integrator wind-up and, therefore, it is recommended that separate weighting be used with a modified integrating component predictive controller.

The separate weighting also improves the designers intuition with respect to tuning the controller, significantly reducing the time required to generate desired closed loop responses.

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Robust State Estimation for Linear Systems

S. S. Garimella[1,2] and K. Srinivasan[2]

Real-time state estimation of a linear dynamic system using an observer, in the presence of modeling errors in the system model used by the observer and uncertainty in the initial system states, is considered here. A guideline for designing observers for multioutput systems is established, based on an expression for an upper bound on the norm of the state estimation error derived in this paper. An example is presented to illustrate the usefulness of this guideline.

Introduction

The usefulness of observers for real-time state estimation of linear dynamic systems based on measured system outputs is well known. Procedures for designing observers (Luenberger, 1971; Gopinath, 1971) are based on the system model used by the observer being accurate. The presence of errors in the system model used by the observer makes the robustness of the estimate state to these errors a significant consideration in observer design. The effect of modeling error on the state estimate provided by the observers has been studied by Thau and Kestenbaum (1974) and bounds on state estimation errors obtained. However, no guidelines are available for observer design to reduce the state estimation error in cases where there is adequate freedom of design, for instance, for multi-output systems where eigenvectors are assignable even after eigenvalue assignment. Furuta et al. (1976) and Bhattacharyya (1980) have considered one class of robust observers, namely, reduced order observers which satisfy the condition of asymptotic convergence of the estimated state vector to the exact state vector despite parametric errors in the state space model of the system used by the observer. Such observers have been termed zero sensitivity observers or parameter invariant observers. Based on geometric considerations, conditions for the existence of such observers have been derived, and procedures for their design have been described. However, these conditions are highly restrictive in nature, because of the requirement of asymptotic convergence of the state estimates to their exact values despite modeling error.

Another approach to robust state estimation has centered upon the fact that the estimated state is often used for feedback control. Hence, the criterion for observer design in these cases is to reduce the effect of modeling errors on the controlled system response. The work of Doyle and Stein (1979) on robust observers falls in this category. The observer is designed to achieve full-state loop transfer recovery, in this case the emphasis being on recovery of the robustness of the full-state feedback design. Thus, the robustness of the controlled system to modeling errors is characterized by robustness measures of the full-state feedback design. Bongiorno (1973), Thau and Kestenbaum (1974) and Furuta et al. (1976) have also considered the effect of modeling errors in the observer on different aspects of the overall controlled system performance.

The current work on robust state estimation using observers is motivated by the need to estimate pressure and temperature fields in thermoplastic injection molding processes, based on a few measurement locations in the mold cavity. Robustness of the estimate to errors in the process model is essential for this application given the complexity of the process. The initial use of the estimated pressure and temperature fields is for more effective process monitoring rather than for feedback control.

The robustness of the state estimates obtained using observers, in the presence of system modeling error, is examined in this paper following the procedure of Thau and Kestenbaum (1974). Since the estimated state is to be used only for process monitoring in the application of interest, the consequences of modeling error in the observer for control action are not examined. An upper bound on the norm of the state estimation error is obtained and shown to be related to the eigenstructure of the observers. Simple guidelines for observer eigenstructure assignment to lower the estimation error bound are offered and illustrated by an example. Concluding remarks are given at the end of the paper.

Determination of State Estimation Error Bound

Consider the linear time-invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) \tag{1}$$

subject to the initial condition

$$x(0) = x_0$$

where A, B, and C are $(n \times n)$, $(n \times p)$, and $(m \times n)$ matrices, respectively, and $x(t)$, $u(t)$, and $y(t)$ are $(n \times 1)$, $(p \times 1)$ and $(m \times 1)$ vectors, respectively. A full order observer is designed

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based on this model to estimate the state $x(t)$. The observer is described by (Luenberger, 1964)

$$\dot{\hat{x}}(t) = A_c\hat{x}(t) + B_c u(t) + L(y(t) - \hat{y}(t))$$

$$\hat{y}(t) = C\hat{x}(t) \tag{2}$$

subject to the initial condition

$$\hat{x}(0) = \hat{x}_0$$

Note that modeling errors are permitted only in the A and B matrices and not in the C matrix. Let the estimation error be defined by

$$e(t) = x(t) - \hat{x}(t) \tag{3}$$

Manipulation of (1), (2), and (3) leads to

$$\dot{e}(t) = (A_c - LC)e(t) + (A - A_c)x(t) + (B - B_c)u(t)$$

$$= F_c e(t) + \Delta A x(t) + \Delta B u(t) \tag{4}$$

subject to the initial condition

$$e(0) = x(0) - \hat{x}(0) = e_0 \tag{5}$$

The eigenvalues of the augmented system described by (1) and (4) are those of A and F_c. We assume that the input $u(t)$ is bounded in magnitude and that all the eigenvalues of A have negative real parts, thus ensuring that the estimation error is bounded if all the eigenvalues of F_c also have negative real parts.

The solution of (4) yields, for the case of distinct eigenvalues, (Brogan, 1982)

$$e(t) = Me^{\Lambda t}M^{-1}e_0 + \int_0^t Me^{\Lambda(t-\tau)}M^{-1}\Delta A e^{A\tau}x_0 d\tau$$

$$+ \int_0^t Me^{\Lambda(t-\tau)}M^{-1}(\Delta A \int_0^\tau e^{A(\tau-\tau_1)}Bu(\tau_1)d\tau_1 + \Delta B u(\tau))d\tau \tag{6}$$

where

$$F_c = M\Lambda M^{-1} \text{ and } e^{F_c t} = Me^{\Lambda t}M^{-1} \tag{7}$$

M being the modal matrix corresponding to F_c and Λ a diagonal matrix with the eigenvalues of F_c as the diagonal elements. Extension of the results obtained here to the case of repeated eigenvalues is relatively straightforward. Taking norms of both sides of Eq. (6), we get

$$|e(t)| \le \|M\| \cdot \|M^{-1}\| \{ \|e^{\Lambda t}\| \cdot |e_0| + \int_0^t \|e^{\Lambda(t-\tau)}\| \cdot |\Delta A e^{A\tau}x_0| d\tau$$

$$+ \int_0^t \|e^{\Lambda(t-\tau)}\| \cdot |\Delta A \int_0^\tau e^{A(\tau-\tau_1)}Bu(\tau_1)d\tau_1 + \Delta B u(\tau)| d\tau \}$$

$$= k(M)\{ e^{c_1 t} \cdot |e_0| + \int_0^t e^{c_1(t-\tau)} \cdot |\Delta A e^{A\tau}x_0| d\tau$$

$$+ \int_0^t e^{c_1(t-\tau)} \cdot |\Delta A \int_0^\tau e^{A(\tau-\tau_1)}Bu(\tau_1)d\tau_1 + \Delta B u(\tau)| d\tau \} \tag{7}$$

$$\|e^{\Lambda t}\| = e^{c_1 t} \tag{8}$$

c_1 being the real part of the observer pole farthest to the right in the complex plane, assumed to be negative here. $|v|$ represents the Euclidean norm of any $(n \times 1)$ vector v and $\|P\|$ represents the spectral norm of any $(n \times n)$ matrix P above. Also, $k(M)$ is the condition number of the $(n \times n)$ matrix M and is equal to $\|M\|$. $\|M^{-1}\|$ (Wilkinson, 1965). The spectral norm $\|P\|$ is induced by and compatible with the Euclidean vector norm $|v|$ allowing us to write (Morari and Zafiriou, 1989)

$$|Pv| \le \|P\| \cdot |v| \tag{9}$$

Note that the expression within curly brackets on the right hand side of Eq. (7) depends on the observer eigenvalues and not on the eigenvectors associates with these eigenvalues. The dependence of the state estimation error bound on these eigenvectors is solely via the condition number $k(M)$ of the modal matrix corresponding to F_c. Therefore, for competing observer designs with the same eigenvalues, the only difference is in the modal matrix M. The other terms within the curly brackets would be identical for such competing designs.

Equation (7) is the basis of the observer design guideline formulated here. Observer eigenvalues are selected to ensure more rapid decay of the estimation error transients as compared to the state transients (Luenberger, 1971; Gopinath, 1971). For multi-output cases, eigenvalue selection does not determine observer gains uniquely. Equation (7) suggests that in these cases,the observer gains should be chosen to minimize the condition number $k(M)$, in addition to yielding the desired eigenvalues. The condition number $k(M)$ is minimized to unity if the eigenvectors of the matrix F_c, which are also the columns of the modal matrix M, form a mutually orthonormal set. In general, however, the eigenvectors corresponding to specified eigenvalues are not arbitrarily assignable. Consequently, the objective in eigenvector selection here should be to make them as nearly mutually orthogonal as possible, to reduce the estimation error bound. Furthermore, since the condition number of a matrix is almost invariably reduced by equilibration, that is, by scaling the elements of each eigenvector to get a Euclidean norm of nearly unity (Wilkinson, 1965), the columns of the modal matrix are equilibrated before the condition number $k(M)$ is determined. The bound on the estimation error norm is then lower than if the columns of the modal matrix were not equilibrated.

The result obtained here that the eigenvectors corresponding to the observer eigenvalues be chosen to be as nearly mutually orthogonal as possible to reduce the norm of the state estimation error seems to be a natural extension of a result obtained by Gilbert (1984) relating to eigenvalue sensitivity. In the cited reference, Gilbert has shown that a Euclidean matrix norm of the sensitivity of the eigenvalues of a matrix with respect to its elements is minimized for all the eigenvalues if the eigenvectors of the matrix are mutually orthogonal. It should be noted that sensitivity of the observer eigenvalues is not of direct interest here since they depend only on model parameters and observer gains. The result presented here suggests, however, that the factors governing eigenvalue sensitivity of the matrix F_c are also significant in determining the size of the state estimation error though they are not exclusively so. Furthermore, the result presented here shows clearly the dependence of the state estimation error on other factors.

The suggested observer design guideline does not address the issue of observer eigenvalue selection despite the fact that eigenvalue selection affects the estimation error. Thus, selection of observer eigenvalues without reference to consequences for estimation error may well lead to more robust observer designs being overlooked. Futhermore, Eq. (7) provides only a bound on the estimation error norm. Therefore, it is possible that even if two observer designs differ only in their eigenvector selections, the actual state estimation error norm may in some cases be lower for the design which yields a higher value of $k(M)$ and hence of the error bound. This is less likely to occur, however, if the difference in the values of $k(M)$ for the competing designs is large. Finally, the results obtained here are valid only for cases where the C matrix is known exactly.

The procedure for eigenvector selection and observer gain computation follows that of D'Azzo and Houpis (1988). Since the eigenvectors and reciprocal eigenvectors of a matrix are known to be mutually orthogonal, the procedure begins with selection of the reciprocal eigenvectors of F_c to be as nearly orthogonal as possible and normalized to have Euclidean norms of unity. Let w_i^T be the reciprocal eigenvector corresponding

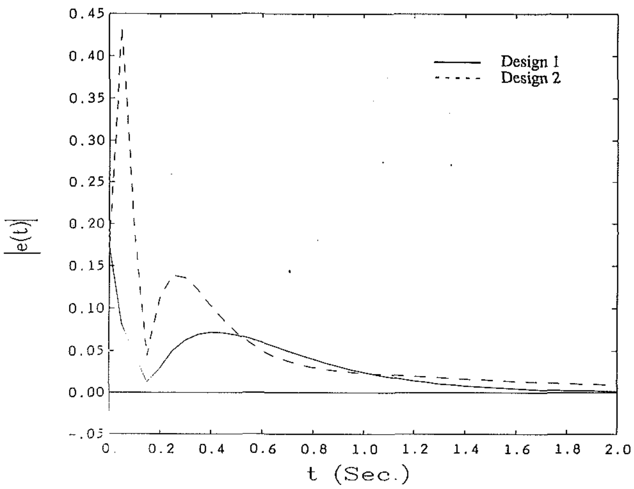

Fig. 1 Estimation error when both the initial estimation error and the observer model error are considered

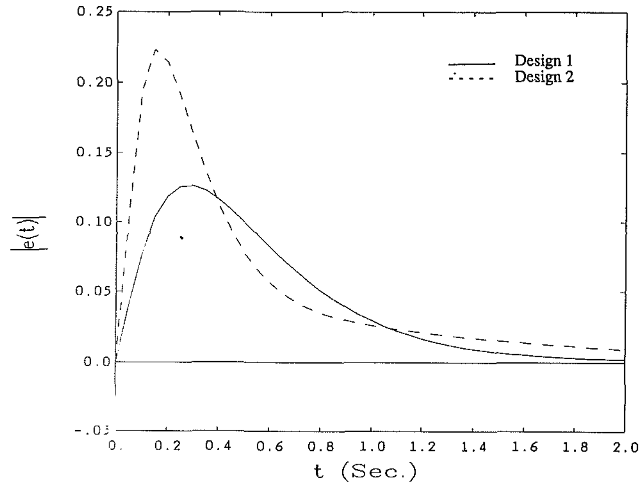

Fig. 3 Estimation error when only the observer model error is considered

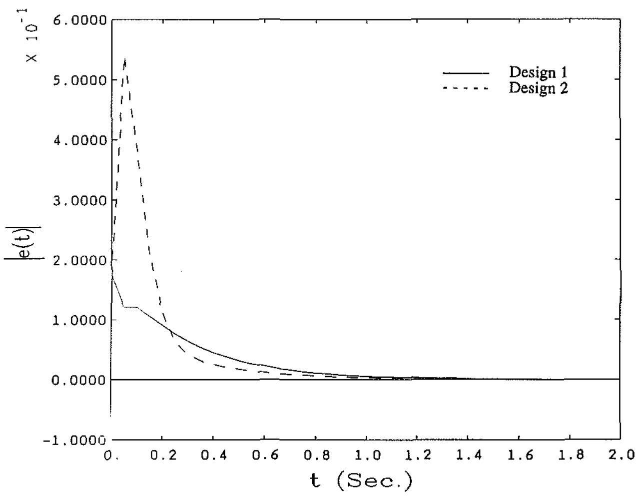

Fig. 2 Estimation error when only the initial estimation error is considered

to the eigenvalue λ_i of F_c. Then, it can be shown that the vector $(w_i^T\ \xi_i^T)^T$ lies in the null space of the matrix

$$S(\lambda_i) = (A_c^T - \lambda_i I\ C^T) \tag{10}$$

for the n specified eigenvalues of F_c. At this point in the observer design, the available freedom in eigenvector assignment is used to obtain as nearly mutually orthogonal a set of reciprocal eigenvectors as is possible. The observer gain matrix is then given by

$$L^T = -(\xi_1\ \xi_2 \ldots\ldots \xi_n)(w_1\ w_2 \ldots\ldots w_n)^{-1} \tag{11}$$

Example of Observer Design

Consider one dimensional heat conduction in a bar insulated at both ends, governed by the equation

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial r^2} \qquad r\epsilon(0,\ 1)$$

$$\frac{\partial u}{\partial r}(r,\ t) = 0 \quad r = 0,\ 1 \tag{12}$$

where c is the thermal diffusivity of the bar and $u(r,\ t)$ is the temperature at the location r and time t. It is assumed here that two temperature sensors are located on the bar, one at each end. Using the two measurements provided by the sensors, we need to estimate the temperature distribution in the bar. It is also assumed that the initial temperature distribution in the bar may be unknown.

A third order lumped parameter approximation of the distributed parameter system is developed using the modal expansion method. This lumped parameter model is described in a normalized form by

$$\dot{x} = Ax = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -c'\pi^2 & 0 \\ 0 & 0 & -4c'\pi^2 \end{bmatrix} x$$

$$y = Cx = \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} \\ 1 & -\sqrt{2} & \sqrt{2} \end{bmatrix} x \tag{13}$$

The elements of x are the normalized weighting factors on the responses of the corresponding modes. c' is a normalized version of c. It is assumed that the actual value of c' is 0.11, while for observer design, a value of 0.09 is assumed, indicating about 18 percent error. The elements of the C matrix depend only on the boundary conditions and the form of the partial differential Eq. (12), and are assumed to be accurately known. The eigenvalues of the A matrix are 0, -1.086 and -4.343 rad/s. The observer eigenvalues are chosen to be -3.5, -15 and -30 rad/s.

Two observers are designed using the procedure described above. Design 1 has the observer gain matrix

$$L_1 = \begin{bmatrix} 14.7760 & 14.7760 \\ 4.9893 & -4.9893 \\ 0.1396 & 0.1396 \end{bmatrix} \tag{14}$$

and yields a condition number of the modal matrix of F_c, after equilibration, of 3.43. In design 2, the reciprocal eigenvectors are chosen to get a poorer condition number of the modal matrix of F_c, equal to 31.44. The observer gain matrix for this design is given by

$$L_2 = \begin{bmatrix} 63.3193 & 63.3193 \\ 0.9234 & -0.9234 \\ -30.1205 & -30.1205 \end{bmatrix} \tag{15}$$

It should be noted here, as an indication of the restricted nature of the results of Furuta et al. (1976), that zero-sensitivity reduced order observers of the type discussed in the cited reference do not exit for this simple example.

Figures 1–3 show the simulated time histories of the norm $|e(t)|$ of the state estimation error for the two observer designs. The initial system state x_0 is assumed to be $(1.0\ 1.0\ 1.0)^T$. The error in the initial system state estimate is set to be (0.1 0.1

$0.1)^T$. Figure 1 represents the case where the effects of both the error in the estimated initial state and the observer model error are included. In Fig. 2, the observer model error is set to zero, the parameter c' in the system Eqs. (13) also being set to 0.09. Thus, only the effects of initial condition mismatch on state estimation error are included here. In Fig. 3, the system initial condition is assumed to be known exactly, the figure representing only the effects of the observer model error indicated before. In all the three cases indicated, the observer design 1 gives lower estimation error than the observer design 2, in qualitative agreement with the relative magnitudes of the condition numbers of the modal matrices corresponding to the two F_c matrices. Similar results were obtained for a variety of initial state estimation errors and assumed model errors. The usefulness of the observer design guideline proposed in this paper is thus clearly illustrated.

There is no guarantee, however, that the norm of the state estimation error will always be lower if the observer is designed as indicated here. In fact, if the initial state estimation error vector is dominated by one component, or if the errors in some of the parameters of the A and B matrices are dominant over the others, the relationship between the state estimation error norms may not be the same as the relationship between the error bounds indicated by Eq. (7). If the structure of the initial estimation error is known, such a priori information can be used to guide the observer eigenstructure assignment effectively to reduce the resulting state estimation error size (Andry et al., 1984). Similarly, if a priori information is available regarding model errors, it can be used to modify the eigenstructure assignment procedure appropriately. The results of the present paper are therefore primarily applicable to cases where such a priori information is not available. It is also expected that normalizing of the system state space equations such that the normalized outputs and state variables have maximum magnitudes of nearly unity prior to the observer design, enhances the utility of the results presented here.

Conclusions

In this paper, we have derived an expression for an upper bound on the norm of the estimation error for an observer, in the presence of errors in the system A and B matrices and in the estimated initial conditions. It is shown that, in designing observers for multi-output systems using eigenstructure assignment, if the eigenvectors of the F_c matrix are chosen to be as nearly mutually orthogonal as possible, a smaller bound on the state estimation error is obtained and thus may lead to more accurate state estimation. This is demonstrated by means of an example. The approach presented seems most appropriate in the absence of any a priori information on the initial state or the nature of the modeling errors.

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Neural Networks and Identification of Systems With Unobserved States

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Consider a nonlinear control system, whose structure is not known (apart from the order of the system) and whose states are not observed. We observe the output of the system for a period of time using persistently exciting input, and use the observation to train a neural network emulator whose output approximates that of the original system. We point out that such an explicit dynamical relationship between the input and the output is useful for the purpose of construction of output feedback controller for nonlinear control systems. Specialization of the method to linear systems allows swift convergence and parameter identification in some cases.

1 Introduction

This paper is concerned with the problem of identifying the input-output relationship of an unknown nonlinear dynamical system. Classical adaptive control of deterministic linear systems whose state variables are not all observed makes use of the separation principle (Narendra and Annaswamy, 1989) which says, in effect, that the problems of constructing an observer and parameter estimator can be considered separately. When the system is not observable it is not possible to construct an observer to recover the full state. Furthermore, when the system is nonlinear the separation principle no longer applies, and hence conventional adaptive identification and control techniques offer little hope of effective control of partially observed nonlinear systems. In this paper we show that these difficulties can be avoided by using neural networks instead.

Neural networks are already successfully applied in control theory and system identification. In a recent paper, Narandra and Parthasarathy (1990) formalized a unified approach to solving nonlinear identification and control problems using multilayered neural networks. Chen (1990) applied multilayer neural network to nonlinear self-tuning tracking problems. Chu et al. (1990) implemented a Hopfield network on identifying time-varying linear systems. Various learning architectures for training neural net controller are outlined in Psaltis et al. (1988) and some interesting applications of neural networks in adaptive control can be found in Goldenthal and

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