$, $j = 1,\cdots,M$, while noting that C' is monotonically increasing. Further, we increase denominator by extending the sum to $\sum_{j,k} \chi$. Equality (b) follows through multiplying both the numerator and denominator by $N^{\delta}B_t(m)$.

Now, by Kolmogorov's strong law of large numbers, we define the quantities:

$$c_1(m) := \lim_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{\mathbb{I}_{n,t}(m)}{B_t(m)}$$

$$= \lim_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{P_{n,t}(m)}{B_t(m)} \quad \text{almost surely},$$

$$c_2(j) := \lim_{N\to\infty} \sum_{k=1}^{M} \frac{\mathbb{I}_{k,t-1}(j)}{N}$$

$$= \lim_{N\to\infty} \sum_{k=1} \frac{P_{k,t-1}(m)}{N} \quad \text{almost surely,}$$

and

$$c_3(j) := \lim_{N\to\infty} \sum_{k=1}^{M} \frac{\mathbb{I}_{k,t}(j)}{N}$$

$$= \lim_{N\to\infty} \sum_{k=1} \frac{P_{k,t}(m)}{N} \quad \text{almost surely.}$$

Since $q_{k,\tau}(j) < 1$, $\forall k, j, \tau$, $\beta_t(m) > 0$, and $P_{k,t}(j) > 0$, $\forall k \in \mathcal{B}_t(m)$, we conclude

$$c_1(m) > 0 \quad \text{almost surely}$$
$$c_2(j) > 0 \quad \text{almost surely}, j = 1, \cdots, M$$
$$c_3(j) > 0 \quad \text{almost surely}, j = 1, \cdots, M.$$

Hence,

$$\frac{\mathbb{E}\left[\liminf_{N\to\infty} \sum_{n\in\mathcal{B}_t(m)} \frac{\mathbb{I}_{n,t}(m)}{B_t(m)} \left(\sum_{j=1}^{M}\sum_{k=1}^{N} \frac{(S(j)-\chi)\mathbb{I}_{n,t}(j)}{N}\right)^{\delta}\right]}{\mathbb{E}\left[\liminf_{N\to\infty} \left(\sum_{j=1}^{M}\sum_{k=1}^{N} \frac{S(j)\mathbb{I}_{k,t-1}(j)}{N} + M\chi\right)^{\delta}\right]} \stackrel{(c)}{=}$$

$$\frac{c_1(m)\left(\sum_{j=1}^{M}(S(j)-\chi)c_3(j)\right)^{\delta}}{\left(M\chi + \sum_{j=1}^{M} S(j)c_2(j)\right)^{\delta}}.$$

The right hand side (RHS) of Equality (c) is strictly greater than 1 if and only if

$$\chi < \frac{\left(c_1(m)^{\frac{1}{\delta}}\sum_{j=1}^{M} c_3(j)S(j)\right) - \sum_{j=1}^{M} c_2(j))S(j)}{\left(M + \sum_{j=1}^{M} c_3(j)\right)}.$$

Now, to show that $\chi > 0$, it suffices to prove that

$$\left(c_1(m)^{\frac{1}{\delta}}\sum_{j=1}^{M} c_3(j)S(j)\right) - \sum_{j=1}^{M} c_2(j)S(j) > 0.$$

We have by the definition of set $\mathcal{B}_t(m)$ that

$$\sum_{n\in\mathcal{B}_t(m)} \mathbb{E}\left[\mathbb{I}_{n,t}(m)C'(L_t) - C'(L_{t-1})\right] > 0.$$

By Condition (15), and for sufficiently large N, we obtain

$$\mathbb{E}\left[\sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)(L_t)^{\delta}\right] > \mathbb{E}\left[\sum_{n\in\mathcal{B}_t(m)} (L_{t-1})^{\delta}\right],$$

which implies

$$\liminf_{N\to\infty} \frac{\mathbb{E}\left[\sum_{n\in\mathcal{B}_t(m)} \mathbb{I}_{n,t}(m)(L_t)^{\delta}\right]}{\mathbb{E}\left[\sum_{n\in\mathcal{B}_t(m)} (L_{t-1})^{\delta}\right]} > 1,$$

for otherwise $\beta_t(m) = 0$ which contradicts the hypothesis that $\beta_t(m) > 0$.

By multiplying both the numerator and denominator by $N^{\delta}B_t(m)$, we obtain

$$\frac{c_1(m)\left(\sum_{j=1}^{M} S(j)c_3(j)\right)^{\delta}}{\left(\sum_{j=1}^{M} S(j)c_2(j)\right)^{\delta}} > 1$$

$$\Rightarrow \frac{(c_1(m))^{\frac{1}{\delta}}\sum_{j=1}^{M} S(j)c_3(j)}{\sum_{j=1}^{M} S(j)c_2(j)} > 1.$$

APPENDIX H
PROOF OF THEOREM 4

We use proof by contradiction as follows. Suppose that $(\bar{\mathbf{p}}, \bar{\mathbf{x}})$ with

$$\bar{\mathbb{I}}_{n,t}(m) := \begin{cases} 1, & \text{with probability } \bar{P}_{n,t}(m), \\ 0, & \text{with probability } 1 - \bar{P}_{n,t}(m), \end{cases}$$

satisfies

$$C^{\mathcal{J}}(\bar{\mathbf{p}}, \bar{\mathbf{x}}) < C^{\mathcal{J}}(\mathbf{p}^*, \mathbf{x}^*) \tag{34}$$

where $\bar{P}_{n,t}(m) < 1 - q_{n,t}, \forall m \in \mathcal{M}^*$.

Now, by considering the difference $D := C^{\mathcal{J}}(\bar{\mathbf{p}}, \bar{\mathbf{x}}) - C^{\mathcal{J}}(\mathbf{p}^*, \mathbf{x}^*)$, we have

$$D =$$

$$C^{\mathcal{J}}(\bar{\mathbf{p}}, \bar{\mathbf{x}}) - \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left[C\left(\sum_{n=1}^{N}\left(S(m^*) - x^*_{n,t}(m^*)\right) \times \right.\right.$$

$$\left.\left. \mathbb{I}^*_{n,t}(m^*) + x^*_{n,t+1}(m^*)\right)\right] \stackrel{(a)}{\geq}$$

$$C^{\mathcal{J}}(\bar{\mathbf{p}}, \bar{\mathbf{x}}) - \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left[C\left(\sum_{n=1}^{N}\left(S(m^*) - \bar{x}_{n,t}(m^*)\right) \times \right.\right.$$

$$\left.\left. \mathbb{I}^*_{n,t}(m^*) + \bar{x}_{n,t+1}(m^*)\right)\right] \stackrel{(b)}{\geq}$$

$$\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left[C'(Y_t) \cdot \left(\sum_{m,n}\left(S(m) - \bar{x}_{n,t}(m)\right)\bar{\mathbb{I}}_{n,t}(m)\right.\right.$$

$$+\bar{x}_{n,t+1}(m) - \sum_{n=1}^{N}((S(m^*) - \bar{x}_{n,t}(m^*))\mathbb{I}^*_{n,t}(m^*) +$$

$$\left.\left. \bar{x}_{n,t+1}(m^*))\right)\right] \stackrel{(c)}{\geq}$$

$$\frac{1}{T}\min_{t=0,\cdots,T-1} C'(\inf Y_t) \cdot \sum_{t=0}^{T-1} \mathbb{E}\left[\left(\sum_{m,n}(S(m) - \right.\right.$$

$$\bar{x}_{n,t}(m))\bar{\mathbb{I}}_{n,t}(m) + \bar{x}_{n,t+1}(m) -$$

$$\sum_{n=1}^{N}\left((S(m^*) - \bar{x}_{n,t}(m^*))\mathbb{I}^*_{n,t}(m^*) + \bar{x}_{n,t+1}(m^*)\right)\right] \geq$$

$$\frac{1}{T} \min_{0 \leq t \leq T-1} C' \left(\inf Y_t\right) \times$$

$$\left(\sum_{t=0}^{T-1} \sum_{n=1}^{N} \left(S(m^*) - \bar{x}_{n,t}(m^*)\right) \left(P_{n,t}^*(m^*) - \bar{P}_{n,t}(m^*)\right) +\right.$$

$$\left. \sum_{m=1, m \neq m^*}^{M} \left(S(m) - \bar{x}_{n,t}(m)\right) \bar{P}_{n,t}(m) + \bar{x}_{n,t+1}\right) =$$

$$\frac{1}{T} \min_{0 \leq t \leq T-1} C' \left(\inf Y_t\right) \times$$

$$\left(\sum_{t=0}^{T-1} \sum_{n=1}^{N} \left(S(m^*) - \bar{x}_{n,t}(m^*)\right) \left(P_{n,t}^*(m^*) - \bar{P}_{n,t}(m^*)\right) +\right.$$

$$\left. \sum_{m=1, m \neq m^*}^{M} S(m) \bar{P}_{n,t}(m) + \bar{x}_{n,t}(m)(1 - \bar{P}_{n,t}(m))\right) \quad,$$

where the first equality follows since $x_{n,t}^*(m) = 0, \forall m \neq m^*$ as $P_{n,t}^*(m) = 0, \forall m \neq m^*$. Inequality (a) holds by replacing $x_{n,t}^*(m^*)$ with $\bar{x}_{n,t}(m^*)$ while noting that $\bar{x}_{n,t}(m^*)$ does not necessarily minimize the expected cost under $\mathbf{p}^*$. Inequality (b) holds by the mean value theorem for random variables [32] since Y_t is a random variable satisfying

$$Y_t > \min\left\{\sum_{n=1}^{N} \left(S(m^*) - \bar{x}_{n,t}(m^*)\right) \mathbb{I}_{n,t}^*(m^*) + \bar{x}_{n,t}(m^*),\right.$$

$$\left. \sum_{m=1}^{M} \sum_{n=1}^{N} \left(S(m) - \bar{x}_{n,t}(m)\right) \bar{\mathbb{I}}_{n,t}(m) + \bar{x}_{n,t}(m)\right\} \geq 0,$$

$$Y_t < \max\left\{\sum_{n=1}^{N} \left(S(m^*) - \bar{x}_{n,t}(m^*)\right) \mathbb{I}_{n,t}^*(m^*) + \bar{x}_{n,t}(m^*),\right.$$

$$\left. \sum_{m=1}^{M} \sum_{n=1}^{N} \left(S(m) - \bar{x}_{n,t}(m)\right) \bar{\mathbb{I}}_{n,t}(m) + \bar{x}_{n,t}(m)\right\},$$

on the entire space of events for all $t = 0, \cdots, T-1$. Inequality (c) is straightforward since we consider $\min$ operator instead of $\mathbb{E}$ operator, while noting that $C'(Y_t) > 0$ as C is an increasing function. The last equality follows by rearranging the terms of the preceding inequality.

Now, since

$$\frac{1}{T} \min_{0 \leq t \leq T-1} C' \left(Y_t\right)) > 0,$$

$1 - \bar{P}_{n,t}(m) \geq 0, \forall m, n, t$, and for every n, we have

$$\left(S(m^*) - \bar{x}_{n,t}(m^*)\right) \left(P_{n,t}^*(m^*) - \bar{P}_{n,t}(m^*)\right)$$

$$+ \sum_{m=1, m \neq m^*}^{M} S(m) \bar{P}_{n,t}(m) > 0$$

by the definition of $\mathbf{p}^*$. It follows that $D < 0$ which contradicts the main hypothesis in (34).

If $|\mathcal{M}^*| = 1$, the uniqueness of $(\mathbf{p}^*, \mathbf{x}^*)$ follows since we have proved that $D < 0$ for any $m^* \in \mathcal{M}^*$.

APPENDIX I
PROOF OF THEOREM 5

First, we note that $\hat{f}^k$ is convex in $(\mathbf{p}, \mathbf{x})$ since $f_0(\mathbf{p}^{k-1}, \mathbf{x})$ is convex in $\mathbf{x}$ by the definition of the cost function C, the term $\sum_{m,n,t} \frac{\partial f_0(\mathbf{p}, \mathbf{x}^{k-1})}{\partial P_{n,t}(m)}\bigg|_{\mathbf{p}=\mathbf{p}^{k-1}} \cdot P_{n,t}(m)$ is affine in $\mathbf{p}$, hence convex, and the superposition of convex functions is also convex.

Second, we consider the three conditions specified in Lemma 7. Since f_0 is continuous in $(\mathbf{p}, \mathbf{x})$ and is defined over a bounded and closed feasible set, then it has a global maximum value $U > 0$. Such a value can be added to $\hat{f}^k$ defined above to keep Condition 1) of Lemma 7 satisfied. However, adding a constant to the objective function does not affect the solution, which is main point of interest. Therefore, Condition 1) of Lemma 7 is not necessary in this case.

For Condition 2) of Lemma 7, we have

$$\frac{\partial \hat{f}(\mathbf{p}, \mathbf{x})}{\partial x_{n,t}(m)}\bigg|_{(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})} = \frac{\partial \hat{f}(\mathbf{p}^{k-1}, \mathbf{x})}{\partial x_{n,t}(m)}\bigg|_{(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})}$$

$$= \frac{\partial f_0(\mathbf{p}, \mathbf{x})}{\partial x_{n,t}(m)}\bigg|_{(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})}, \quad \forall m, n, t.$$

Likewise,

$$\frac{\partial \hat{f}(\mathbf{p}, \mathbf{x})}{\partial P_{n,t}(m)}\bigg|_{(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})} = \frac{\partial f_0(\mathbf{p}, \mathbf{x})}{\partial P_{n,t}(m)}\bigg|_{(\mathbf{p}^{k-1}, \mathbf{x}^{k-1})}, \quad \forall m, n, t.$$

Thus Condition 2) of Lemma 7 is satisfied.

Finally, Condition 3) of the same lemma need not be satisfied since it is mainly stated in Theorem 1 [34] for non-convex constraint function that has to be replaced by a convex approximate. Condition 3) mainly implies the satisfaction of the complementary slackness conditions by both the approximate and the original constraint functions. Since we are interested only in the objective function, Condition 3) of Lemma 7 is not necessary for convergence to a KKT point.

APPENDIX J
PROOF OF THEOREM 6

We use proof by contradiction. Suppose that there exists a user n_0 and a time slot t_0 for which

$$\|\hat{\mathbf{p}}_{n_0,t_0} - \tilde{\mathbf{p}}_{n_0,t_0}\| < \alpha_{n_0}(1 - q_{n_0,t_0}) H(\tilde{\boldsymbol{\pi}}_{n_0,t_0}).$$

By the local optimality of $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$, $\exists r > 0$ such that

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[C\left(L_t + \sum_{n=1}^{N} \sum_{m=1}^{M} \hat{x}_{t+1}(m) - \hat{x}_t(m)\hat{\mathbb{I}}_{n,t}(m)\right)\right] <$$

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[C\left(L_t + \sum_{n=1}^{N} \sum_{m=1}^{M} \hat{x}_{t+1}(m) - \hat{x}_t(m)\mathbb{I}_{n,t}(m)\right)\right], \tag{35}$$

for any $\mathbf{p}$ such that $\|\mathbf{p}_{n_0,t_0} - \hat{\mathbf{p}}_{n_0,t_0}\| < r$, $\mathbf{p}_{n_0,t_0} \in \mathcal{F}_{n,\tilde{\mathbf{p}}_{n_0,t_0}}$, and $\mathbf{p}_{n,t} = \tilde{\mathbf{p}}_{n,t}$ for any other $(n,t) \neq (n_0,t_0)$, with $\mathbf{p}_{n_0,t_0} = (P_{n_0,t_0}(m))_m$, and $P_{n,t}$ is the probability of the random variable $\mathbb{I}_{n,t}(m)$.

Now, we construct the profile $\bar{\mathbf{p}} = (\bar{\mathbf{p}}_{n,t})_{n,t}$ as follows. We set $\bar{\mathbf{p}}_{n,t} = \hat{\mathbf{p}}_{n,t}$ for all $(n,t) \neq (n_0, t_0)$, but $\hat{\mathbf{p}}_{n_0,t_0}$ is chosen such that

$$\|\hat{\mathbf{p}}_{n_0,t_0} - \tilde{\mathbf{p}}_{n_0,t_0}\| = \min\{r, \alpha_{n_0}(1-q_{n_0,t_0})H(\tilde{\boldsymbol{\pi}}_{n_0,t_0})\}. \quad (36)$$

Consider now the difference between the expected costs obtained under $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$, and $(\bar{\mathbf{p}}, \hat{\mathbf{x}})$. We have

$$\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[C\left(L_t + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t+1}(m) - \hat{x}_t(m)\hat{\mathbb{I}}_{n,t}(m)\right) - \right.$$
$$\left.C\left(L_t + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t+1}(m) - \hat{x}_t(m)\bar{\mathbb{I}}_{n,t}(m)\right)\right] \overset{(a)}{=}$$
$$\frac{1}{T}\cdot\mathbb{E}\left[C\left(L_{t_0} + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t_0+1}(m) - \hat{x_{t_0}}(m)\hat{\mathbb{I}}_{n,t_0}(m)\right) - \right.$$
$$\left.C\left(L_{t_0} + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t_0+1}(m) - \hat{x}_{t_0}(m)\bar{\mathbb{I}}_{n,t_0}(m)\right)\right] \overset{(b)}{=}$$
$$\frac{1}{T}\cdot\mathbb{E}\left[C'(Y_{t_0})\left(\sum_{m=1}^{M}\sum_{n=1}^{N}(S(m) - \hat{x}_{n,t_0}(m))\right)\right] \overset{(c)}{\geq}$$
$$\frac{1}{T}\cdot C'(\inf Y_{t_0})\cdot(\mathbf{s} - \hat{\mathbf{x}}_{n_0,t_0})^{\dagger}(\hat{\mathbf{p}}_{n_0,t_0} - \bar{\mathbf{p}}_{n_0,t_0}) \quad (37)$$

where Equality (a) holds by the construction of $\bar{\mathbf{p}}$. Equality (b) holds by the mean values theorem for random variables [32], where Y_{t_0} is a positive random variable (see also the proof of Theorem 4). Inequality (c) follows by taking $\inf$ of the random variable Y_{t_0} while noting that C' is a positive function, and taking the expectation of the sum. The vectors $\mathbf{s}$ and $\hat{\mathbf{x}}_{n_0,t_0}$ are constructed respectively as $\mathbf{s} := (S(m))_m$, and $\hat{\mathbf{x}}_{n_0,t_0} := (\hat{x}_{n_0,t_0}(m))_m$, and the notation $\mathbf{x}^{\dagger}$ is the transpose of $\mathbf{x}$.

Now, it suffices to show that the RHS of (37) is positive. To do so, we construct another collection of demand profiles $\check{\mathbf{p}} = (\check{\mathbf{p}}_{n,t})_{n,t}$ with $\check{\mathbf{p}}_{n,t} = \hat{\mathbf{p}}_{n,t}$ for all $(n,t) \neq (n_0, t_0)$ and $\check{\mathbf{p}}_{n_0,t_0} = \tilde{\mathbf{p}}_{n_0,t_0}$. Also, we use $\check{P}_{n,t}(m)$ as the probability that a binary random variable $\check{\mathbb{I}}_{n,t}(m)$ equals one.

By the optimality of $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$, the difference between the costs incurred under $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ and $(\check{\mathbf{p}}, \hat{\mathbf{x}})$ must be negative. That is,

$$D :=$$
$$\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[C\left(L_t + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t+1}(m) - \hat{x}_t(m)\hat{\mathbb{I}}_{n,t}(m)\right) - \right.$$
$$\left.C\left(L_t + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t+1}(m) - \hat{x}_t(m)\check{\mathbb{I}}_{n,t}(m)\right)\right] < 0. \quad (38)$$

After conducting a similar analysis to this used in deriving (37), we obtain the following necessary condition on the local optimality of $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$:

$$(\mathbf{s} - \hat{\mathbf{x}}_{n_0,t_0})^{\dagger}(\hat{\mathbf{p}}_{n_0,t_0} - \tilde{\mathbf{p}}_{n_0,t_0}) < 0. \quad (39)$$

Now, by the construction of $\bar{\mathbf{p}}$, where $\bar{\mathbf{p}}_{n_0,t_0}$, $\hat{\mathbf{p}}_{n_0,t_0}$, and $\tilde{\mathbf{p}}_{n_0,t_0}$ lie on the same line, $\exists\ \xi > 1$ such that

$$\bar{\mathbf{p}}_{n_0,t_0} - \tilde{\mathbf{p}}_{n_0,t_0} = \xi\cdot(\hat{\mathbf{p}}_{n_0,t_0} - \tilde{\mathbf{p}}_{n_0,t_0}).$$

Hence, we can write

$$\hat{\mathbf{p}}_{n_0,t_0} - \bar{\mathbf{p}}_{n_0,t_0} = (1 - \xi)(\hat{\mathbf{p}}_{n_0,t_0} - \tilde{\mathbf{p}}_{n_0,t_0}).$$

Substituting with this result in the RHS of (37), it turns out that

$$\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[C\left(L_t + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t+1}(m) - \hat{x}_t(m)\hat{\mathbb{I}}_{n,t}(m)\right) - \right.$$
$$\left.C\left(L_t + \sum_{n=1}^{N}\sum_{m=1}^{M}\hat{x}_{t+1}(m) - \hat{x}_t(m)\bar{\mathbb{I}}_{n,t}(m)\right)\right] > 0$$

since $\xi > 1$. This, therefore, contradicts the optimality condition (35). As a result, $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ must satisfy (28).

REFERENCES

[1] Cisco Visual Networking Index: Forecast and Methodology, $2010-2015$. http://www.cisco.com/en/US/solutions/collateral/ns341/ns525/ns537/ns705/ns827/white_paper_c11-481360.pdf

[2] Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, $2010-2015$. http://www.cisco.com/en/US/solutions/collateral/ns341/ns525/ns537/ns705/ns827/white_paper_c11-520862.pdf

[3] FCC. Spectrum policy task force report, FCC 02-155. Nov. 2002.

[4] FCC. Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order, FCC 03-322. December 2003.

[5] J. Mitola III,"Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio," Doctor of Technology Dissertation, Royal Institute of Technology (KTH), Sweden, May, 2000

[6] I. Akyildiz, W. Lee, M. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks Journal (Elsevier)*, vol. 50, Issue 13, pp. 2127-2159, September 2006.

[7] S. A. Jafar, S. Srinivasa, I. Maric, and A. Goldsmith, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proceedings of the IEEE*, May 2009.

[8] J. Tadrous, A. Eryilmaz, and H. El Gamal, "Proactive resource allocation: harnessing the diversity and multicast gains," http://arxiv.org/abs/1110.4703

[9] H. El Gamal, J. Tadrous and A. Eryilmaz, "Proactive resource allocation: Turning predictable behavior into spectral gain," *2010 48th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, vol., no., pp.427-434, Sept. 29 2010-Oct. 1 2010.

[10] J. Tadrous, A. Eryilmaz and H. El Gamal, "Proactive multicasting with predictable demands," *2011 IEEE International Symposium on Information Theory Proceedings (ISIT)*, vol., no., pp.239-243, July 31 2011-Aug. 5 2011.

[11] J. Tadrous, A. Eryilmaz, H. El Gamal, and M. Nafie, "Proactive resource allocation in cognitive networks," *2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, vol., no., pp.1425-1429, 6-9 Nov. 2011.

[12] N., Eagle and A. Pentland, "Reality mining: sensing complex social systems", *Personal and Ubiquitous Computing*, vol. 10, pp. 255-268, 2006.

[13] K. Farrahi and D. Gatica-Perez, "Discovering human routines from cell phone data with topic models," *The 12th IEEE International Symposium on Wearable Computers*, pp. 29-32, 2008.

[14] B.S. Jensen, J.E. Larsen, K. Jensen, J. Larsen and L. K. Hansen, "Estimating human predictability from mobile sensor data", *2010 IEEE International Workshop on Machine Learning for Signal Processing (MLSP)*, pp. 196 - 201, Sept. 2010.

[15] R. Kwok, "Personal technology: phoning in data", *Nature*, vol. 458, pp. 959-961, 2009.

[16] C. Song, Z. Qu, N. Blumm, and A. Barabas, "Limits of predictability in human mobility," *Science*, vol. 327, pp. 1018-1021, Feb. 2010.

[17] http://oss.oetiker.ch/rrdtool/gallery/index.en.html

[18] Zan Huang, D. Zeng and H. Chen, "A Comparison of Collaborative-Filtering Recommendation Algorithms for E-commerce," *IEEE Intelligent Systems*, vol.22, no.5, pp.68-78, Sept.-Oct. 2007.

[19] T.L. Wickramarathne, K. Premaratne, M. Kubat and D. T. Jayaweera, "CoFiDS: A Belief-Theoretic Approach for Automated Collaborative Filtering," *IEEE Transactions on Knowledge and Data Engineering*, vol.23, no.2, pp.175-189, Feb. 2011.

[20] J. Salter and N. Antonopoulos, "CinemaScreen recommender agent: combining collaborative and content-based filtering," *IEEE Intelligent Systems*, vol.21, no.1, pp. 35- 41, Jan.-Feb. 2006.

[21] L. Kozma, A. Ilin, and T. Raiko, "Binary principal component analysis in the Netflix collaborative filtering task," *IEEE International Workshop on Machine Learning for Signal Processing, 2009. MLSP 2009.*, vol., no., pp.1-6, 1-4 Sept. 2009.

[22] O.B. Fikir, I.O. Yaz and T. Ozyer, "A Movie Rating Prediction Algorithm with Collaborative Filtering," *2010 International Conference on Advances in Social Networks Analysis and Mining (ASONAM)*, vol., no., pp.321-325, 9-11 Aug. 2010.

[23] G. Adomavicius and A. Tuzhilin, "Toward the next generation of recommender systems: a survey of the state-of-the-art and possible extensions," *IEEE Transactions on Knowledge and Data Engineering,* vol. 17, no.6, pp. 734- 749, June 2005.

[24] V. Glass and P. U. Edge Lab, "United States broadband goals: managing spillover effects to increase availability, adoption and investment," white paper, 2010. Available: http://scenic.princeton.edu/paper/NECAPrincetonPaperJune2010.pdf

[25] "The mother of invention: network operators in the poor world are cutting costs and increasing access in innovative ways," *The Economist*, Special Report, September 24 2009.

[26] S. Higginbotham, "Mobile operators want to charge based on time and apps," *gigaOm*, December 14 2010.

[27] S. Ha, C. J.-Wong, S. Sen and M. Chiang, "Pricing by timing: innovating broadband data plans," Proceedings of *SPIE 8282, Broadband Access Communication Technologies*.

[28] J. Tadrous, A. Eryilmaz, and H. El Gamal, "Pricing for demand shaping and proactive download in smart data networks," proceedings of *The 2nd IEEE International Workshop on Smart Data Pricing (SDP 2013)*, INFOCOM 2013.

[29] A. K. Pathan, and R. Buyya, A taxonomy and survey of content delivery networks, *Tech Report*, University of Melbourne, 2007.

[30] J. Kangasharju, J. Roberts, and K. Ross, "Object replication strategies in content distribution networks," *Computer Communications*, vol. 25, no. 4, pp. 376383, March 2002.

[31] S. Boyd and L. Vandenberge, "Convex Optimization," *Cambridge University Press*, 2004.

[32] A. Di Crescenzo, "A probabilistic analogue of the mean value theorem and its applications to reliability theory," *Journal of Applied Probability*, vol. 36, no. 3, pp. 706-719, Sept. 1999.

[33] T. Cover and J. Thomas, "Elements of information theory," *Wiley − Interscience*, 2006.

[34] B. Marks and G. Wright, "A general inner approximation algorithm for nonconvex mathematical programs, *Operations Research*, vol. 26, Issue 4, pp. 681-683, 1978.