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Reply to the comment by W.R. Peltier

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1. Introduction

Peltier (2002) raises two issues concerning our earlier paper (Yokoyama et al., 2000) in which we addressed the question of ice volumes at the time of the LGM as inferred from new sea-level data from the Bonaparte Gulf of northwestern Australia. These issues concern the reliability or otherwise of (i) the glacio-hydro-isostatic calculation and (ii) the inferred volume of land-based, including shelf-grounded, ice. On the first, he states that we violate the principle of mass conservation, and on the second, he concludes that while our estimates of the LGM lowstand are likely to be correct our arguments are based on faulty logic. We respond to these two points separately below and then make a comment about the use of observations in sea level analyses. The response to the first point has been published (Yokoyama et al., 2001a) and in it, contrary to Peltier's statement, we did address the consequences concerning estimates of ice volume and concluded that these estimates were correct. We are pleased that, contrary to earlier versions of his criticism, he now accepts our estimates of this volume, although he now argues that the way we reached the estimates is based on faulty logic.

2. The glacio-hydro-isostatic calculation

In the original Yokoyama et al. (2000) paper, as well as in Lambeck et al. (2000), an error had been introduced—as discussed in Yokoyama et al. (2001a) and Lambeck et al. (2001)—that made an erroneous distinction between the ice-volume equivalent sea level (defined below) and eustatic sea level. In the absence of other factors contributing to sea-level change (thermal expansion, melting of mountain glaciers not included in the ice models, or changes in ground- and surface-water storage), these two terms are the same. This error has been addressed in the two corrections cited above but because Peltier ignores this we repeat the comments here.

The distinction drawn was a consequence of a programming error introduced to the section of our code that calculated the mean change in sea level over the oceans. This portion of the program was altered in 1997 to incorporate refinements to the evaluation of the ocean function, specifically an improved treatment of ice shelves in the load history and in the definition of the ocean-basin margin. For each iteration of the solution of the sea-level equation, the mean sea-level change over the oceans was output in the form of the Y_{00} coefficient of the spherical harmonic expansion of sea-level change. When this value was compared with the ice-volume equivalent sea level calculated directly from the ice volumes of the input ice sheets (Eq. (3) below), a difference was found but, instead of ringing alarm bells, this was interpreted as the two estimates not being the same thing. In our formulation (see Nakada and Lambeck, 1987; Johnston, 1993; Lambeck and Johnston, 1998), sea-level change is mathematically non-zero over land. This permits, for example, the calculation of water-level change along narrow canals, of changes in leveling networks, the tilting of lakes, and the time-dependence of the elevations of thresholds to inland lakes. If the sea-level equation is formulated such that sea-level change is everywhere zero on land then the Y_{00} component of sea-level change is correspondingly equal to the mean sea-level change over the ocean. However, this results in a loss of flexibility in calculating terrain changes on the continents and in our formulation the average sea-level change at any time is determined by evaluating the integral over the ocean as defined at that time by the shoreline and the ice-grounding line. However, in the actual calculation of Y_{00} the integration

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was mistakenly carried out over the entire globe but in all sea-level calculations the second estimate, based directly on the input ice model, was used."

Recent checks, in which the averaging is now done correctly, lead to identical results for the two estimates of ice-volume equivalent sea level, the one evaluated directly from the ice volumes, the other from Y_{00}. The conceptual error arose because, rather than recognizing, that these results violated the condition of conservation of mass imposed in the core-program, it appeared to offer an explanation for the distinction that has sometimes been made between the equivalent sea-level and eustatic sea level (e.g. Denton and Hughes, 1981, p. 274).

The theory used by us is essentially that of Farrell and Clark (1976), iterated so as to allow for the loading by the spatially variable water load and for the time dependence of the ocean basins, and including rotation effects (Milne, 1998). This formulation has evolved through three phases as successive improvements were introduced: first by Nakiboglu et al. (1983), then independently by Nakada and Lambeck (1987) and again, for the water loading part, independently by Johnston (1993, 1995). The presently used formulation is discussed in Lambeck and Johnston (1998). Intercomparisons indicated consistency at each stage. Parts of the code were checked against independent results by G. Kaufmann and recent applications of the latter, using comparable ice sheet models and sea-level observations, led to agreement in terms of inferred mantle response parameters (Kaufmann and Lambeck, 2000). Also, checks against the wholly independent code developed by J. X. Mitrovica and G. Milne indicates good agreement (Mitrovica, personal communication) although exact comparisons based on identical ice- and earth-model input parameters have not yet been completed. We note that although we do not give names to specific parts of the surface load, the definition of the ice–water load includes ice on the shelves when grounded and the changing ice–water load when the ice lifts off the shelf due to either ice thinning or to sea-level rise (see, for example, Lambeck et al., 1998, p. 115; Lambeck and Johnston, 1998, p. 471). After all, the implementation of these contributions is more a matter of programming than a new bit of physics! We also emphasize that the model includes the time dependence of the shorelines and the consequential loading of shallow shelves during the sea-level transgression. Note that this was already introduced by Chappell (1974) for regional models and by Nakada and Lambeck (1990) for global models, although the more recent models are based on the formulation by Johnston (1995) (see also Milne, 1998; Milne and Mitrovica, 1998). In view of the long history of this treatment of loading of the shelf, it is curious that Peltier now appears to be taking credit for the discovery of its importance. The preliminary comparisons referred to above made between our theory and those of the other 'Toronto Group' indicate the same relationship between the local sea level at continental margins and eustatic sea level. This is the relationship that Peltier appears to have belatedly discovered (see his conclusion). Since we do have a rigorous and internally consistent description of the ice–water load through time there is no validity in the statement made by Peltier at the end of paragraph 1, Section 2, following Eq. (5).

3. The inferred ice volume

Contrary to Peltier's statement, the impact of the above-mentioned error on the ice volume estimates was discussed in the corrections (Yokoyama et al., 2001a; Lambeck et al., 2001). In particular, it was noted that there was no impact because the ice-volume equivalent sea level was calculated directly from the ice volumes and not from the integrated local sea levels.

The sea-level equation for a tectonically stable area can be written schematically as

$$\Delta\zeta_{\mathrm{rsl}} = \Delta\zeta_{\mathrm{i}} + \Delta\zeta_{\mathrm{e}} = \Delta\zeta_{\mathrm{load}} - \langle\Delta\zeta_{\mathrm{load}}\rangle_0 + \Delta\zeta_{\mathrm{e}}, \tag{1}$$

where $\Delta\zeta_{\mathrm{rsl}}$ (φ, t) is the height of the palaeo sea surface relative to present sea level and is a function of position φ and time t. $\Delta\zeta_{\mathrm{e}}(t)$ is the "ice-volume-equivalent sea-level change" (defined below) associated with the change in ocean volume resulting from the melting or growth of land-based ice sheets. This includes ice grounded on shelves, including the ice that is below coeval sea level, but not floating ice. The $\Delta\zeta_{\mathrm{i}}$ term represents the combined glacio- and hydro-isostatic contributions to sea-level change from the isostatic crustal displacement and associated geoid change. It is a function of the earth rheology, of the ice mass distribution through the glacio isostatic part, and of the spatial and temporal distribution of the water load including the migration of shorelines through the hydro-isostatic part. $\Delta\zeta_{\mathrm{i}}$ may also be expanded into the alternative form given above where $\Delta\zeta_{\mathrm{load}}$ represents the direct effects of glacio- and hydro-isostatic loading and $\langle\Delta\zeta_{\mathrm{load}}\rangle_0$ the integral of this quantity over the area of the ocean, often called the mass balance term (cf. Farrell and Clark, 1976).

The full solution of (1) relies on accurately determining the distribution of melt water across the oceans and therefore requires knowledge of the ocean depths $h(\varphi, t)$ through time which are determined from

$$h(\varphi, t) = h(\varphi, t_0) - \Delta\zeta_{\mathrm{rsl}}(\varphi, t), \tag{2}$$

where $h(\varphi,\ t_0)$ is the present-day (t_0) bathymetry or topography at location φ. The ocean basin geometry at time t is determined by the contour $h(\varphi, t) = 0$ and by the location of the grounding lines of the ice sheets on the shelves. The grounding line is defined at the positions where the thickness of the shelf ice at time t is equal to $h\rho_o/\rho_i$ where ρ_o and ρ_i are the average densities of ocean and ice, respectively. Thus, the ocean function is dependent on the ice model as well (see also Milne (1998) and Milne et al. (1999) in this regard who have used a correct formulation since at least 1998 and described it as 'water dumping'). To simply state that because we do not mention the influence of 'implicit ice' (Peltier's terminology) we do not treat the mass fluxes correctly is clearly unwarranted, particularly as we were incorporating these factors before the terminology was invented (see Lambeck et al., 1998).

When shelf ice has thinned enough for it to float, the mass of this ice becomes part of the ocean load and the ocean function is shifted to the new grounding line. The solution of the sea-level equation is then determined by iterating between (1) and (2) and the time dependence of the ice sheet margins and thickness of ice on the shelf. Numerical tests indicate that this approach is accurate to at least 1–2 m at the time of the LGM. This is because the time steps at which the ice sheet is defined are short and because the load history between successive time steps is defined by linear functions (Nakada and Lambeck, 1987).

The term $\Delta\zeta_e(t)$ relates to the total change in land-based ice volume V_i according to

$$\Delta\zeta_e(t) = -\frac{\rho_i}{\rho_o}\int_t \frac{1}{A_o(t)}\frac{dV_i\,dt}{dt}, \tag{3}$$

where $A_o(t)$ is the ocean surface area. $A_o(t)$ is a function of time because of the advance or retreat of shorelines as the relative position of land and sea is modified and because of the retreat or advance of grounded ice over shallow continental shelves and seas. With this definition, the ice mass includes grounded shelf ice at all times.

The process for estimating the ice volumes from local observations of sea level $\Delta\zeta_{obs}$ is to predict the isostatic term, $\Delta\zeta_i$, for the location and time of observation, based on the initial earth and ice models, and subtract this and the corresponding $\Delta\zeta_e$ from the observed value. The residual then represents the sum of a correction to the ice-volume equivalent sea level, any correction to the isostatic term $\Delta\zeta_i$ that may be due to earth- and ice-model limitations, and any error in the observed value. As noted by Lambeck and Nakada (1990) and confirmed by Peltier in his present note, the dependence of $\Delta\zeta_i$ on the earth rheology parameters is not very strong and, in the case of the Bonaparte Gulf analyses of Yokoyama et al., rheological parameters derived from analyses of Holocene sea level data for the Australian region are used (Lambeck and Nakada, 1990; Lambeck, 2001). The accuracy estimate of this correction is based on a range of predicted values for different plausible earth models as well as for different ice models.

If the resulting correction to the ice-volume equivalent sea level is substantial, the difference from the starting ice model is distributed between the ice sheets, consistent with rebound analyses in the neighborhood of the ice sheets, and the solution is iterated. Once the solution for the ice-volume equivalent sea level has converged, the ice volume function $V_i(t)$ follows from the inverse of (3), using the same time dependence of the ocean area as before.

This is the procedure used by Yokoyama et al. (2000) to produce their Fig. 2b which represents the observed sea level minus the local isostatic correction $\Delta\zeta_i$ (Eq. (1)). At no stage has some average correction been applied as suggested by Peltier in his Section 3 of the comment. The resulting ice volumes include any shelf ice that is grounded. Thus, all results in Fig. 2 are correct.

Since we did not receive the paper referred to by Peltier as P02 until very recently, and because the nature of his criticism changed even more recently, we cannot comment further on where the differences lie, but it appears from his Fig. 3 that we essentially produce the same predictions of local relative sea level at the coastal sites. We argue, following the original Farrell and Clark model (but including the higher order effects), that an isostatic correction is required to reduce the local levels to an estimate of ice volume. For example, at sites far from the ice margins, the principal isostatic effect is from the water load. During the melting phase, the rise in sea level loads and the ocean floor which subsides in response and drags the continental shelf down with it. Far offshore, the isostatic effect would be small (ignoring glacio-isostatic effects) and the sea-level signal would be essentially equal to the ice-volume equivalent sea-level estimate. But on-shore or near-shore the local sea level will depart from the latter and this is the isostatic effect we apply allowing for the shifting shorelines as sea level rises and falls. Our original estimates for the ice volumes are robust.

Two other comments about the critique are worth making. (i) There is now an acknowledgement by Peltier that Barbados may not be the best target for tuning the theory, a point previously made by the present authors (see their paper, this volume). (ii) There is finally a recognition that all data sets from locations far from the ice margins can be reconciled. However, there is no need for a new line of argument as stated by Peltier because the 'reconciliation' has existing models include the effects. The 'reconciliation' has already been made (Lambeck et al., 2002).

4. A comment on observations

The data set used in Fig. 2 of Yokoyama et al. (2000) represents those data points that are indicative of brackish-water and marginal marine conditions at the time of faunal growth. In their Fig. 1, ages for other sediments in the core are given but these correspond to deeper water environments and for which the error bars are correspondingly much larger and one-sided. For example, a number of the ages correspond to open marine or shallow marine facies for which sea level may have been tens of meters higher at the time. A full discussion of the interpretation of the data is given by Yokoyama et al. (2001b) and anyone using these additional data points should look carefully at the discussion therein. It is certainly inappropriate to use these latter data points as done by Peltier in his Fig. 3 without introducing error bars that reflect the depositional environments at the time of formation. If this was also done, for example, for the complete data set from the South China Sea (see Hanebuth et al., 2000b) as given in Hanebuth (2000) then a similar large scatter of lower-limit sea level estimates would result.

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