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FATIGUE DAMAGE EVALUATION OF TURBINE GENERATOR DUE TO MULTI-MODE SUBSYNCHRONOUS OSCILLATION

Chao Liu
State Key Laboratory of Control and Simulation of Power System and Generation Equipments,
Department of Thermal Engineering,
Tsinghua University, Beijing, 100084, China
Email: chao-liu08@mails.tsinghua.edu.cn

Dongxiang Jiang*
State Key Laboratory of Control and Simulation of Power System and Generation Equipments,
Department of Thermal Engineering,
Tsinghua University, Beijing, 100084, China
Email: jiangdx@tsinghua.edu.cn

ABSTRACT

In recent years, Subsynchronous Resonance (SSR) and Subsynchronous Oscillation (SSO) are increasingly attracting more and more researchers' interests in China. The network is encountering great changes and large-scale networks are increasingly implemented for long distance power transmission as well as various kinds of power electronic devices. Several SSO phenomena were monitored in a fossil-fired power plant in China in 2008. They were determined as complex factors' co-activation between the network and the turbine generator. Multi-mode torsional vibration is one significant feature of torsional vibration caused by SSO. The paper simulates the multi-mode SSO based on the practical situation in China. The torsional vibration is studied to analyze the torsional vibration features under multi-mode SSO and the differences caused by different peak values and phases of electromagnetic torques. Based on some type of 600MW steam turbine generator, the fatigue damage of the shafts is studied.

NOMENCLATURE

SSR Subsynchronous Resonance
SSO Subsynchronous Oscillation
TCSC Thyristor Controlled Series Capacitors
HVDC High Voltage Direct Current Transmission
HCF High Cycle Fatigue
FEM Finite Element Method
ZoV Zone of Vulnerability
CMM Continuous mass model
LP-GEN low pressure cylinder and generator

INTRODUCTION

The interaction between the turbine generator and network was widely studied in 1970s to 1980s due to several severe accidents caused by network disturbance or grid faults [1-3]. SSR and SSO are two main kinds of interactions while SSO including more disturbances sources and more devices in the network. The torsional vibration problems caused by SSR or SSO can play significant impact on the turbine generator shafts which may induce the fatigue damage of the shafts and other attached components. So the fatigue problems are also studied in that period with practical cases [4-6]. After that, severe accidents caused by network disturbances are decreased and kinds of mitigation devices to the coupled interactions are proposed which protect the network effectively as well as the turbine generator. However, cracks emerged again in several plants around the world recent years that the causes are determined to be the high cycle fatigue (HCF) [7-9]. These cracks are monitored and detected in the early stage and don't induce severe accidents as the ones in 1970s, yet the loss of the units is large and the measures to prevent such accidents are needed to be further analyzed.

*Professor and author of correspondence, Phone: (86)10-62794046, Fax: (86)10-62770209

In China, the electric power industry has encountered great developments. The unit capacity of the plant is increasing quickly where the network access may be a long distance from the units and kinds of devices are used to improve the transmission capacity like series capacitor, Thyristor Controlled Series Capacitors (TCSC), High Voltage Direct Current Transmission (HVDC) and so on. These circumstances containing large scale network, high parameter unit and kinds of devices increase the risk of the interaction between the turbine generator and network. The SSR and SSO problems are analyzed in China in several units and networks where the risks of the interaction are validated. However, the researchers pay more attention to the network stability where the safety of units is less considered [9-12].

Cracks emerged in a power plant in 2008 and the possible causes of the accident are analyzed in [13]. Further on, the authors analyzed more phenomena in torsional vibration due to network disturbances and multi-mode SSO is a significant one whose possibilities are analyzed in [12]. The paper simulates the multi-mode SSO based on practical cases, and then the torsional vibrations under multi-mode SSO is studied using the complex shafts model which is based on the continuous mass model. The torques response and peak torques are acquired using transient response calculation. Finally, the fatigue damage is evaluated based on the shafts structure and material's property.

SIMULATION OF MULTI-MODE SSO

In order to simulate the SSO phenomenon emerged between the generator and network, time domain simulation method is usually applied while the method needs detailed network parameters. In [12], multi-mode torsional vibration is acquired which is a typical multi-mode SSR. However, SSO often contains more widely oscillation frequency spectrum range. It is analyzed in the paper. To acquire more detailed information of torsional vibration of the turbine generator, the network is simplified and the electromagnetic torques are simulated.

The torques needs to be determined by the natural torsional frequency of turbine generator shafts for the purpose of evaluating the SSO impacts. In the paper, the turbine generator is from some 600MW steam turbine generator and the characteristics of the shafts are analyzed in [13] which is using Riccati Transfer matrix method. Finite Element Method (FEM) is used in this paper to get the transient response solution. The detailed method of modeling was analyzed in [14] and the low-order natural torsional vibration frequencies are listed in Table 1.

Here, the simulated oscillation frequencies are a little larger than the natural frequency to avoid resonance frequency which will induce significantly sharp increase at the response peak torques. The peak torques and frequencies are ascertained which are listed in Table 2.

The attenuation in amplitude depends on the mitigation effect of the network devices and the damping characteristics of the turbine generator. Usually, the damping coefficient of the torsional vibration is small to the turbine generator as the damping consumes the kinetic energy which is adverse for unit efficiency. The networks mitigation devices are the main mitigation sources. While the mitigation mechanisms of the devices are complex, the paper chooses the conclusion in [12] to simulate the mitigation effect. The simulated torques are listed in Fig. 1. The spectrum of torques varying with time is shown in Fig. 2.

TABLE 1: THE NATURAL TORSIONAL VIBRATION FREQUENCIES OF 600MW TURBINE GENERATOR

Mode	Mode1	Mode2	Mode3
Frequency (Hz)	12.525	21.306	25.876

TABLE 2: THE CHARACTERISTICS OF SIMULATED ELECTROMAGNETIC TORQUES

Item	C_1	C_2	C_3
Peak torque (p.u.)	0.02	0.05	0.02
Frequency (Hz)	12.625	21.406	25.976

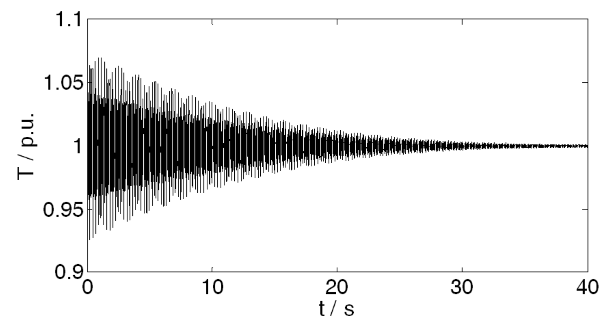


FIGURE 1: TIME HISTORY OF SIMULATED ELECTROMAGNETIC TORQUES ACTING ON GENERATOR

Besides the peak torques and the frequencies, phase of the different components is another possible influence factor. The phase differences between the first component and the second component is defined as φ_1. The phase differences between the first component and the third component is defined as φ_2. Five situations are analyzed in the paper containing different conditions of peak torques and phases. They are listed in Table 3.

Using the simulated results, the transient response is to be ascertained to acquire the peak torques at the Zone of Vulnerability (ZoV).

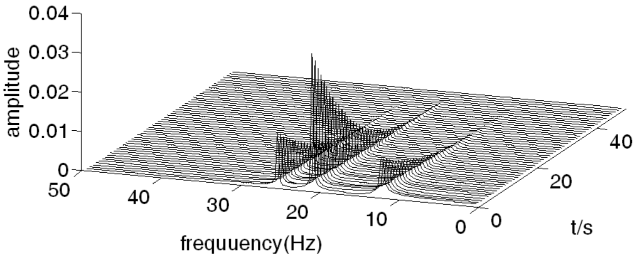


FIGURE 2: TIME HISTORY OF SIMULATED ELECTROMAGNETIC TORQUES ACTING ON GENERATOR

TABLE 3: FIVE SITUATIONS OF MULTI-MODE SSO

Condition	Peak torque (p.u.)	Phase (rad)
1	C_1=0.02, C_2=0.02, C_3=0.02	φ_1=0, φ_2=0
2	C_1=0.01, C_2=0.02, C_3=0.02	φ_1=0, φ_2=0
3	C_1=0.01, C_2=0.02, C_3=0.01	φ_1=0, φ_2=0
4	C_1=0.02, C_2=0.02, C_3=0.02	φ_1=π/3, φ_2=π/3
5	C_1=0.02, C_2=0.02, C_3=0.02	φ_1=π/3, φ_2=2π/3

TORSIONAL VIBRATION RESPONSE OF TURBINE GENERATOR UNDER MULTI-MODE SSO

The response under multi-mode SSO is usually acquired by the network and the lumped mass model of the turbine generator where more details are considered in network than the mechanical subsystem. The accuracy meets the network's standard, but it's not detailed enough for torsional vibration of the turbine generator or fatigue evaluation, especially for the high parameter units containing long and thin rotors as well as complex local structures. Continuous mass model (CMM) is studied in several researches which can improve the accuracy of the calculation of mechanical subsystem. The contradiction lies in the coupled solution of both network and mechanical subsystem. Another aspect is that the transient response of the CMM needs high computational scale which is rarely adopted in the research [15].

Finite Element Method (FEM)

The paper analyzed the CMM based on FEM. Firstly, the dynamic equations of continuous entity are analyzed. The basic equations of three dimensional elastodynamics are [16]:

Equilibrium equation:

$$\sigma_{ij,j} + f_i = \rho u_{i,u} + \mu u_{i,t} \tag{1}$$

Deformation compatibility equation:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

Constitutive equation:

$$\sigma_{ij} = D_{ijkl}\varepsilon_{kl} \tag{3}$$

Boundary conditions:

$$u_i = \overline{u_i} \tag{4}$$

$$\sigma_{ij} n_j = \overline{T_i} \tag{5}$$

Using FEM, the continuous entity are discretized by nodes and elements. The equivalent inertia forces and damping forces can be acquired inside the elements:

$$\mathbf{P_T(t)^e} = -\int\int\int \mathbf{N^T}\rho\mathbf{N}dxdydz\ddot{\mathbf{u}}^e = -\mathbf{M}^e\ddot{u}^e \tag{6}$$

$$\mathbf{P_c(t)^e} = -\int\int\int \mathbf{N^T}\mu\mathbf{N}dxdydz\dot{\mathbf{u}}^e = -\mathbf{C}^e\dot{u}^e \tag{7}$$

Where, N is the shape function matrix,M^e is the mass matrix of element, C^e is the damping matrix of element.

Then, the equations of dynamics based on FEM are established according to the equilibrium equation and deformation compatibility equation:

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = \mathbf{P}_f(t) \tag{8}$$

Where, M is the mass matrix, C is the damping matrix, K is the stiffness matrix, P is the excitation force matrix.

Transient Response Using Continuous Mass Model

The finite element equations are established in the previous part and the solution procedures of transient response are to be analyzed in this part. Here, the Newmark method is used to analyze the solution procedures containing two steps [16]:

Step 1: Initial value calculation

(1) Establish the initial matrices of Stiffness, mass and damping;

(2) Determine the initial values of displacement, velocity and acceleration;

(3) Choose the time step Δt and parameters α and δ , calculate the integral constant:

$$\begin{aligned} c_0 = \frac{1}{\alpha \Delta t^2}, \qquad c_1 = \frac{\delta}{\alpha \Delta t} \\ c_2 = \frac{1}{\alpha \Delta t}, \quad c_3 = \frac{1}{2\alpha - 1} \\ c_4 = \frac{\delta}{\alpha} - 1, \quad c_5 = \frac{\Delta t}{2}(\frac{\delta}{\alpha} - 1) \\ c_6 = \Delta t(1-\delta), \quad c_7 = \delta \Delta t, \end{aligned} \tag{9}$$

(4) Form the effective stiffness matrix $\hat{K}$:

$$\hat{\mathbf{K}} = \mathbf{K} + c_0\mathbf{M} + c_1\mathbf{C} \tag{10}$$

(5) Triangular decomposition $\hat{K}$:

$$\hat{\mathbf{K}} = \mathbf{KDL^T} \tag{11}$$

Step 2: Iterative calculation

(1) Calculation of the effective load at $t+\Delta t$:

$$\begin{aligned} \hat{\mathbf{Q}}_{t+\Delta t} = \mathbf{Q}_{t+\Delta t} + \mathbf{M}(c_0\mathbf{q_t} + c_2\dot{\mathbf{q}}_\mathbf{t}) + c_3\ddot{\mathbf{q}}_\mathbf{t}) \\ +\mathbf{C}(c_1\mathbf{q_t} + c_4\dot{\mathbf{q}}_\mathbf{t}) + c_5\ddot{\mathbf{q}}_\mathbf{t}) \end{aligned} \tag{12}$$

(2) Calculation of the effective displacement at $t+\Delta t$:

$$\mathbf{LDLq}_{t+\delta t} = \hat{\mathbf{Q}}_{t+\triangle t} \tag{13}$$

(3) Calculation of the velocity and acceleration at $t+\Delta t$:

$$\begin{aligned} \ddot{\mathbf{q}}_{t+\Delta t} = c_0(\mathbf{q}_{t+\Delta t} - \mathbf{q}_t) - c_2\dot{\mathbf{q}}_t - c_3\ddot{\mathbf{q}}_t \\ \dot{\mathbf{q}}_{t+\Delta t} = \dot{\mathbf{q}}_t + c_6\ddot{\mathbf{q}}_t + c_7\ddot{\mathbf{q}}_t \end{aligned} \tag{14}$$

The Newmark method is a widely adopted method in applications, another widely used method is Wilson-theta method which is illustrated in [13]. Based on the solution procedures, ANSYS is used in the paper to acquire the transient response under multi-mode SSO [17], where the model is established using CMM and characteristics of the model are same as the Table 1.

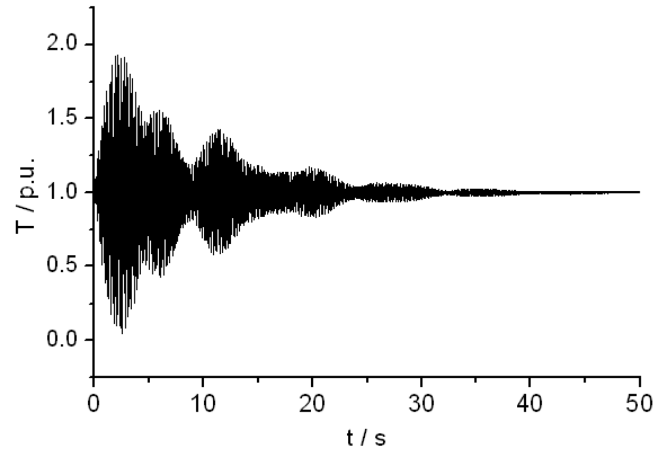


FIGURE 3: THE TORQUES RESPONSE UNDER MULTI-MODE SSO CONDITION 1

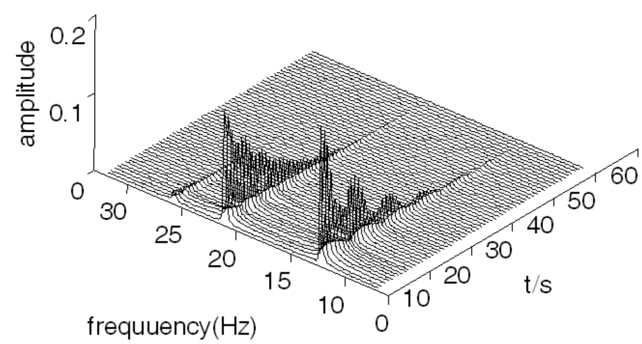


FIGURE 4: THE SPECTRUM OF LP-GEN TORQUES VARYING WITH TIME

The transient response under multi-mode SSO listed in condition 1 in Table 3 is shown in Fig. 3 which signifies the torques between the low pressure cylinder and generator (LP-GEN) by per-unit system. Here, the damping coefficient is selected as 0.069 1/s uniformly. The time varying spectrum of torques is acquired using FFT analysis which is shown in Fig. 4.

It is concluded that the response of multi-mode SSO has different characteristics of different components:

(1) The three components close to the natural torsional vibration frequencies are decreasing with time;

(2) The three components have different response amplitudes, although the excitation amplitude are the same;

(3) The first component emerges significant vibration atten-

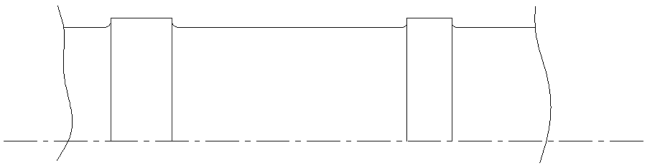


FIGURE 5: THE GEOMETRIC STRUCTURE OF THE JOURNAL IN ZoV

uation phenomenon while the second component is not so obvious.

The different characteristics may vary as the excitation conditions change which may have influences on the fatigue evaluation.

Zone of Vulnerability of Shafts System

As the transient response of multi-mode SSO may have different characteristics at different positions of shafts, determination of the dangerous point is critical to the fatigue evaluation for the whole shafts system.

According to the stress distribution rule of the circular shaft, the smallest cross section encounters the highest stress. For shafts of turbine generator, the journals of the shafts are the slender parts which are considered as the Zone of Vulnerability (ZoV). And then different journals are analyzed and the stresses are compared. Finally the end journal of the low pressure cylinder is ascertained as the dangerous part among all journals. The geometric structure of the journal is shown in Fig. 5.

Torques in ZoV under Multi-mode SSO

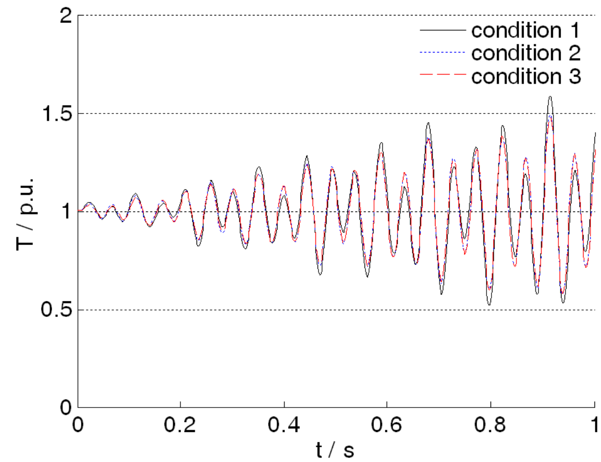
Torques between LP-GEN has been presented in the previous part where the torsional vibration is compared in condition 1 and characteristics of different components are analyzed. Here, torques at the dangerous part in ZoV are analyzed under different conditions of multi-mode SSO. The five conditions are listed in Table 3, using the transient response solution procedures, torques are extracted in ZoV where the decreasing trends are similar. Small differences lie in the amplitudes and phases in different conditions which are shown in Fig. 6.

In (a) and (b) of Fig. 6, three conditions have the same phases of excitation torques and different peak excitation torques. The responses of the three conditions also have the same phases while the amplitudes are different.

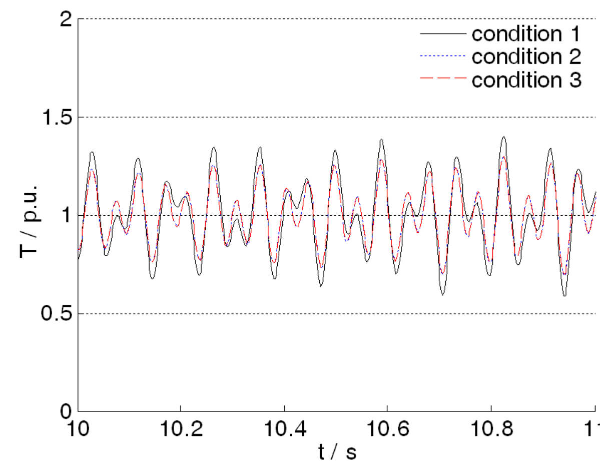
In (c) and (d) of Fig. 6, three conditions have the different phases of excitation torques and same peak excitation torques. The responses of the three conditions also have the different phases while the amplitudes are same. The third component close to the third natural torsional vibration frequency plays a significant impact on the response phase.

TABLE 4: FIVE SITUATIONS OF MULTI-MODE SSO

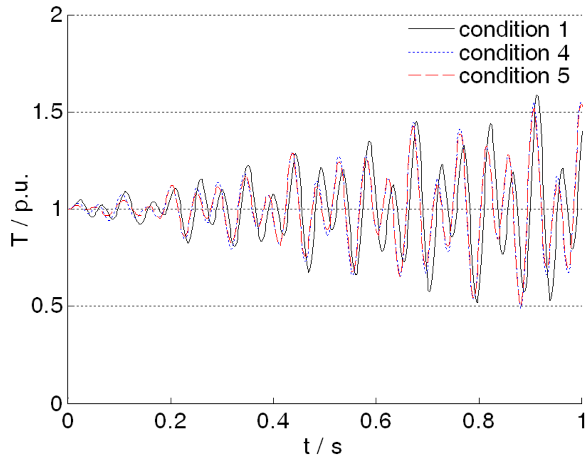
Condition	Maximal torque (p.u.)	Minimum torque (p.u.)
1	1.9259	0.0536
2	1.7471	0.2625
3	1.7296	0.2771
4	1.9215	0.0514
5	1.9030	0.0553



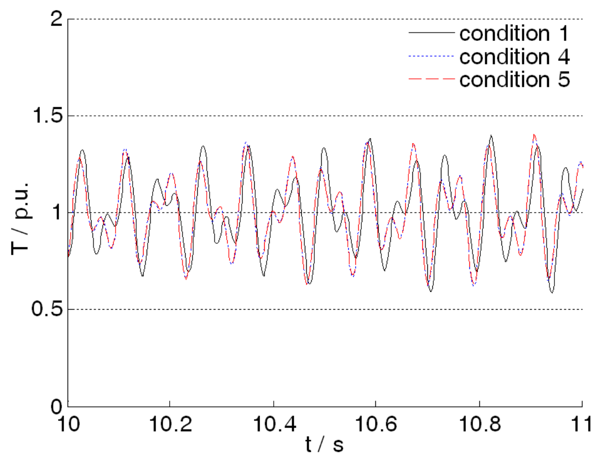
(a) The first three conditions at 0-1s



(b) The first three conditions at 10-11s



(c) Condition 1, 4 and 5 at 0-1s



(d) Condition 1, 4 and 5 at 10-11s

FIGURE 6: RESPONSES OF THE FIVE MULTI-MODE SSO CONDITIONS

It is still confused that whether these differences will affect the fatigue evaluation of them. The fatigue estimation is being analyzed in the next part to find out the impacts of different conditions and the leading factor of fatigue. Here, the peak torques under the five conditions are listed in Table 4 which will be used in the next part.

FATIGUE DAMAGE ACCUMULATION DUE TO MULTI-MODE SSO

To ascertain the fatigue damage of the dangerous part, several steps are needed to be analyzed: the stress distribution of the structure, the fatigue property of the material, the fatigue rule

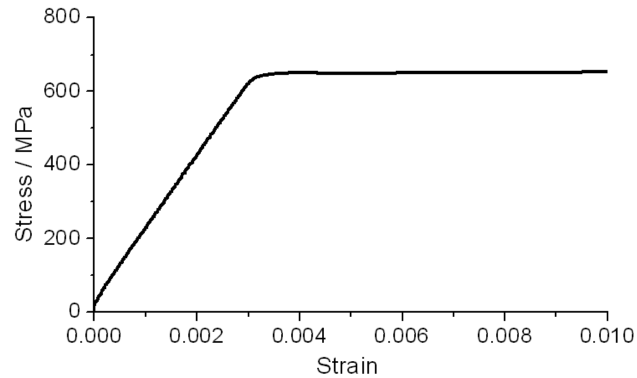


FIGURE 7: THE CONSTITUTIVE RELATIONSHIP OF THE ALLOY STEEL USED IN TURBINE GENERATOR SHAFTS

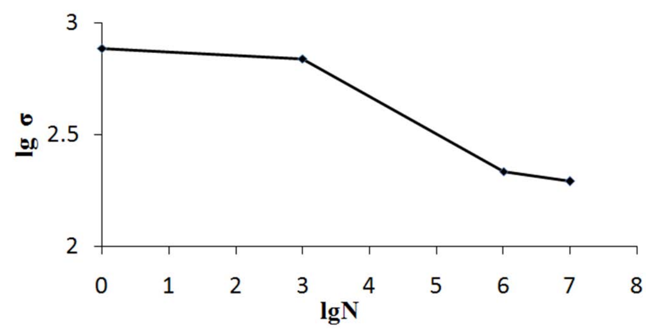


FIGURE 8: THE SIMPLIFIED S-N CURVE

for proper evaluation, the fatigue cumulation rule for continuous loads and so on. Many factors can influence the fatigue property of the fixed part with their own dispersion and uncertainty. In this section, the main aspects of fatigue are considered: material property, the fatigue rule, the counting method and the cumulation method for continuous loads.

Material Property

The material used in the turbine generator shafts is alloy steel containing Cr, Ni, Mo and V with good strength and toughness. The constitutive relationship is shown in Fig. 7.

According to the constitutive relationship, the stress distribution of the dangerous part in ZoV can be acquired under the local torques calculated in the previous part which can be used for fatigue evaluation.

Fatigue Cumulation Rule

Usually, the practical fatigue curve of the material used in the turbine generator is hard to acquire. Here, the simplified S-N curve is used in the analysis which is shown in Fig. 8. The S-N curve has relatively good accuracy for High Cycle Fatigue (HCF).

The maximal torque of the five condition listed in Table 4 is 1.9259 p.u. and the correspondingly maximal stress of the structure is below 460 MPa. The minimum torque of the five condition listed in Table 4 is 0.0536 p.u. and the correspondingly minimum stress of the structure is about 15 MPa. The fluctuation value of the stress is 445 MPa and the consequent number of cycles for fluctuated load fatigue is between 10^4-10^5. So the HCF rule is adaptive for the fatigue evaluation in the conditions listed in the paper. Furthermore, for most situations of SSO, the HCF rule is adaptive.

In fatigue evaluation of complex continuous loads circumstances, Rainfolw method is widely used for counting the number of cycles which is adopted in the paper.

Palmgren-Miner linear cumulative rule is used for the fatigue cumulation:

$$D = \sum \frac{n_i}{N_i} \tag{15}$$

Where, D is the fatigue damage of the whole load history, N_i is the number of cycles to failure in each counted cycle by Rainfolw rule which is signified by mean stress and stress amplitude, n_i is the number of cycles counted by Rainfolw method.

Gerber correction method is applied to consider the influences of the mean stresses:

$$\frac{\sigma_{ra}}{\sigma_{-1}} + (\frac{\sigma_{rm}}{\sigma_b})^2 = 1 \tag{16}$$

Where, σ_{ra} is the stress amplitude limit, σ_{-1} is the fatigue limit under symmetry cyclic load, σ_{rm} is the mean stress limit, σ_b is the strength limit.

Case Study

Using the procedures in the previous parts, the fatigue damage of the five multi-mode SSO conditions are acquired which are listed in Table 5.

From the fatigue cumulation results, the influence factors can be ascertained:

(1) The phases of the excitation torques don't influence the fatigue cumulation, although they influence the response torques in ZoV;

TABLE 5: FATIGUE CUMULATION RESULTS OF THE FIVE MULTI-MODE SSO CONDITIONS

Condition	Fatigue cumulation rule
1	0.0067
2	0.0061
3	0.0060
4	0.0067
5	0.0067

(2) The peak torques influence the fatigue cumulation while the second component has the most significant influence;

(3) Multi-mode SSO has combined effects on the fatigue cumulation while the torsional vibration of mode 2 is the most dangerous component.

Results

Based on the practical situation in some plant, the electromagnetic torques under multi-mode SSO are simulated to calculate the transient response of the shafts system. The simulated multi-mode SSO torques contains three components that they are close to the three natural torsional vibration frequencies, different peak torques and phases.

Using the simulation results, the transient responses of the shafts system are acquired under different conditions. The response shows that the attenuation processes of the three components are different: the first component has significant oscillation process and slower attenuation; the second component decreasing almost all the way; the third component has small amplitude of vibration. Then the ZoV is analyzed in the shafts system where the journal at the end of the low pressure cylinder is ascertained as the dangerous position.

Fatigue evaluation is analyzed in ZoV based on the transient response and the material property. HCF is ascertained for fatigue calculation rule, Rainfolw counting method is chosen as the counting and Palmgren-Miner linear cumulative rule is chosen as the fatigue cumulation rule. Then the fatigue cumulation results are given out and the dominant factor of multi-mode SSO is ascertained.

CONCLUDING REMARKS

The electric power industry is developing rapidly in China. Crack accidents happened in the power plants need close attention. The fatigue life loss of turbine generator caused by network disturbances needs more researches. The paper analyzes the fatigue damage due to multi-mode SSO. The torsional vibration, Zone of Vulnerability, stress and fatigue are analyzed in detail.

Although several aspects are considered in the fatigue calculation process, the fatigue of the specific structure is hard to evaluate accurately. The detailed fatigue curve, the more accurate cumulation rule, more backgrounds of the network and turbine generator and more practical experiences are needed to be further analyzed.

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