

Multi Target Optimization of Turbojet Engine with Multi Target Genetic Algorithm

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Abstract: In this paper, Turbojet engine will be optimized in ideal condition by multi target genetic algorithm. Target functions are specific thrust (ST), specific fuel consumption (SFC) and thermal efficiency (η_t) that will be optimized simultaneously according to design variables and in two by two way and their Pareto points will be showed. Design variables included inlet Mach number and total compressor pressure ratio. Then according to Pareto points important relations between target functions will be introduced. It is obvious that these relations without using these methods are inaccessible.

Key words: Genetic Algorithm, Pareto, Multi target optimization, Crossover, Mutation, Turbojet engine.

INTRODUCTION

As a matter of fact optimization procedure is defined as a way to find numerical collection for design vector variables. These are various numerical methods included Gradient methods to find optimum points. However some basic problems such as their great dependency to first assumptions can move the problem toward local optimization than absolute optimization. On the other hand in non continues or non derivative functions by using gradient methods seems improbable so other optimization methods especially genetic algorithm can solve this problem (Goldberg, D.E., 1989; Back, T.D.B., Z. Fogel, 1997).

These evolutionary algorithms are inspired by nature and their main deference with old ones is that in these methods, we do optimization by function not by their gradients. And they will help us to escape from local optimization. In multi target optimization, several targets will be optimized simultaneously. These targets may be in disagreement with each other, thus optimum of a target may deteriorate that of another target. Pareto was an Italian economist that revealed the context of multi target optimization (Pareto, V., 1896). Pareto points do not have any superiority toward each other but comparing to other points, they are superior in research. NSGA method was suggested by Deb (1994) and SPEA method was introduced by Zitzler and Thiele (Fonseca, C.M., 1993).

In this paper, design variables such as inlet Mach number and total compressor pressure ratio are considered.

Selective multi target in ideal subsonic turbo jet included specific thrust, specific fuel consumption and thermal efficiency and with considering design variables will be optimized two by two. The results will be revealed by Pareto curves. In this paper, our goal is decreasing fuel consumption and increasing thrust and thermal efficiency. Can we find a design vector that is minimum in fuel consumption and maximum in thrust and thermal efficiency?

2. Turbo Jet Thermodynamic Model:

Operating fuel in turbo jet engine is air which by changing in kinetic energy in inlet comparing with outlet can create thrust.

Ideal turbojet engine equations are shown in table A (Jack D. Mattingly, 1996). Inlet parameters in this cycle included flight Mach number (M_0), inlet air temperature (T_0 , K), temperature coefficient(γ), heating value (h_{pr} , kj.kg⁻¹), burner exit total temperature (T_{t4} ,K), total compressor pressure ratio (π_c).

Outlet parameters involves specific thrust (ST, N.kg⁻¹.S⁻¹), fuel/air ratio (f), thrust specific fuel consumption(TSFC, kg.S⁻¹.N⁻¹) and thermal efficiency (η_t).

In this paper $h_{pr}=48000$ kj.kg⁻¹, $\gamma=1.4$, $T_{t4}=1666$ K, $T_0=216.6$ K. Flight Mach number $0 < M_0 \leq 1$ and total compressor pressure ratio $1 \leq \pi_c \leq 40$ are considered as design variables (Jack D. Mattingly, 1996).

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3. Multi Target Genetic Algorithm:

3.1. Multi Target Optimization:

In multi target optimization problems, we are looking for vector design $X^*=[x^*_1, x^*_2, \dots, x^*_n]^T$ which is member of R_n that target functions are

$$F=[f_1(X), f_2(X), f_3(X)]^T \tag{1}$$

Member of R_k according to m number condition

$$g_i(X) \leq 0, i=1,2, \dots, m \tag{2}$$

And p number of equal condition

$$h_j(X)=0, j=1,2, \dots, p \tag{3}$$

Will optimize (Pareto, 1896; Oseyezka, A., 1985).

3.2. Defining Predominant Pareto:

The vector $U=[u_1, u_2, \dots, u_k] \in R_k$ is predominant to vector V if and if

$$\forall i \in \{1,2, \dots, k\}, u_i \leq v_i \wedge \exists j \in \{1,2, \dots, k\} : u_j < v_j \tag{4}$$

3.3. Defining optimum Pareto:

A point like $X^* \in \Omega$ (Ω is an accepted design region which satisfy 2, 3 equations) is called optimum Pareto if and only if $F(X^*) < F(X)$ or on the other hand

$$\begin{aligned} \forall i \in \{1,2, \dots, k\}, \forall X \in \Omega - \{X^*\} \quad f_i(X^*) \leq f_i(X) \\ \wedge \exists j \in \{1,2, \dots, k\} : f_j(X^*) < f_j(X) \end{aligned} \tag{5}$$

3.4. Defining Pareto Collection:

In multi target optimization problems, a Pareto collection (Θ^*) included all design vectors of optimum Pareto. On the other hand

$$\Theta^* = \{X \in \Omega \mid \exists X' \in \Omega : F(X') < F(X)\} \tag{6}$$

3.5. Defining Pareto Front:

Vectors including target functions which are made from vectors of Pareto collections (Θ^*) are called Pareto Front.

The results of multi target optimization have no superiority toward each other and are called non superior results.

In figure (1) for example can see the Pareto points, in this figure by moving from A to B (or vice versus), any improvement in condition of any target functions can deteriorate the condition of at least one target function of problem, (the goal is to minimize or maximize both target functions).

Pareto optimum points almost are located in boundary lines of design region or are over lapped points of target functions. In figure (1) the bold line shows such boundary line of two target functions which its component points are called Pareto Front.

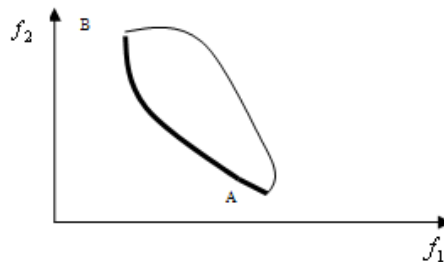


Fig. 1: Pareto points in a curve form.

3.6. Multi Target Genetic Algorithm (NSGAI):

First stage of this method, N is the primary population which is generated randomly (R_t). Then these populations will be categorized and vectors which satisfy the condition of equation (4) will be categorized in lower levels, then among these populations, some populations will be selected randomly for crossover and mutation. The population which is created by crossover and mutation (Q_t) will be added to primary population and again the total population will be classified.

The base of NSGAI is that while reproduction is continuing, the number of population must be constant, so the population in higher levels will be eliminated.

The end of a reproduction is an improved population. Diagram (a) shows a reproduction in NSGAI.

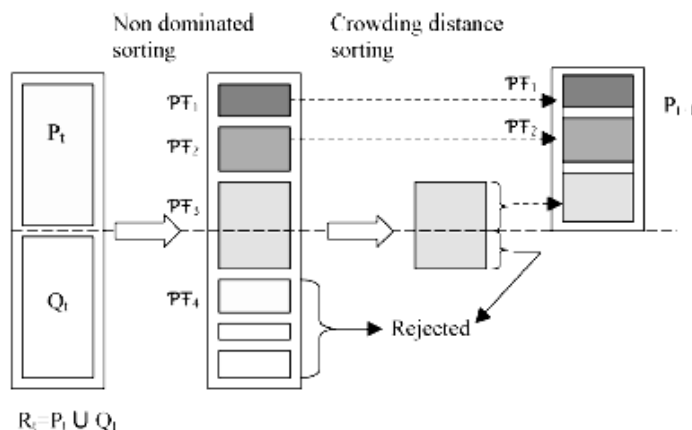


Fig. a: Reproduction procedures.

4. Multi Target Optimization of Turbo Jet Engine With Multi Target Genetic Algorithm Two by Two Way:

4.1. Optimization According to Thermal Efficiency And Specific Fuel Consumption Target Functions (η_t , SFC):

Figure (2) is the collection of points resulting from optimization according to two target functions SFC and η_t which is generated by NSGAI. In this figure five optional design vectors are shown.

By comparing design vectors 2 and 3, it can be proved that by increasing 94% in SFC, thermal efficiency will be increased only 13.6%. Since our target is decreasing SFC and increasing η_t , so it obvious that design vector 2 comparing with 3 is more important in design. Also by comparing design vectors 2 and 4, we can show that by increasing 16.1% in SFC, thermal efficiency will be increased 29.8%.

Why multi target optimization is superior to single target optimization?

In optimization which was according to η_t and SFC, we see that increasing thermal efficiency can lead to increasing fuel consumption, since the target is increasing η_t and decreasing SFC, if we do single target optimization, the results will be points one and five. While the point one has the minimum SFC and thermal efficiency and point five has the maximum SFC and thermal efficiency. It should be mentioned that point one is according to single target optimization of SFC and point five is according to single target optimization of η_t .

4.2. Optimization According to Specific Thrust and Thermal Efficiency Target Functions (ST, η_t):

Figure (3) shows the Pareto points according to ST and η_t . With comparing the design vectors one and two, we can see that with increasing 21.6% in specific thrust, thermal efficiency will increase only 5.68%. Also by comparing design vectors two and three, we consider that with increasing 11.9% in specific thrust, thermal efficiency will decrease 36.2%. So in design viewpoint, point two is more valuable than point three. Design vector one amount all design vectors has the least ST and the most η_t , while design vector seven has the most ST and the least η_t . The design vector one is made in single target optimization η_t and design vector seven is made in single target optimization ST.

4.3. Optimization According to Specific Thrust And Specific Fuel Consumption Target Functions (ST,SFC):

Our goal is finding design vector with the least SFC and the most ST. Figure (4) shows the Pareto points according these two target functions that obtained from multi genetic algorithm. Considering this figure, we understand that increasing ST will happen with increasing SFC.

On the other hand optimization a target function can cause to deteriorate another one. (Decreasing SFC can cause decreasing ST). With comparing design vectors two and three, we find that 0.1% increasing in ST, SFC will increase 0.93% and it means that points two and three almost have a same value in design.

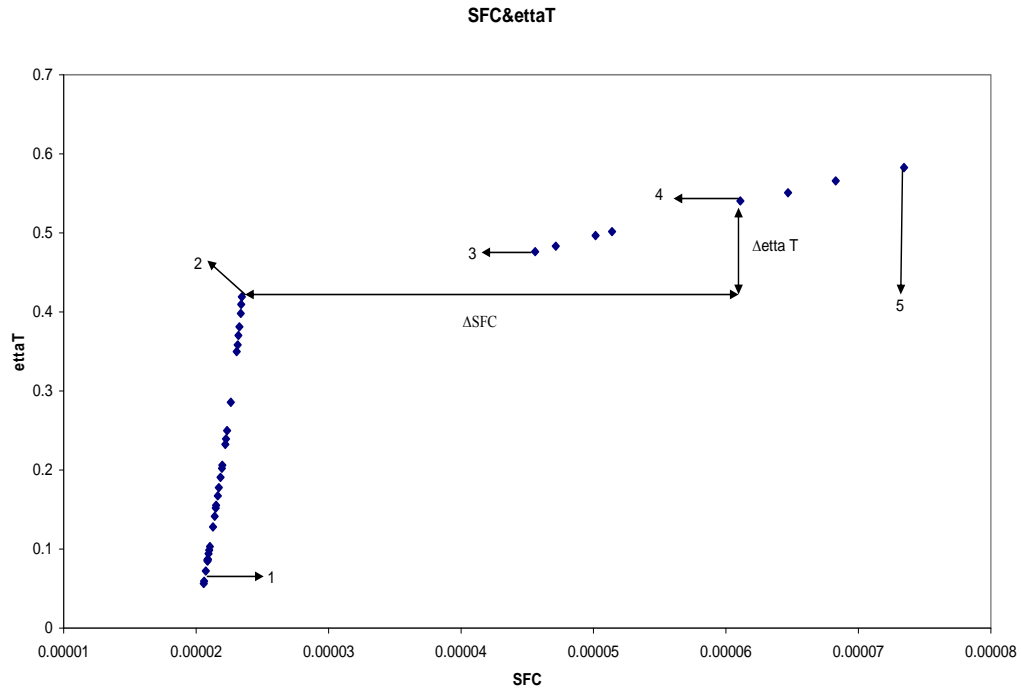


Fig. 2: Pareto points of thermal efficiency and fuel consumption.

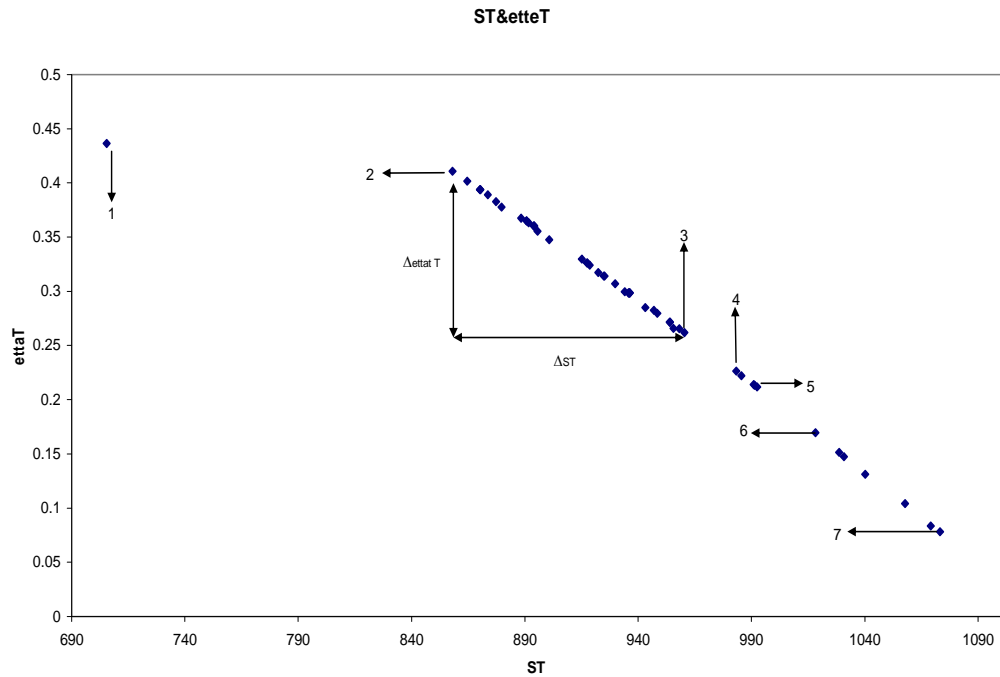


Fig. 3: Pareto points of thrust and thermal efficiency.

5. Optimization According to Three Target Functions (ST, SFC, η):

Pareto points are shown two by two way. It's obvious that these design vectors which are product of optimization are representative of 3D curve that are showed two by two way. Similar points to one number are shown in figures (5), (6), (7).

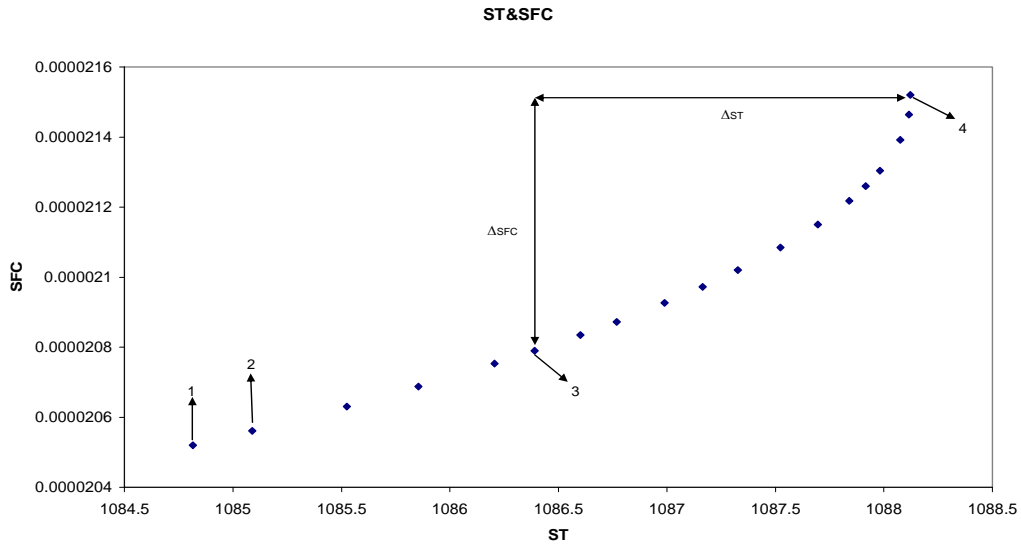


Fig. 4: Pareto points of thrust and fuel consumption.

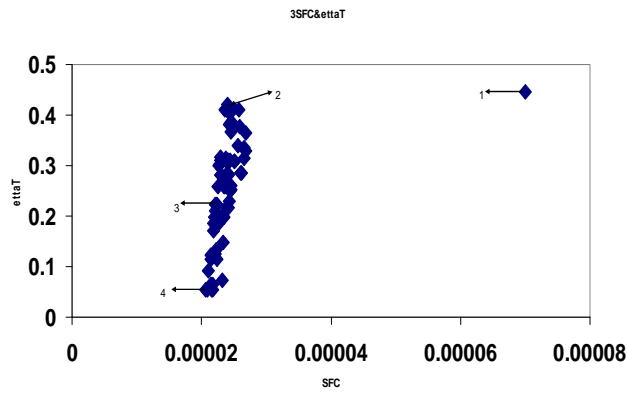


Fig. 5: Pareto points of thermal efficiency and fuel consumption according to three target functions.

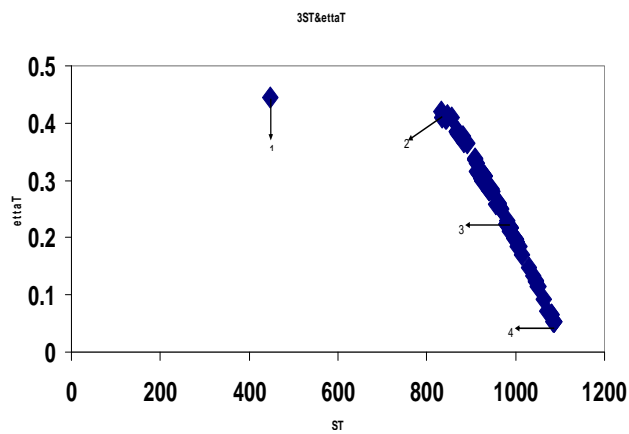


Fig. 6: Pareto points of thermal efficiency and thrust according to three target functions.

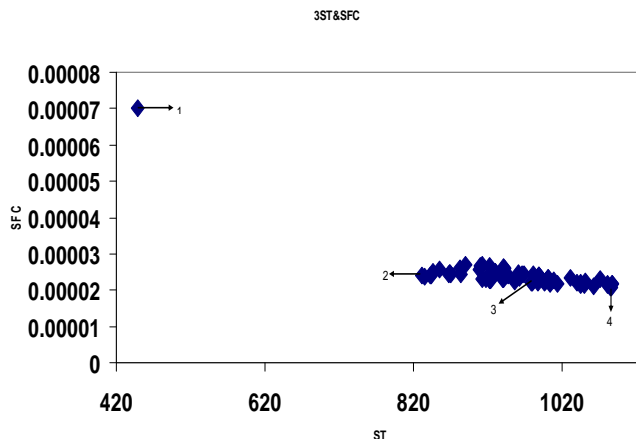


Fig. 7: Pareto points of thrust and fuel consumption according to three target functions.

Comparing two and three design vectors, it can be inferred that by 18% increasing ST, SFC will decreased to 8% but thermal efficiency will fall to 47%.

By comparing one and two design vectors, we conclude that increasing 85% in ST will be accompanied with loss of 5.6% in thermal efficiency and fall of 66% in SFC. So it seems that design vector two is considerably more important than design vector one.

Conclusions:

Several functions may optimize simultaneously in optimization. Now if we consider optimization problem as a single target, we can not see the results of the other functions in optimum vector. So the optimum point of a target function may be the weak point of others.

In high pressure coefficient $30 < \pi_c \leq 40$ and $0 < M_0 \leq 0.98$, specific thrust and fuel consumption will be improved but in this range, we can't expect that thermal efficiency be more than 42%.

Around $M_0=1$ and in low pressure coefficient ($1 \leq \pi_c < 2.1$), we can reach to the highest thermal efficiency (60%), however in this range fuel consumption will be in its worst condition ($4.55897 \leq SFC \times 10^5 \leq 7.3457$).

It obtained from Pareto points that fuel consumption is effected by pressure coefficient than Mach number (Table 1).

According to Pareto figures which are shown in this paper, we can conclude that SFC is almost proportional with square of ST ($SFC \propto ST^2$) and thermal efficiency is proportional with ST ($\eta_t \propto ST$). Other important results are summarized two by two in table (1).

Table 1:

η_t, ST	
$0 < \eta_t < 0.44$	
$705 \leq ST \leq 1074$	
M_0	$0.1 < M_0 \leq 1$
π_c	$2.5 < \pi_c < 34$
SFC, ST	
$2.05 \leq SFC \times 10^5 < 2.2$	
$1084 < ST < 1089$	
M_0	$M_0 = 0.1$
π_c	$30 < \pi_c \leq 40$
η_t, SFC	
$0 < \eta_t < 0.42$	$0.47 < \eta_t < 0.6$
$2.06 < SFC \times 10^5 < 2.34$	$4.56 \leq SFC \times 10^5 \leq 7.34$
Flight Mach number (M_0)	$0 < M_0 \leq 0.98$
Pressure ratio (π_c)	$\pi_c = 40$
	$M_0 \cong 1$
	$1 \leq \pi_c < 2.1$

Finding results included specific boundaries for target functions and design variables. For example if optimization is according to η_t , SFC: if $0 < \eta_t < 0.42$, fuel consumption must be to [2.056e-5, 2.344e-5] and flight Mach number must be [0, 0.98] and total compressor pressure ratio must be 40.

Index

a_0	Velocity of sound at inlet.....m/s
\dot{m}_0	Mass flow rate.....kg/s
T_{t4}	Burner exit total temperature.....K
T_0	Inlet temperature.....K
h_{pr}	Heating valuekJ/kg
V_0	Air velocity at inlet.....m/s
g_c	Newton's constants.....kg-m.
R	Gas constants.....J/kg.K
F	Thrust.....N
M_0	Flight Mach number
τ_r	Total static temperature ratio at inlet
τ_t	Burner exit/inlet total temperature ratio
τ_λ	Burner exit total enthalpy/inlet total enthalpy
τ_c	Compressor exit total temperature/Compressor inlet temperature
f	Fuel/air ratio
ST	Specific thrust
SFC	Specific fuel consumption
η_t	Thermal efficiency
X^*	Vector of optimal design variables
π_c	Total compressor pressure ratio
F(X)	Vector of objective function

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Table (A):

Turbojet equations (Oseyezka, A., 1985).

$$R = \frac{\gamma - 1}{\gamma} C_p$$

$$a_0 = \sqrt{\gamma R g_c T_0}$$

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2$$

$$\tau_\lambda = \frac{T_{t4}}{T_0}$$

$$\tau_c = (\pi_c)^{(\gamma-1)/\gamma}$$

$$\tau_t = 1 - \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$$

$$\frac{V_9}{a_0} = \sqrt{\frac{2}{\gamma - 1} \frac{\tau_\lambda}{\tau_r \tau_c} (\tau_r \tau_c \tau_t - 1)}$$

$$ST = \frac{F}{\dot{m}_0} = \frac{a_0}{g_c} \left(\frac{V_9}{a_0} - M_0 \right)$$

$$f = \frac{C_p T_0}{h_{pr}} (\tau_\lambda - \tau_r \tau_c)$$

$$SFC = \frac{f}{ST}$$

$$\eta_t = 1 - \frac{1}{\tau_r \tau_c}$$