TAPE-TETHER DESIGN FOR DE-ORBITING FROM GIVEN ALTITUDE AND INCLINATION

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ABSTRACT

The product Π of the tether-to-satellite mass ratio and the probability of tether cuts by small debris must be small to make electrodynamic bare tethers a competitive and useful de-orbiting technology. In the case of a circular orbit and assuming a model for the debris population, the product Π can be written as a function that just depends on the initial orbit parameters (altitude and inclination) and the tether geometry. This formula, which does not contain the time explicitly and ignores the details of the tether dynamics during the de-orbiting, is used to find design rules for the tape dimensions and the orbit parameter ranges where tethers dominate other de-orbiting technologies like rockets, electrical propulsion, and sails.

Key words: space debris; electrodynamic tethers.

1. INTRODUCTION

Since a notable agreement about the importance of space debris remediation and mitigation exists, proactive actions by the space community are required. The request to limit the orbital lifetime of spacecrafts to a period no longer than 25 years is an important step in this direction. Whether a global decision about the disposal of launched satellites at the end of their lives is taken and/or missions to capture and eliminate current space debris will be carried out, an efficient de-orbiting technology is needed. Among several characteristic, such a technology should be light, cover a broad range of satellite masses, and work for the altitudes and inclinations where a high collision risk exists (1000 km and 82^o, 800 km and 98^o and 850 km and 71^o) [1].

Electrodynamic bare tether is a very promising concept that satisfies the above conditions. Since sails are not effective for the altitudes and the high masses of interest and electric propulsion presents several drawbacks (in term of reliability, attitude control and power supply), rockets seems to be the best device to compare with. On the other hand, the de-orbiting device-to-satellite mass ratio should be the main figure of merit and it may govern the orbital parametric domain (inclination and altitude) where tethers or rockets dominate. In addition to the mass ratio, tethers should satisfy an additional requirement, which rockets are free of; the cut probability of tapes along the full de-orbiting should be below certain design threshold.

Past missions are not conclusive about tether survivability [2]. In SEDS-2 a tether about 20 km was cut after 3.7 days after its deployment. The remaining 7.2 km attached to the upper stage of the Delta II survived 54 days until the re-entry. It is remarkable that SEDS-2 flied at a low orbit of 350 km altitude where the debris flux is expected to be low and the sever probability should be small. Up to now it is unclear whether the early cut of SEDS-2 is representative or just an unfortunate event. On the other hand, in TiPS mission a 4 km long tether survived during about 10 years. This period is one and half orders of magnitude larger than the characteristic time needed by an electrodynamic tether to complete a full de-orbiting mission (typically few months).

A previous work discussed the performances and design criteria for bare tethers in terms of drag efficiency (defined as the Lorentz force versus the tether mass) [3]. Drag efficiency, however, is not the only important figure of merit and cut probability should also be considered. This work combines the cut probability equation for tape tethers derived in Ref. [4] with a simple de-orbiting model to find a formula for the product of the tether-to-satellite mass ratio and the cut probability, Π. This tape-tether design formula, which does not involve the time, only depends on the orbital parameters and the tether geometry. By looking for its minimum, the optimal tape geometry for an specific mission can be found and determine whether or not the tethers dominate the rocket in term of mass cost.

The physical reason underlying the optimum value for the tether geometry is a competition between several factors. For instance, given width and thickness values, long tethers are more massive and have large front areas exposed to the debris flux. However, since the collected current is higher, they complete the deorbiting faster (a good feature from the tether survival point of view). The quantitative weight of all these factors are included in the cut probability and the de-orbiting equations that, if combined, provide a simple formula for the dimensionless product of the mass ratio and the sever probability.

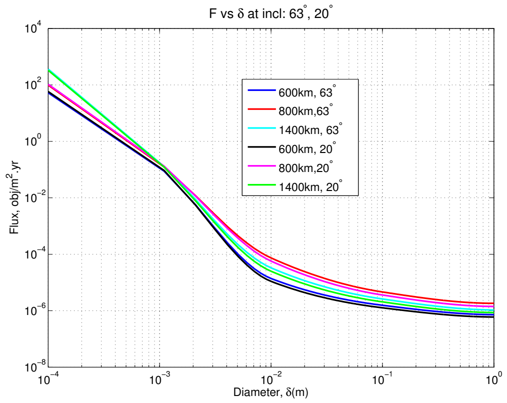

Figure 1. Flux versus debris diameter at different LEO altitudes and inclinations (ORDEM 2000).

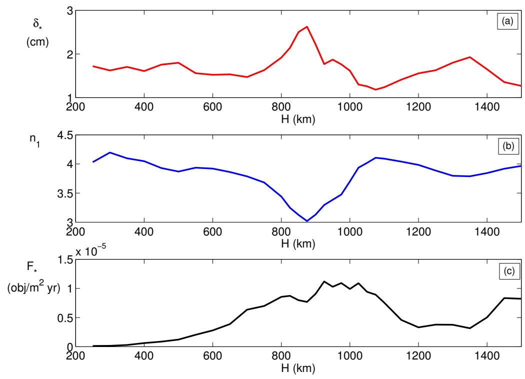

Figure 2. Debris diameter δ_* [panel (a)], exponent n_1 [panel (b)], and F_* [panel (c)] versus the altitude at inclination 63^0.

The paper is organized as follows. Section 2 briefly summarizes the results of Ref. [4] and presents the cut probability formula for a tape tether. A simple de-orbiting model for a bare electrodynamic tether is presented in Sec. 3. The results of Sec. 2 and 3 are used in Sec. 4 to derive the tape-tether design formula. The procedure to choose the optimum tape geometry is discussed. Conclusions are summarized in Sec. 5.

2. SURVIVABILITY OF A TAPE TETHER

Due to its geometry, with its length much larger than cross-section dimensions, tethers have a non-negligible sever probability. Here we consider a tape tether of length L, width w and thickness h that, as shown in Ref [4], has a probability of survival of about one and half orders of magnitude higher than a round tether of equal mass and length. The fatal impact rate per unit length is estimated as

$$\dot{n}_c \equiv \frac{1}{L}\frac{dN_c}{dt} = -\int_0^{\pi/2}\frac{d\theta}{\pi/2}\int_{\delta_m}^{\delta_\infty} D_{eff}(\theta,\delta)\frac{dF}{d\delta}d\delta \tag{1}$$

where D_{eff} is an effective diameter given by

$$D_{eff} = W'(\theta) + \delta - \delta_m \tag{2}$$

with

$$W'(\theta) \equiv w\cos\theta + h\sin\theta \tag{3}$$

In Eq. 1, δ represents the debris diameter and $F(\delta)$ the cumulative debris flux, given by ESA's Master or NASA's ORDEM models. Figure 1 shows an example of the fluxes versus the debris diameter given by ORDEM model at two different inclinations and several altitudes.

For our calculation we set $\delta_\infty = 1m$ in Eq. 1, assuming satellites will maneuver to avoid collision with larger object. We emphasize that, unlike other de-orbiting technology like sails, tethers could have certain maneuver capability by controlling the hollow-cathode behavior. The lower limit in the integral of Eq. 1, δ_m, is estimated by energy considerations as [4]

$$\delta_m = \begin{cases} f_m W'(\theta) & \theta < \theta^* \\ \delta_m^* \equiv \frac{1}{3}\sqrt{\frac{4wh}{\pi}} & \theta > \theta^* \end{cases} \tag{4}$$

where θ^* is given by the condition $\delta_m^* = f_m W'(\theta^*)$ and f_m is a constant about 0.25 and 0.33. Here we shall use the value $f_m = 1/3$.

Equation 1 is general and it allows to compute the fatal impact rate per unit length $\dot{n}_c$ for a given flux $F(\delta)$ and a tape width and thickness. In the particular case of ORDEM model and for preliminary design considerations, the flux is approximated by two power laws (see Fig. 1)

$$F(\delta) = \begin{cases} F_*\left(\frac{\delta_*}{\delta}\right)^{n_1} & \delta < \delta_* \\ F_*\left(\frac{\delta_*}{\delta}\right)^{n_2} & \delta > \delta_* \end{cases} \tag{5}$$

where the exponents n_1 and n_2, the debris diameter δ_* where the two power laws meet and the flux value F_* depend mainly on the altitudes. Figure 2 shows some of this quantities for an orbit of 63^0 of inclination and 2013. The exponent n_1 takes values from 3 to 4.3 and the debris diameter δ_* is about $2cm$ (close to the maximum width for the OML collection).

Using Eq. 5 in Eq. 1 yields the following close expression for the fatal impact rate per unit length [5]:

$$\dot{n}_c \approx A(n_1)\frac{h^{1-\frac{n_1}{2}}}{w^{n_1/2}}\delta_*^{n_1}F_* \tag{6}$$

with

$$A(n_1) \equiv \frac{4}{\pi^2}\left(\frac{3}{2}\sqrt{\pi}\right)^{n_1}\frac{3n_1+2}{3(n_1-2)} \tag{7}$$

We remark that the above equation does not depend on the cumulative flux beyond δ_*, as characterized by the exponent value n_2.

3. DE-ORBITING MODEL

The energy equation of a bare tether of density ρ and conductivity σ_c carrying the length averaged current I_{av} is

$$\frac{d}{dt}\left(\frac{1}{2}M_S v^2\right) = L I_{av}\left(\mathbf{u}_t \times B\right)\cdot \mathbf{v} \tag{8}$$

where M_S is the mass of the satellite and we took a tether perfectly aligned with the local vertical $\mathbf{u}_t$. For simplicity we will also assumed that the orbit evolves in a quasi-circular manner with velocity $v^2 = GM_E/(R_E + H)$. Here M_E and R_E are the Earth mass and radius, respectively.

Neglecting the potential drop along the plasma contactor and assuming a bare tether following the OML current collection law, the average current I_{av} normalized with the short circuit current $I_{sc} \equiv \sigma_c E_m w h$

$$i_{av}(\xi) \equiv \frac{I_{av}}{I_{sc}} \tag{9}$$

is a function of just a dimensionless parameter ξ that involves tether geometry and ambient parameters [6]

$$\xi \equiv \frac{L}{h^{2/3}l^{1/3}}, \quad l \equiv \frac{9\pi^2}{128} \times \frac{m_e \sigma_c^2}{e^2} \times \frac{E_m}{N_\infty^2} \tag{10}$$

Here N_∞ is the plasma density and $E_m = (\mathbf{v}\times\mathbf{B})\cdot\mathbf{u}_t$ is the motional electric field that typically takes values about $100 - 150V/km$. The function i_{av} versus ξ can be seen in Fig. 3. It behaves as $i_{av} \sim 0.3\xi^{3/2}$ for $\xi << 1$ and it takes the exact value $i_{av} = 1 - 1/\xi$ for $\xi > 4$.

Now, introducing the tether mass $m_t = \rho L w h$ and using the definitions of i_{av} and E_m, Eq. 8 becomes

$$\frac{M_S}{m_t} \times \frac{dH}{dt} = -2 i_{av}\frac{\sigma_c}{\rho} \times (R_e + H) \times \frac{E_m^2}{v^2} \tag{11}$$

4. TAPE-TETHER DESIGN FORMULA

The tape-tether design formula is obtained by making the ratio of Eq. 1 and 11 and integrating from the initial to the final altitudes H_0 and H_F:

$$\Pi \equiv \frac{m_t}{M_S}N_c = \int_{H_F}^{H_0} \frac{(R_e + H_F)dH}{2(R_e+H)^2}\,\frac{\rho v_f^2}{\sigma_c E_m^2} \times \times \frac{\xi}{i_{av}(\xi)}\, l^{1/3}h^{2/3}\,\dot{n}_c \tag{12}$$

This formula does not involve the time explicitly and ignores the detail of the de-orbiting trajectory. Its left hand side should be as small as possible to make tethers competitive (m_t/M_S small) and operational (N_c small). The right hand side (RHS) only involves the tether geometry (L, w and h) and the orbit properties (altitudes and inclination), allowing an optimum design for each mission.

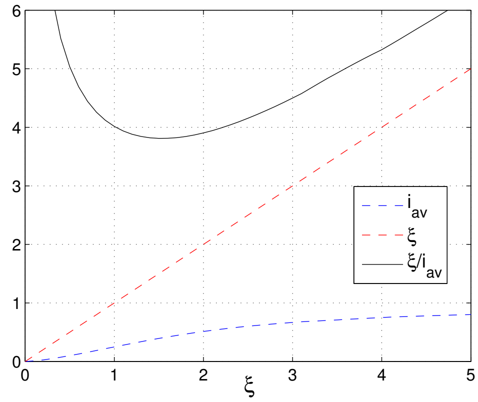

Figure 3. The functions i_{av}, ξ and ξ/i_{av} versus the dimensionless variable $\xi \equiv L/h^{2/3}l^{1/3}$

We remark that the RHS also depends on: (i) the epoch due to the plasma density (appearing in ξ) variations during the solar cycle and (ii) the full position vector $\mathbf{r}$ and not just on its modulus $r = R_E + H$ through the motional electric field E_m and the plasma density.

The factor $\xi/i_{av}(\xi)$ appearing in the RHS of Eq. 12 deserve some attention. As Fig. 3 shows, the function $\xi/i_{av}(\xi)$ presents a minimum, indicating that an optimum value of the tether geometry (the ratio $L/h^{2/3}$) exists. We remark that this result is universal; other effects here ignored, such as plasma contactor impedance or making the tether operates beyond the OML regime because its width would be large, does not qualitatively affect to this optimum because the main features of the i_{av} function are not modified. Fig. 3 also indicates that a well-designed tether should operate in a regime where ohmic effects are neither dominant but nor negligible.

For preliminary mission analysis one can use the approximation of $\dot{n}_c$ found in Sec. 2 in the case of ORDEM model. Substituting Eq. 6 in Eq. 12 yields

$$\Pi = \int_{H_F}^{H_0} \frac{(R_e + H_F)dH}{2(R_e+H)^2}\,\frac{\rho v_f^2}{\sigma_c E_m^2}\,\frac{\xi}{i_{av}(\xi)} \times \times F_* A(n_1)\frac{l^{1/3}\delta_*^{n_1}}{w^{n_1/2}}h^{(10-3n_1)/6} \tag{13}$$

Given a certain tether geometry, epoch, and orbit inclination, the RHS in Eq. 13 can be computed as follows. For an altitude within the range $H_0 - H_F$, we find the averaged value of the function inside the integral for many periods along a circular orbit of such an altitude. Since a single hollow-cathode is assumed, we set to zero the value of the function during the orbit segment where the motional electric field does not point away from the hollow cathode, a situation that may occur for orbits with high inclinations as in Sun-synchronous orbits. In this calculations the magnetic field and the plasma density are taken from the IGRF [7] and the IRI [8] models respectively. Using this procedure a table with the altitude and

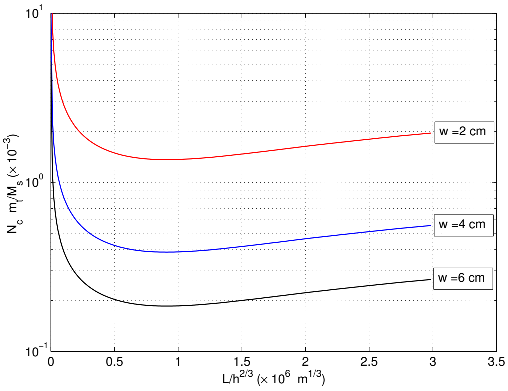

Figure 4. Mass ratio by cut probability product versus $L/h^{2/3}$ for orbital parameters $i = 63^o$, $H_0 = 800km$, $H_F = 300km$ and $h = 50\mu m$

the averaged value of the function is found and used afterwards to compute the integral in Eq. 13.

Figure 4 shows the product Pi versus the ratio $L/h^{2/3}$ for a given orbit and different widths. The minimum of the function, consequence of the ξ/i_{av} factor, happens about $L/h^{2/3} \approx 0.8 \times 10^6 m^{1/3}$. The minimum, however, is not very pronounced, indicating that a margin for the tether dimensions exists. Clearly, wider tapes present better performances; the effect of the thickness (not shown) is not very important.

5. DISCUSSION

The tether-to-satellite mass ratio and the cut probability are two figure of merits for electrodynamic tethers. The product of both quantities can be written as a time-independent function, just involving the tether geometry and the orbit properties. The analysis shows that an optimum choice of the ratio $L/h^{2/3}$ minimizes such a product. The tether would operate in a regime with ohmic effect not dominant but neither negligible.

The cut probability model shows that tether survivability is enhanced for wider tapes. However, tape width is limited by different factors, like the electron current collection. We recall that, for a given electron-to-ion temperature ratio T_e/T_i and normalized bias $e\Phi_p/kT_e$, there is a maximum tape width to operate within the OML regime and collect the maximum current I_{OML}[9]. Beyond this regime the ratio I/I_{OML} falls below one [10]. Current collection, however, is not expected to be the most limiting factor (Fig. 4 in Ref. [10] shows that the current drop will be below 10% for reasonable tether widths).

In addition to the date and the orbital parameters, the full tether dimensioning requires two additional data. The first one, N_c, is a design decision that is found by a balance of risk and economical considerations. The second is the mass of the satellite M_S. The design scheme is as follows. For a given thickness value h, the width w and the length L is found by solving two coupled equations (i) the minimum of the RHS of Eq. 13 and (ii) the condition $m_t = \rho L w h$ together with Eq. 13 itself. This procedure can be repeated for several h values within a reasonable range (30-200 μm, say). The thickness could be used to find a better value of the ratio m_t/M_S, although its impact is not expected to be very important.

In addition to the conductive tether mass m_t, other subsystems should be taken into account. The most important are the deployer, the hollow-cathode, the power system and (possibly) an inert tether or a damper to prevent dynamical instabilities. All these subsystem may represent a mass below two or three times m_t. We also remark that the presented design scheme should be understood as a preliminary analysis to bound the main parameter of the tether and carry out trade off analysis. For a specific mission, detailed calculations including the full dynamic of the tether would be required.

Preliminary analysis taking into account the full mass of the tether system indicates that tethers are the best technology at the third high risk orbit pointed out in the Introduction (altitude 850 km and inclination 71^o). For higher inclinations they are still competitive, thanks to the contributions of the eccentricity of the Earth magnetic field and its high harmonics [11]. First computations of the cut probability using ESA's MASTER model seems to be more optimistic than ORDEM. A detailed paper about the optimization method in tape-tether sizing is currently under preparation [12]. It will include a parametric survey and examples of tether dimensioning for specific missions.

Under the FP7 Space Project *BETs* we are currently developing the computer program *BETsMA*. This software is aimed to carry out preliminary Mission Analysis using Bare Electrodynamic Tethers. It will implement the algorithm here presented to find the optimum tether geometry with the full debris model and also a simple deorbiting simulator including the Lorenz force and the aerodynamic drag. Using its friendly user interface the user will be able to obtain useful information for an specific mission like the deorbit time, masses of the different components of the tether system or the survival probability of tether. Parametric analysis will be also possible as well as the selection of different models for the debris flux (ORDERM or MASTER). Software registering is planned at the end of 2013.

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