Fig. 4(a) shows the parameter estimates under the existence of the measurable disturbance ($d_1 = 5$, $d_2 = 0$) with the parameter estimates under the ideal condition ($d_1 = d_2 = 0$) overlaid. Since the inserted DDR's remove the disturbance from the input-output relation, the disturbance does not slow down the identification speed. Fig. 4(b) shows the parameter estimates under the existence of the unmeasurable disturbance ($d_1 = 0$, $d_2 = 1$) with the parameter estimates under the ideal condition ($d_1 = d_1 = 0$) overlaid. There exists no difference between the two cases as far as the identification speed is concerned.

In the simulation, the step disturbances, d_1 and d_2, were injected to the plant at $k = 0$. Thus, strictly speaking, at $k = 0$, $d_1(k)$ and $d_2(k)$ did not satisfy equation (15). Also, it took a certain number of steps before the DC component was removed from the plant output $y(k)$. This implies that some transient existed and that equation (16) was asymptotically satisfied. A consequence of these facts is reflected in the initial portion of the plotted results in Fig. 4. If identification were to have started after the transient was over, the DDR would not have caused such effect.

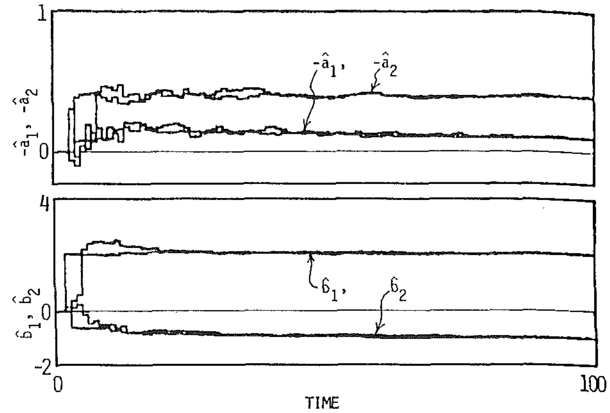

Fig. 4(a) Measurable constant disturbance

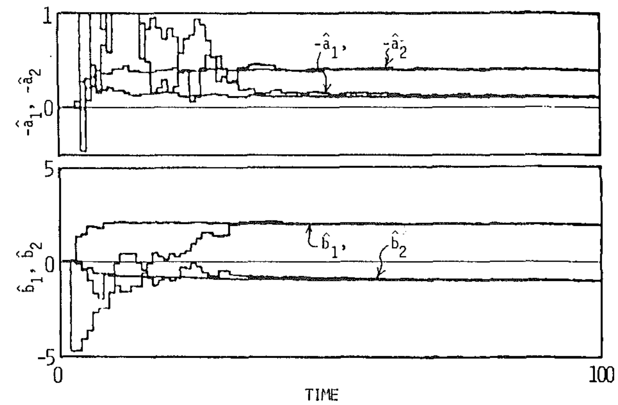

Fig. 4(b) Unmeasurable constant disturbance

Fig. 4 Removal of adverse disturbance effects by DDR

V Conclusions

Adverse effects of deterministic disturbances in linear identification have been pointed out, and a method to remove such effects has been presented. This method works for measurable and unmeasurable disturbances which can be regarded as the outputs of free systems with known dynamics. The unmeasurable disturbance must always be removed to achieve successful identification. When the disturbance is measurable, however, it does not have to be removed if it can provide a positive contribution to identification. A constant disturbance was shown to slow down the identification speed. The best results will be obtained if one selects a DDR which removes only undesirable disturbances. In this technical brief, discrete series-parallel and parallel identification schemes for single-input, single-output systems were considered. The same principle, however, can be extended to other situations including the continuous time case and multi-input, and multi-output case.

References

1 Astrom, K. J., and Eykhoff, P., "System Identification — A Survey," *Automatica*, Vol. 7, 1971, pp. 123–162.

2 Landau, I. D., "A Survey of Model Reference Adaptive Techniques — Theory and Applications," *Automatica*, Vol. 10, 1974, pp. 353–379.

3 Young, P. C., "An Instrumental Variable Method for Real Time Identification of a Noisy Process," *Automatica*, Vol. 6, 1970, pp. 271–288.

4 Landau, I. D., "Unbiased Recursive Identification Using Model Reference Adaptive Techniques," *IEEE Transactions on Automatic Control*, Vol. AC-21, No. 2, Apr. 1976, pp. 194–202.

5 Tomizuka, M., "Series-Parallel and Parallel Identification Schemes for a Class of Continuous Nonlinear Systems," Journal of Dynamic Systems, Measurement, and Control, Trans. ASME, Vol. 99, No. 2, June 1977, pp. 137–140.

Transient Response of Fluid Lines by Frequency Response Conversion

S. Katz[1]

A method of frequency response conversion to transient response is applied to terminated fluid lines. The method can be used to solve transmission line problems with various input waveforms and terminations. Some typical results for circular lines are presented in non-dimensional terms. The results show that the transient response is a function of a characteristic number which depends on the fluid properties and the geometry of the line.

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Nomenclature

a = speed of sound, m/s
d = diameter of line, m
$G(j\omega)$ = systems function in frequency domain
j = complex operator ($\sqrt{-1}$)
J_0 = Bessel function of first kind zeroth order
J_1 = Bessel function of first kind first order
k = integer variable in summations
l = length of line, m
M = magnitude of systems function
M_0 = magnitude of systems function at $\omega = 0$
M_k = magnitude of systems function at $\omega = \omega_k$
N_K = characteristic number ($= \omega_N/\omega_\nu$)
N_p = Prandtl number
$p_i(t)$ = input signal pressure (fct. of time), kN/m^2
$p_0(t)$ = output signal pressure (fct. of time), kN/m^2
t = time, s
T_p = half-period of square wave, s
T_s = settling time, s

V_L = volume of transmission line ($= \pi d^2 l/4$), m^3
V_T = volume of terminating tank, m^3
Z_c = characteristic impedance of line, kN-s/m^5
Z_L = load impedance, kN-s/m^5
γ = ratio of specific heats
Γ = complex propagation factor
ϕ = phase of systems function, radians
ϕ_k = phase of systems function at $\omega = \omega_k$, radians
ω = angular velocity, rad/s
ω_k = angular velocity at discrete frequencies, rad/s
ω_N = wave transport frequency, (a/l), rad/s
ω_ν = viscous characteristic frequency, $(32\ \nu/d^2)$, rad/s
ν = kinematic viscosity, m^2/s

1 Introduction

The dynamic behavior of pressure signals in fluid lines is important in fluidics, instrumentation, and control. There have been a large number of papers on this subject. Most of these papers deal with the frequency response of fluid transmission lines. Investigations of transient response are less common.

Schuder and Binder [1][2] developed a theory and performed experiments for the step response of long pneumatic lines with terminations. Heat transfer effects and the time dependency of laminar friction were neglected. Nichols [2] derived a frequency dependent propagation operator that included heat transfer and the change of the laminar velocity profile. Simultaneously, Brown [3, 4] developed the propagation operator in terms of Laplace Transforms. However, the inverse Laplace Transform is not available in analytical form.

Time domain solutions have been obtained for special terminations, such as in semi-infinite (matched) lines by Kantola [5] and Karam [6]. Superposition of semi-infinite line results have been used by Brown and Nelson [7] and Karam and Leonard [8] to formulate the transient response of terminated lines.

The difficulty of an analytical solution for the transient response of terminated fluid lines has led many investigators to computer methods. Zielke [9] and Brown [10] have presented a quasi-method of characteristics that accounts for frequency dependent shear and heat transfer. This method has been applied by Kirshner and Katz [11] to find the step and pulse response of blocked lines. However the method is time consuming and requires a computer with large storage capacity. Hausner [12] has determined the step response of blocked lines by using numerical inversion of the Laplace Transform. Here again the procedure requires much computing time.

The approach considered here is the conversion of the frequency response formulation for terminated transmission lines into transient response. The foundation for this approach is based on the work of Leonhard [13] and has been suggested for application to lines by Streeter and Wylie [14]. An analogous method has been applied to Laplace Transform inversion by Dubner and Abate [15].

This paper presents the frequency conversion method with respect to terminated line response for a wide range of inputs.

2 Frequency Response Conversion to Transient Response

This process requires that the input function be described in terms of a Fourier series. When input functions are non-periodic, therefore, they must be considered in a periodic form. For example, to obtain the step response, the step input is modelled as a square wave with half-period longer than the system settling time. Once the input is expressed as a sum of discrete frequency terms, the output response is merely the superposition of the system responses to these discrete frequencies.

Suppose a system function, $G(s)$, has no known analytic inverse Laplace Transform. To find the step response of this system we apply a square wave with half-period, T_p, which is longer than the settling time, T_s. When the settling time is unknown it must be estimated and then adjusted as required by the subsequent results. The Fourier series representing the square wave input, $p_i(t)$ is:

$$p_i(t) = {}^1/_2 + \sum_{k=1}^{\infty} \frac{2}{\pi(2k-1)} \sin \omega_k t \qquad (1)$$

where

$$\omega_k = (2k-1)\pi/T_p$$

To normalize the time variable we introduce the wave transport frequency, ω_N, as the ratio of the acoustic velocity, "a," to the line length, "l" so that equation (1) can be written as:

$$p_i(\omega_N t) = {}^1/_2 + \sum_{k=1}^{\infty} \frac{2}{\pi(2k-1)} \sin\left[\frac{\omega_k}{\omega_N}(\omega_N t)\right] \qquad (2)$$

Now in the frequency domain the system function for the fluid line is $G(j\omega)$ and it may be represented by magnitude, $M(\omega/\omega_N)$, and phase, $\phi(\omega/\omega_N)$. For the discrete frequencies of the input we designate $M_k = M(\omega_k/\omega_N)$ and $\phi_k = \phi(\omega_k/\omega_N)$. We may express the step response, $[p_0(\omega_N t)]_s$, by multiplying the amplitude in equation (2) by M_k and shifting the phase by ϕ_k. Thus

$$[p_0(\omega_N t)]_s = \frac{M_0}{2} + \sum_{k=1}^{\infty} \frac{2M_k}{\pi(2k-1)} \sin\left[\frac{\omega_k}{\omega_N}(\omega_N t) + \phi_k\right] \qquad (3)$$

where M_0 is the magnitude of the system function at zero frequency. Equation (3) represents the Fourier series for the step response of the system. The ramp response $[p_0(\omega_N t)]_R$, is merely the integral of equation (3) and is:

$$[p_0(\omega_N t)]_R = \frac{M_0 \omega_N t}{2} + \sum_{k=1}^{\infty} \frac{2M_k\omega_N}{\pi(2k-1)\omega_k}\left[\cos\phi_k - \cos\left(\frac{\omega_k}{\omega_N}\omega_N t + \phi_k\right)\right] \qquad (4)$$

With the formulation of equations (3) and (4) we may use superposition to find the output response to composite input functions that consist of step and ramp segments. The response to other input waveforms can be obtained by describing the input in terms of an appropriate Fourier series and following the procedure indicated.

The number of terms required to achieve a particular computational accuracy depends on the system function. A system with a narrow bandwidth requires fewer terms for the same accuracy.

3 Transient Response of Circular Lines

For the case under consideration, the system is a circular line of length, l, (Fig. 1). The system function for fluid lines is generally presented in terms of the angular frequency ratio, ω/ω_ν, where ω_ν is the characteristic viscous frequency. Since ω_N is the normalizing factor in the time domain we introduce a characteristic number $N_K (= \omega_N/\omega_\nu)$ into the system function

[2]Numbers in brackets designate References at end of Brief.

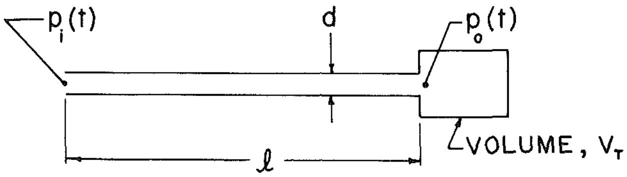

Fig. 1 Circular transmission line terminated by volume

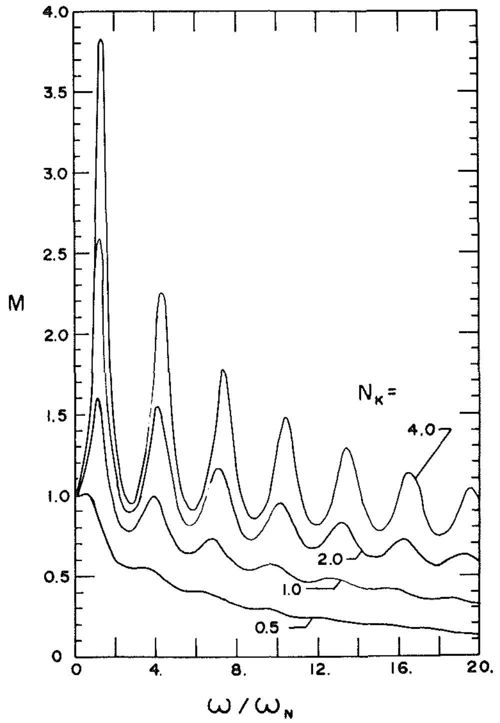

Fig. 2 Frequency response magnitude for blocked circular line

to bring both domains into correspondence. The system function for a terminated transmission line is:

$$G(j\omega) = \frac{1}{\frac{Z_c}{Z_L}\sinh\Gamma + \cosh\Gamma} \tag{5}$$

where Z_c is the characteristic impedance, Z_L is the load impedance and Γ is the propagation operator. Nichols [2] and Brown [3, 4] have developed expressions for Γ and Z_c. We may use these expressions to derive for a volume terminated line:

$$\Gamma = j\left(\frac{\omega}{\omega_N}\right)\left(\frac{D}{B}\right)^{1/2} \tag{6a}$$

$$\frac{Z_c}{Z_L} = j\left(\frac{\omega}{\omega_N}\right)\left(\frac{V_T}{V_L}\right)\left(\frac{1}{DB}\right)^{1/2} \tag{6b}$$

where

$$D = 1 + \frac{2(\gamma - 1)\, J_1[j^{3/2}(8N_K N_p\omega/\omega_N)^{1/2}]}{j^{3/2}(8N_K N_p\omega/\omega_N)^{1/2}\, J_0[j^{3/2}(8N_k N_p\omega/\omega_N)^{1/2}]}$$

$$B = 1 - \frac{2\, J_1[j^{3/2}\,(8N_K\omega/\omega_N)^{1/2}]}{j^{3/2}\,(8N_K\omega/\omega_N)^{1/2}\, J_0[j^{3/2}\,(8N_K\omega/\omega_N)^{1/2}]}$$

and J_0 and J_1 are Bessel functions, N_p is the Prandtl number, γ is the ratio of specific heats, V_L is the volume of the line and j is the complex operator ($\sqrt{-1}$).

The magnitude of the line system function (equation (5)) is shown in Fig. 2 for a blocked line ($V_T = 0$). The effect of N_K is similar to that of a damping factor. Large values of N_K represent resonant lines and small values represent damped lines.

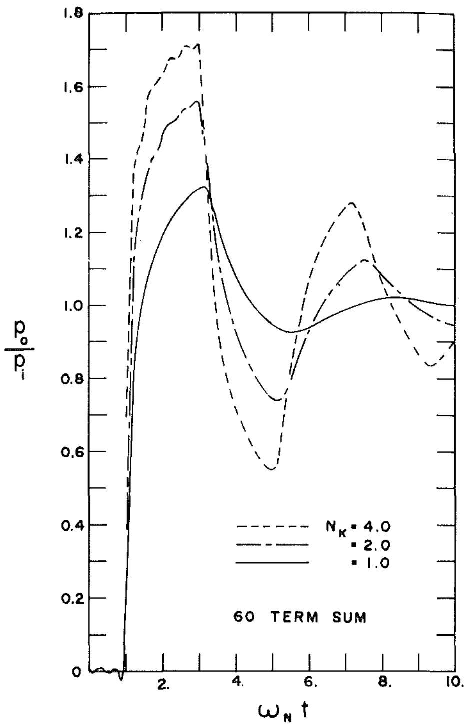

Fig. 3 Step response of blocked circular line

Fig. 3 shows some typical step responses for the blocked line with 60 terms. These results are in excellent agreement with those obtained by Hausner [12] using conventional Laplace Transform inversion methods. There is good agreement also with the experimental results obtained by Kantola [5] for a circular line 15.25 m long and 4.83 mm inner diameter (N_k = 1.07). More detailed results are given in [16] and [17] where annular and rectangular lines are also considered.

4 Summary

Analytic formulations for the transient response of fluid transmission lines are available only for a few special terminations. Frequency response conversion to transient response provides answers to a wide variety of line problems that would be impractical by any other means.

Some typical results are given for the case of a blocked circular line. The type of transient response depends on the characteristic number, N_k.

References

1 Schuder, C. B., and Binder, R. C., "The Response of Pneumatic Transmission Lines to Step Inputs," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 81, No. 4, Dec. 1959, pp. 578–584.

2 Nichols, N. B., "The Linear Properties of Pneumatic Transmission Lines," *Transactions of the Instruments Society of America*, Vol. 1, No. 1, Jan. 1962, pp. 5–14.

3 Brown, F. T., "The Transient Response of Fluid Lines," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 84, No. 4, 1962, pp. 547–553.

4 Brown, F. T., "Pneumatic Pulse Transmission with Bistable-Jet-Relay Reception and Amplification," ScD thesis, M.I.T., May 1962.

5 Kantola, R., "Transient Response of Fluid Lines Including Frequency Modulated Inputs," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 93, No. 2, 1971.

6 Karam, J. T., Jr., "A Simple But Complete Solution for the Step Response of a Semi-Infinite Circular Fluid Transmission Line," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 94, No. 2, 1972.

7 Brown, F. T., and Nelson, S. E., "Step Responses of Liquid Lines with Freqeency-Dependent Effects of Viscosity," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 87, No. 2, 1965, pp. 504–510.
8 Karam, J. T., Jr., and Leonard, R. G., "A Simple yet Theoretically Based Time Domain Model for Fluid Transmission Line Systems," JOURNAL OF FLUIDS ENGINEERING, TRANS. ASME, Series I, Vol. 95, No. 4, Dec. 1973, pp. 498–504.
9 Zielke, W., "Frequency Dependent Friction in Transient Pipe Flow," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 90, No. 1, 1968, pp. 109–115.
10 Brown, F. T., "A Quasi-Method of Characteristics with Application to Fluid Lines with Frequency Dependent Wall Shear and Heat Transfer," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 91, No. 2, 1969, pp. 217–227.
11 Kirshner, J. M., and Katz, S., *Design Theory of Fluidic Components*, Academic Press, 1975, pp. 123–124.
12 Hausner, A., "A Program for the Numerical Inversion of the Laplace Transform," Harry Diamond Laboratories Technical Report, 1975.
13 Leonhard, A., "Determination of Transient Response from Frequency Response," *Frequency Response*, ed. R. Oldenburger, MacMillan Company, N. Y., 1956.
14 Streeter, V. L., and Wylie, E. B., *Hydraulic Transients*, McGraw-Hill, New York, 1967.
15 Dubner, H., and Abate, J., "Numerical Inversion of Laplace Transforms by Relating Them to the Finite Fourier Cosine Transform," *Journal of the Association for Computing Machinery*, Vol. 15, No. 1, Jan. 1966, pp. 115–123.
16 Katz, S., "Transient Response of Terminated Pneumatic Transmission Lines by Frequency Response Conversion," ASME Paper 75-WA/Flcs-4, Dec. 1975.
17 Katz, S., and Blach, W., "Transient Response of Rectangular Pneumatic Transmission Lines with Blank Chamber Terminations," *Fluidics Quarterly*, Vol. 8, No. 4, Oct. 1976, pp. 85–103.

Comparison of Continuous and Discrete Adaptive Identification Algorithms[1]

Bruce K. Colburn[2] and Joseph S. Boland, III[3]

Discretization of a popular continuous-time control algorithm is effected and an equivalent discrete-time identification law developed and compared to a published discrete identification algorithm developed from Lyapunov Theory. Results are compared as regards asymptotic stability as insured using Lyapunov theory. Some analysis and design guidelines are proposed as regards implementation and practical utility.

I Introduction

Stable adaptive identification and control schemes for both continuous and discrete time systems have been suggested by many authors [1–6].[4] Each method is developed essentially independent of the other, although the resulting equations are similar. Little definitive work on discrete-time adaptive control/identification was done until the publications of Mendel [7], Landau [8], and Narendra [4]. Previous to this time, most published work in adaptive control and adaptive identification theory dealt with continuous time systems. This is due in part to the fact that obtaining the necessary conditions for Lyapunov stability for discrete adaptive systems generally requires further functional analysis theorem proving than is required for the continuous-time case. More recent results have dealt with the use of hyperstability theory [9] for variations in continuous-time adaptive laws.

In this paper a popular continuous-time adaptive control algorithm is discretized and results compared and analyzed with respect to a discrete identification algorithm developed previously from Lyapunov Theory. The analysis is effected using a linearized error characteristic equation (LECE) approach. The contributions in this paper are the analysis of results, stability comparison via the LECE technique, the advancement of some design guidelines for implementation of the discretized continuous-time law, and synthesis procedures for developing other adaptation laws.

II Discretization

From the Kudva, Narendra discrete adaptive observer form in [4], the adaptive gain, or "identifier," terms are of the form

$$K_{ij}(l+1) = K_{ij}(l) - C_{ij}(l-1) \sum_{k=1}^{n} e_k(l) q_{ki} x_{pj}(l-1) \qquad (1)$$

where $\mathbf{x}_p$ is an nth order plant vector, K_{ij} are the adaptive gains, q_{ij} are entries of a constant matrix $Q = Q^T > 0$, $\mathbf{e}$ is a system error between the plant and model, and l is a time counter, $l = 0, 1, 2, \ldots$. They show that in order to guarantee asymptotic stability, one form of $C_{ij}(l-1)$ is

$$C_{ij}(l-1) = \frac{\beta}{\left[\sum_{i=1}^{n} x_{pi}^2(l-1) + u^2(l-1) \right]} \qquad (2)$$

where β is a constrained function, $\mathbf{x}_p$ are plant states, and u is the plant input. The form in equation (2) is important because it is not suggested by direct comparison of the continuous and the discrete-time cases. The important points to note in equation (2) are that (*a*) plant state and input magnitudes appear as a division factor in the weighting value C_{ij}, and (*b*) a fixed constant β is constrained to lie within certain bounds, $\beta > 0$ in all cases however. The division factor in item (*a*) has been shown to occur in some other methods, such as instrument variable and recursive least squares [8], but little discussion of its significance has been presented.

Rewriting the Winsor and Roy [1] control law in a form compatable with that in equation (1) (minus signs appear based on the definition of how the adaptive gain entries are added or subtracted from plant or model dynamics) yields

$$K_{ij}(t) = -\alpha_{ij} \int_{t_0}^{t} \sum_{m=1}^{n} e_m q_{mi} x_{pj} dt \qquad (3)$$

where all terms are defined as before, and α_{ij} is an a-priori designer-determined positive constant. This is the same *form* as a continuous time adaptive observer by Narendra [10]. Equation (3) can be written in the complex frequency domain as

$$K_{ij}(s) = -\frac{\alpha_{ij}}{s} \sum_{m=1}^{n} e_m q_{mi} x_{pj}(s) \qquad (4)$$

To convert equation (4) into an "equivalent" discrete system, one of the many s-domain to z-domain mapping functions must be employed. Some of the more common ones include

$$\text{Backward Difference} \quad s \doteq \frac{1 - z^{-1}}{T} \qquad (5)$$

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[4]Numbers in brackets designate References at end of Brief.

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