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The Influence of Axial-Torsional Coupling on the Natural Frequencies of an Aerial Cable

The equations of motion for the small oscillations of a stranded, overhead transmission line are derived and linearized about the static equilibrium position. The influence of axial-torsional coupling on the natural frequencies is studied analytically and an expression is presented for the coupled natural frequency in torsion. In order to verify the analytical results, a finite element analysis of the linearized coupled differential equations is carried out. The results of each analysis are compared and show close agreement. The results of the zero coupling case also closely agree with previous work.

Introduction

For a number of years various investigators have analyzed the problem of free vibrations of suspended cables or chains. Suspended cables can be found in many engineering applications of which electrical power transmission is one. A common problem associated with aerial cables is that in the absence of circular cross-section, the aerodynamic lift characteristic is so altered that it can lead to dynamic instability, even at low wind speeds. The unstable case can result in a large amplitude, low-frequency oscillation, termed as "galloping." Galloping may result in power outages and may cause various kinds of structural damage. Attempts have been made in the past to suppress galloping or, at least, to minimize the galloping amplitude. In order to be effective in controlling galloping, a knowledge of the mechanics of galloping is required.

Routh (1905) presented the equations of motion for an inelastic chain hanging in the form of a cycloid. He obtained exact solutions for the symmetric and anti-symmetric vertical oscillations. Based on the results of some elementary oscillation experiments, Pugsley (1949) presented a simple approximate theory of the oscillations of a uniform suspension chain. He derived semi-empirical formulae for the first three natural frequencies. Saxon and Cahn (1953) obtained an asymptotic solution of the linearized equations of motion for the small vibration of a suspended, in-extensible chain vibrating in the same catenary plane which contains the equilibrium configuration. The results obtained by Saxon and Cahn, which are applicable to cables with a larger sag to span ratio, agreed with those presented by Pugsley which are applicable for sag-to-span ratios between 0.1 and 0.25.

Cheers (1950) used a perturbation analysis for the cable motion which resulted in wave equation analysis. The equations of motion for an elastic cable of symmetric cross-section were presented by Shea (1955) and by Simpson (1963). The equations of motion presented by Shea and Simpson assume an absence of torsional motion. In both of these investigations, motion of the cable in all three Cartesian directions were considered. Shea (1955) derived and solved the linearized equations of motion about the sagged, in-plane equilibrium position for a cable undergoing free vibrations. Shea concluded, on the basis of linearized analysis of the nonlinear partial differential equations that for free vibrations, the out-of-plane motion (normal to the catenary plane) is not coupled to the in-plane motion. Also, for a small sag-to-span ratio, the anti-symmetric vertical natural frequencies are given by the taut string model whereas the symmetric natural frequencies can be obtained by solving a transcendental equation which involves material and geometric parameters of the cable. Shea also mentioned the fact that if the sag-to-span ratio was greater than a critical value, then the first symmetrical mode would have three loops.

Simpson (1966) used a transfer matrix method to determine the in-plane natural frequencies of a shallow elastic catenary. Unlike Shea who considered only fixed-fixed type of end support, Simpson considered other types of end support in addition to the fixed-fixed one. The results for the fixed-fixed case are identical in both of the investigations by Simpson and Shea.

A study of the influence of curvature coupling on the nature of in-plane oscillations of an initially curved cable was carried out by Nariboli and McConnell (1988). As in the works of Shea (1955) and Simpson (1966), torsional motion was not considered in this analysis too. The study is unique in the sense that Nariboli and McConnell used a curvilinear coordinate system to derive the nonlinear equations of motion governing the planar oscillations of an initially curved cable. The nonlinear equations were linearized and the nature of linear oscillations was studied for the case of a shallow catenary geometry. The authors have presesnted dispersion relation for frequency and expressions for mode shapes. The transcendental equation [Eq. (39) in the paper] used for finding the natural frequencies happens to be the same as given by Shea (1955) and Simpson (1966) except for the sign of a term. There appears to be a sign misprint in that equation.

In addition to the three Cartesian directions of motion, torsional motion was also included in the analysis of a bundled

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conductor by Simpson (1972). Thompson (1975) considered the torsional motion of a single conductor with the added assumption that no motion takes place in the cable axial direction.

There are other notable works in the cable vibration area. Irvine and Caughey (1974) derived an approximate solution for the in-plane oscillation of an elastic cable and investigated the effect of elasticity on natural frequencies. For small sag-to-span ratios, the theory provides good results and explains discrepancies caused by the inextensibility assumption. West et al. (1975) carried out a numerical investigation of the natural frequencies and modes of vibration of an elastic cable oscillating in its own plane. Triantafyllou (1984) presented an excellent review of literature on linear dynamics of cables up to the year 1983.

McConnell and Chang (1986) studied the axial-torsional coupling effect on a sagged transmission line. They considered the equations of motion presented by Simpson and included the constitutive model proposed by McConnell and Zemke (1982) which takes into account the stranded geometry. They solved the coupled equations numerically by a combination of finite difference and Runge-Kutta methods. By carrying out a Fast Fourier Transform analysis of the resulting time history of a point on the line, the frequencies were calculated. McConnell and Chang concluded based on their studies that axial-torsional coupling increases vertical fundamental natural frequency and that the ratio of torsional to vertical frequency may be such that it may lead to internal resonance. The equation of motion for torsional oscillation presented by McConnell and Chang is, in the opinion of the authors of the present paper, not dimensionally balanced. Also it was felt that closed form expressions for natural frequencies might give a better insight into the influence of axial-torsional coupling. Hence the present study was carried out.

Following Shea's (1955) approach and including the constitutive model proposed by McConnell and Zemke (1982), the equations of motion of a stranded, sagged cable oscillating in the three Cartesian degrees of freedom and also undergoing torsional motion are derived using Hamilton's principle. The equations of motion are linearized about the sagged equilibrium position. Neglecting motion along the axial direction, closed form expressions for natural frequencies are derived. In order to check the validity of the assumption that the axial motion is negligible, a finite element analysis of the coupled linearized differential equations is carried out without invoking the axial motion assumption. It is found that the assumption is, indeed, valid.

Linearized Equations of Motion

The linearized equations of motion of a stranded, sagged elastic cable, as shown in Fig. 1, are derived using Hamilton's principle. In the figure, XYZ represents the right-handed global coordinate system with the YOZ plane containing the static equilibrium configuration. The origin of the XYZ frame is located at the center span sagged position. The X coordinate direction is normal to the YOZ plane. The Lagrangian representing the complete kinetic and excess potential energies with respect to the equilibrium position is derived by forming the various component energies. Then by taking the first variation and requiring that it vanish, the equations of motion are obtained. Attention will now be focused on deriving the various energies.

Strain Energy. The strain energy due to tension may be written as

$$SE_{Ten}=\frac{1}{2}\int_{-l}^{+l} A\sigma\epsilon ds \tag{1}$$

where A is the cross-sectional area, σ is the axial stress, ϵ is the axial strain, s is the parameter which measures length along

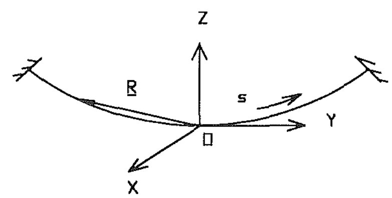

Fig. 1 Conductor geometry

the unstressed line, and l is the semi-free length. Equation (1) may be written as

$$SE_{Ten}=\frac{1}{2}\int_{-l}^{l} P\epsilon ds \tag{2}$$

where P is the axial tension.

The constitutive model presented by McConnell and Zemke (1982) is

$$P=AE\epsilon+B\frac{\partial\theta}{\partial s} \tag{3}$$

and

$$T=B\epsilon+GJ\frac{\partial\theta}{\partial s} \tag{4}$$

where E is the Young's modulus, B is the axial-torsional coupling term, GJ is the torsional rigidity, T is the twisting moment, and θ is the rotational deformation at a point about the unit tangent in the right-handed sense. The unit tangent points in the same direction as the coordinate s. For an aluminum-conductor-steel-reinforced (ACSR) electrical conductor called DRAKE 26/7 (with 26 aluminum wires and 7 steel wires) the value of B is 24900 N-m. This value of B which was given by McConnell and Chang (1986) is used in later calculations. Using the constitutive model in Eq. (2), the strain energy becomes

$$SE_{Ten}=\frac{1}{2}\int_{-l}^{l}\left(AE\epsilon+B\frac{\partial\theta}{\partial s}\right)\epsilon ds. \tag{5}$$

With reference to Fig. 1, the position vector $\mathbf{R}$ of any point on the line located at the center of the cross-section may be written as

$$\mathbf{R}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k} \tag{6}$$

where x, y, and z are the coordinates along X, Y, and Z Cartesian directions. The vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ represent the unit vectors along X, Y, and Z directions, respectively. The compatibility condition is given by

$$(1+\epsilon)^2=\frac{\partial\mathbf{R}}{\partial s}\cdot\frac{\partial\mathbf{R}}{\partial s} \tag{7}$$

where the term on the right-hand side of Eq. (7) is the dot product of the vector which represents the parametric variation of $\mathbf{R}$, with itself. Using Eq. (7), Eq. (5) can be written as

$$SE_{Ten}=\frac{1}{2}\int_{-l}^{l} AE\left(\left|\frac{\partial\mathbf{R}}{\partial s}\right|-1\right)^2 ds+\frac{1}{2}\int_{-l}^{l} B\frac{\partial\theta}{\partial s}\left(\left|\frac{\partial\mathbf{R}}{\partial s}\right|-1\right)ds \tag{8}$$

where $|\partial\mathbf{R}/\partial s|$ denotes the magnitude of the vector $\partial\mathbf{R}/\partial s$.

Similarly, the torsional strain energy may be written as

$$SE_{Tor}=\frac{1}{2}\int_{-l}^{l} B\frac{\partial\theta}{\partial s}\left(\left|\frac{\partial\mathbf{R}}{\partial s}\right|-1\right)ds+\frac{1}{2}\int_{-l}^{l} GJ\left(\frac{\partial\theta}{\partial s}\right)^2 ds. \tag{9}$$

The total strain energy is then the sum of Eqs. (8) and (9). It may be noted here that the influence of bending is not considered in this study since bending stresses in a single conductor span are small owing to small radius of cable cross-section and large radius of curvature of the cable. The total strain energy stored in the equilibrium configuration may be obtained in the same manner.

Denoting quantities related to the static equilibrium configuration by subscript zero, the excess of strain energy with respect to the equilibrium configuration is given by

$$SE-SE_0=\frac{1}{2}\int_{-l}^{l}AE\left\{\left(\left|\frac{\partial \mathbf{R}}{\partial s}\right|-1\right)^2-\left(\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|-1\right)^2\right\}ds$$
$$+\int_{-l}^{l}B\left\{\frac{\partial\theta}{\partial s}\left(\left|\frac{\partial \mathbf{R}}{\partial s}\right|-1\right)-\frac{\partial\theta_0}{\partial s}\left(\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|-1\right)\right\}ds$$
$$+\frac{1}{2}\int_{-l}^{l}GJ\left\{\left(\frac{\partial\theta}{\partial s}\right)^2-\left(\frac{\partial\theta_0}{\partial s}\right)^2\right\}ds. \quad (10)$$

Gravitational Potential Energy. If m represents the mass per unit unstressed length, g the acceleration due to gravity, and z the vertical coordinate of any point on the line, then the excess of gravitational potential energy with respect to the equilibrium position is given by

$$PE_g-(PE_g)_0=-\int_{-l}^{l}mg(z-z_0)ds. \quad (11)$$

Kinetic Energy. The kinetic energy of the line is given by

$$KE=\frac{1}{2}\int_{-l}^{l}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)ds+\frac{1}{2}\int_{-l}^{l}I\dot{\theta}^2ds \quad (12)$$

where I represents the mass moment of inertia per unit length of the cable and the dots represent differentiation with respect to time.

Linearization. Let

$$x=x_0+u$$
$$y=y_0+v$$
$$z=z_0+w$$
$$\theta=\theta_0+\phi \quad (13)$$

where u, v, w, and ϕ represent small perturbations from the equilibrium configuration. Since

$$\left|\frac{\partial \mathbf{R}}{\partial s}\right|=\left\{\left(\frac{\partial x}{\partial s}\right)^2+\left(\frac{\partial y}{\partial s}\right)^2+\left(\frac{\partial z}{\partial s}\right)^2\right\}^{1/2} \quad (14)$$

and

$$\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|=\left\{\left(\frac{\partial x_0}{\partial s}\right)^2+\left(\frac{\partial y_0}{\partial s}\right)^2+\left(\frac{\partial z_0}{\partial s}\right)^2\right\}^{1/2}, \quad (15)$$

we can use the perturbations to write Eq. (14) as

$$\left|\frac{\partial \mathbf{R}}{\partial s}\right|^2=\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|^2+K_1+K_2 \quad (16)$$

where

$$K_1=\left(\frac{\partial u}{\partial s}\right)^2+\left(\frac{\partial v}{\partial s}\right)^2+\left(\frac{\partial w}{\partial s}\right)^2 \quad (17)$$

and

$$K_2=2\left(\frac{\partial y_0}{\partial s}\cdot\frac{\partial v}{\partial s}+\frac{\partial z_0}{\partial s}\cdot\frac{\partial w}{\partial s}\right). \quad (18)$$

Combining Eqs. (10), (11), and (16) the excess of total potential energy (tensile strain energy, torsional strain energy, and gravitational potential energy) with respect to the equilibrium configuration is given by

$$V-V_0=\int_{-l}^{l}\frac{1}{2}AE\left\{K_1+K_2-\frac{K_1+K_2}{\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|}+\frac{1}{4}\frac{K_2^2}{\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|^3}\right\}ds$$
$$+\int_{-l}^{l}B\left[\frac{1}{2}\frac{\partial\theta_0}{\partial s}\frac{K_1+K_2}{\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|}-\frac{1}{8}\frac{\partial\theta_0}{\partial s}\frac{K_2^2}{\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|^3}\right.$$
$$\left.+\frac{\partial\phi}{\partial s}\left\{\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|+\frac{1}{2}\frac{K_2}{\left|\frac{\partial \mathbf{R}_0}{\partial s}\right|}-1\right\}\right]ds$$
$$+\int_{-l}^{l}\frac{1}{2}GJ\left\{\left(\frac{\partial\phi}{\partial s}\right)^2+2\frac{\partial\theta_0}{\partial s}\cdot\frac{\partial\phi}{\partial s}\right\}ds-\int_{-l}^{l}mgw\,ds \quad (19)$$

where Taylor series expansions have been used to express some of the nonlinear terms. It may be noted here that only those terms which, when the first variation is taken, will lead to linear terms in u, v, w, and ϕ are retained. The kinetic energy is given by

$$KE=\frac{1}{2}\int_{-l}^{l}m(\dot{u}^2+\dot{v}^2+\dot{w}^2)ds+\frac{1}{2}\int_{-l}^{l}I\dot{\phi}^2ds. \quad (20)$$

By forming the Lagrangian, taking its first variation, setting the variations of u, v, w, and ϕ at both ends of the line and at either end points in time to zero, equating the resulting variation to zero, and collecting like terms, the equations of motion become

$$\frac{\partial}{\partial s}\left(\frac{P_0}{1+\epsilon_0}\frac{\partial u}{\partial s}\right)=m\frac{\partial^2 u}{\partial t^2}, \quad (21)$$

$$\frac{\partial}{\partial s}\left\{\frac{\partial y_0}{\partial s}\left(\frac{\partial y_0}{\partial s}\cdot\frac{\partial v}{\partial s}+\frac{\partial z_0}{\partial s}\frac{\partial w}{\partial s}\right)+\frac{B}{AE}\frac{\partial y_0}{\partial s}\frac{\partial\phi}{\partial s}\right\}=\frac{m}{AE}\frac{\partial^2 v}{\partial t^2}, \quad (22)$$

$$\frac{\partial}{\partial s}\left\{\frac{\partial w}{\partial s}+\frac{1}{K}\frac{\partial z_0}{\partial s}\left(\frac{\partial y_0}{\partial s}\cdot\frac{\partial v}{\partial s}+\frac{\partial z_0}{\partial s}\frac{\partial w}{\partial s}\right)+\frac{B}{AEK}\frac{\partial z_0}{\partial s}\frac{\partial\phi}{\partial s}\right\}=\frac{m}{AEK}\frac{\partial^2 w}{\partial t^2}, \quad (23)$$

and

$$GJ\frac{\partial^2\phi}{\partial s^2}+B\frac{\partial}{\partial s}\left(\frac{\partial y_0}{\partial s}\frac{\partial v}{\partial s}+\frac{\partial z_0}{\partial s}\frac{\partial w}{\partial s}\right)=I\frac{\partial^2\phi}{\partial t^2} \quad (24)$$

where K represents the stretch per unit length at the span center. The quantity K is the ratio of the horizontal tension to the axial stiffness of the cable. It may be noted here that the out-of-plane motion u is decoupled from the other degrees of freedom, a result already established by Shea (1955) and Simpson (1966). By setting the axial-torsional coupling parameter B to zero, Eqs. (22) and (23) reduce to the corresponding ones given originally by Shea (1955) and Simpson (1966). Also, when B vanishes, the torsional motion is decoupled from the other degrees of freedom as well.

If the sag to span ratio is small (shallow catenary), then the ratio of weight per unit length to the horizontal tension is small compared to one. This implies that the tension in the string is approximately equal to the horizontal tension. Also the ratio of the horizontal tension to the axial stiffness is very small compared to unity for the majority of practical cases. These limits are referred to as the shallow catenary condition. Based on this assumption one may write

$$\frac{\partial y_0}{\partial s}\approx 1+K \text{ and } \frac{\partial z_0}{\partial s}\approx\frac{s}{a}(1+K) \quad (25)$$

where a = AEK/mg. Using Eq. (25), comparing the order of magnitude of coefficients of like terms, neglecting small order coefficients the equations of motion for the axial, vertical, and torsional degrees of freedom may be reduced to

$$\frac{\partial}{\partial s}\left(a\frac{\partial v}{\partial s}+s\frac{\partial w}{\partial s}+\frac{Ba}{AE}\frac{\partial\phi}{\partial s}\right)=\frac{ma}{AE}\frac{\partial^2 v}{\partial t^2}, \quad (26)$$

$$\frac{\partial}{\partial s}\left\{\frac{\partial w}{\partial s}+\frac{s}{Ka^2}\left(a\frac{\partial v}{\partial s}+s\frac{\partial w}{\partial s}+\frac{Ba}{AE}\frac{\partial\phi}{\partial s}\right)\right\}=\frac{m}{AEK}\frac{\partial^2 w}{\partial t^2}, \quad (27)$$

and

$$GJ\frac{\partial^2\phi}{\partial s^2}+\frac{B}{a}\frac{\partial}{\partial s}\left(a\frac{\partial v}{\partial s}+s\frac{\partial w}{\partial s}\right)=I\frac{\partial^2\phi}{\partial t^2} \quad (28)$$

Determination of Natural Frequencies

Assuming normal mode of oscillations, we may write

$$v(s,t)=\bar{v}(s)e^{i\omega t},$$

$$w(s,t)=\bar{w}(s)e^{i\omega t},$$

$$\text{and } \phi(s,t)=\bar{\phi}(s)e^{i\omega t} \quad (29)$$

where $i=\sqrt{-1}$ and ω is the circular frequency. Using Eq. (29) in Eqs. (26), (27) and (28), and canceling out the common terms gives

$$\frac{d}{ds}\left(a\frac{d\bar{v}}{ds}+s\frac{d\bar{w}}{ds}+\frac{Ba}{AE}\frac{d\bar{\phi}}{ds}\right)=-\frac{ma}{AE}\omega^2\bar{v}, \quad (30)$$

$$\frac{d}{ds}\left\{\frac{d\bar{w}}{ds}+\frac{s}{Ka^2}\left(a\frac{d\bar{v}}{ds}+s\frac{d\bar{w}}{ds}+\frac{Ba}{AE}\frac{d\bar{\phi}}{ds}\right)\right\}=-\frac{m}{AEK}\omega^2\bar{w}, \quad (31)$$

and

$$GJ\frac{d^2\bar{\phi}}{ds^2}+\frac{B}{a}\frac{d}{ds}\left(a\frac{d\bar{v}}{ds}+s\frac{d\bar{w}}{ds}\right)=-I\omega^2\bar{\phi}. \quad (32)$$

By assuming that the axial motion is negligible, one may write, using Eq. (30),

$$\frac{d}{ds}\left(a\frac{d\bar{v}}{ds}+s\frac{d\bar{w}}{ds}+\frac{Ba}{AE}\frac{d\bar{\phi}}{ds}\right)\approx 0 \quad (33)$$

or

$$a\frac{d\bar{v}}{ds}+s\frac{d\bar{w}}{ds}+\frac{Ba}{AE}\frac{d\bar{\phi}}{ds}=C_1 \quad (34)$$

where C_1 is a constant. Using Eqs. (34) in Eq. (32) one gets

$$\frac{d^2\bar{\phi}}{ds^2}+\lambda_\theta^2\bar{\phi}=0 \quad (35)$$

where

$$\lambda_\theta=\left\{\frac{I}{(GJ-B^2/AE)}\right\}^{1/2}\omega. \quad (36)$$

The general solution of Eq. (35) can be written as

$$\bar{\phi}=C_2\cos(\lambda_\theta s)+C_3\sin(\lambda_\theta s) \quad (37)$$

where C_2 and C_3 are constants which depend on the boundary conditions. Applying the fixed-fixed boundary conditions the equation for the natural frequency, in Hertz, of torsional oscillation may be written as

$$f=\frac{n}{2(2l)}\left\{\frac{(GJ-B^2/AE)}{I}\right\}^{1/2} \quad (38)$$

where n = 1, 2, 3, ...

From Eq. (38) one may conclude that axial-torsional coupling decreases torsional natural frequency. By using Eqs. (31) and (34), one can find the natural frequencies for vertical oscillations. The expressions for vertical natural frequencies turn out to be identical to those given by Shea (1955) and Simpson (1966). The expression for finding the natural frequencies of anti-symmetrical modes of vertical oscillation is the same as that for a corresponding taut string model whereas the expression for determining the natural frequencies of symmetrical modes of vertical oscillation is

$$\tan\left(\omega\sqrt{\frac{m}{H}}l\right)-\left(\omega\sqrt{\frac{m}{H}}l\right)\left[1-\frac{H/AE}{4(\delta/l)^2}\left(\omega\sqrt{\frac{m}{H}}l\right)^2\right]=0 \quad (39)$$

where δ is the sag at mid span and H is the horizontal component of cable tension. The other quantities appearing in the equation have been defined previously. The axial-torsional coupling parameter "B" does not appear in Eq. (39). This demonstrates the fact that if the axial motion is negligible, then the vertical natural frequency is unaffected and the torsional natural frequency is reduced due to the effect of axial-torsional coupling.

In order to check the validity of the assumption that the axial motion is negligible, a finite element analysis of the Eqs. (30), (31), and (32) is carried out. A Galerkin weighted residual approach is used with linear interpolation polynomials. The formation of element stiffness and mass matrices, the assembly of global stiffness and mass matrices, and the solution of the resulting eigenvalue problem are carried out in the usual manner (Segerlind, 1984; Meirovitch, 1980). A FORTRAN program was written to form the element matrices, build the global matrices, and apply the boundary conditions. The resulting eigenvalue problem was solved using IMSL (International Mathematical and Statistical Library), subroutine EIGRF. The calculation was carried out on an IBM 3084 machine.

Table 1

Mass per unit length	= 0.9669	kg/m
Axial stiffness	= 2.21E7	N
Span	= 853.44	m
Sag	= 70.7136	m

Mode	Vertical Natural Frequencies (radians per second)				
	West et al.		Pugsley	Saxon & Cahn	Present method
	Extrapolation	Continuous method			
1	0.800	0.811	0.811	0.803	0.800
2	1.160	1.175	1.148	1.185	1.189
3	1.630	1.653	1.647	1.671	1.667
4	1.990	2.027	____	2.042	2.046
5	2.450	2.492	____	2.452	2.528

Numerical Examples and Discussion of Results

At this stage, to check the finite element formulation and the FORTRAN program, an example given by West et al. (1975) is used as a test case. West and his coworkers considered only the in-plane (plane containing the static equilibrium configuration) oscillations. Torsional motion and the influence of stranded geometry of the cable were not included in the study. The results are given in Table 1. The last column in the table contains the results obtained for the cable problem using the finite element method described in this paper. The other columns contain results reported and discussed in the paper by West et al. (1975). It can be seen that the finite element results compare very well with those reported elsewhere.

Now, the example given by McConnell and Chang (1986) is examined using the finite element program. Also, calculations for vertical natural frequency for symmetrical modes are performed using the transcendental equation given by Shea (1955). The results are presented in Table 2 which includes the results reported by McConnell and Chang. The frequencies are cal-

Table 2

Mass per unit length	≈ 1.628	kg/m
Axial stiffness	≈ 3.52E7	N
Torsional rigidity	≈ 161.0	N-m^2
Mass moment of inertia per unit length	≈ 0.001543	kg-m
Span	≈ 304.8	m

Symmetrical modes

(Vertical Natural Frequencies)

Sag/Span ratio		Frequency (Hertz) McConnell & Chang 1st	2nd	3rd	Shea/Simpson 1st	2nd	3rd	Present Finite Element method 1st	2nd	3rd
1%	a)	0.333	___	___	0.362	0.953	1.585	0.362	0.955	1.602
	b)	___	___	___	*	*	*	0.361	0.954	1.602
3%	a)	0.467	0.667	0.934	0.475	0.637	0.925	0.477	0.638	0.935
	b)	0.500	___	___	*	*	*	0.477	0.634	0.934
5%	a)	0.467	0.667	0.934	0.402	0.687	0.922	0.404	0.699	0.930
	b)	___	___	___	*	*	*	0.404	0.699	0.932

a) B = 0 N-m
b) B = 24900 N-m

___ Not reported

* *Does not apply*

Table 3

Torsional Natural Frequencies (Hertz)

Sag/Span ratio		Finite Element Method *First*	*Second*	*Third*	Equation (38) *First*	*Second*	*Third*
1%	a)	0.5298	1.0614	1.6023	0.5298	1.0598	1.5897
	b)	0.4988	1.0012	1.5051	0.5000	1.0000	1.5000
3%	a)	0.5287	1.0593	1.5931	0.5289	1.0579	1.5868
	b)	0.4982	0.9995	1.5100	0.4992	0.9983	1.4975
5%	a)	0.5268	1.0555	1.5866	0.5266	1.0532	1.5797
	b)	0.4967	0.9958	1.4990	0.4969	0.9939	1.4908

a) B = 0 N-m
b) B = 24900 N-m

culated for three different values of sag to span ratio. The sag to span ratios chosen are above and below the critical sag which is 2.2 percent as reported by Shea (1955). For a given sag to span ratio there are two rows of entry in the table. The first row represents the natural frequencies without considering the coupling between axial and torsional motion due to stranded geometry ($B = 0$). The other row provides the results taking the effect of coupling into account. As mentioned before, the value of the coupling parameter is 24900 N-m. It can be seen that the results obtained from the finite element method compare well with those obtained using the transcendental equation given independently by Shea and Simpson for the case when B is zero.

It may be noted here that in order to solve the set of coupled ordinary differential equations [Eqs. (30)–(32)] analytically it was assumed that the motion along the span direction could be neglected which implies that the tension in the cable is almost constant throughout the length but varies only with respect to time. However, this assumption was not made for solving the set of equations using the finite element method. This comparison demonstrates the validity of the assumption. Shea and Simpson did not consider the axial-torsional coupling effect and it is noted in Table 2 where their results do not apply.

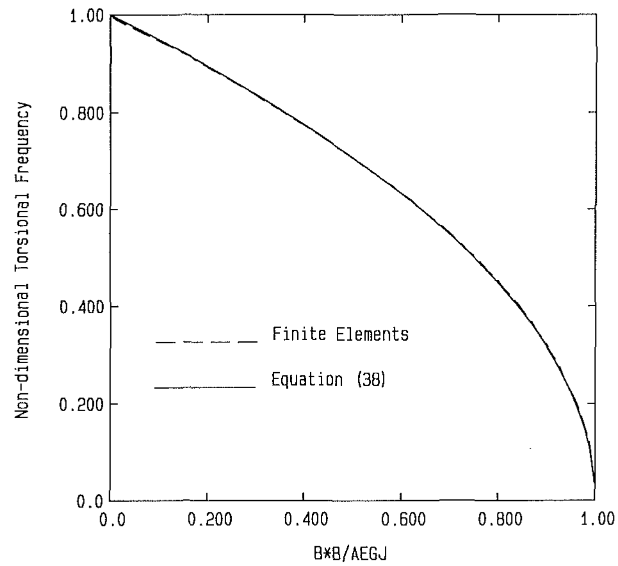

Fig. 2 Fundamental torsional natural frequency as function of B²/AEJG for "DRAKE ACSR" cable: sag-to-span ratio = 1 percent

It has been stated in the paper that even after including the effect of coupling, the frequencies of vertical oscillations which are important in the study of galloping are still given by the same equations as reported by Shea. The entries in the second row for every sag to span ratio confirms this result. This means that the frequencies of vertical oscillation are not affected by the coupling between axial and torsional motion. Also for a value of sag to span ratio greater than the critical sag, the first symmetrical mode (not shown here) obtained using the finite element method has three loops.

The torsional natural frequencies for the same example are calculated using Eq. (38) and using the finite element program. The results are given in Table 3. It can be seen that the results obtained from both the methods agree well with each other. Also it can be seen that the coupling reduces the natural frequencies of torsional oscillations. To the best of the knowledge of the authors, the equation for finding the torsional natural frequencies of a shallow cable, considering coupling due to stranded geometry, has not appeared in literature before. From the results obtained using the finite element method it is observed that the frequencies of oscillations along the span direction are increased due to the coupling. However, the percentage change in the frequencies for along-the-span oscillations for a given value of B, the axial-torsional coupling parameter, is much less than the percentage change in torsional frequencies. With reference to the illustrative cable problem presented by McConnell and Chang, for a 3 percent sag-to-span ratio and a value of 24900 N-m for the coupling parameter, the percentage changes in the fundamental torsional and along-the-span frequency are about 5.6 percent and 0.025 percent, respectively.

Galloping is found to occur in the first few modes. It can be seen from Tables 2 and 3 that the first symmetrical vertical frequency is close to the coupled fundamental torsional frequency. If a torsional frequency is close to a vertical frequency or to an integer multiple of a vertical frequency then it may lead to internal resonance. Internal resonance may, in turn, lead to galloping instability. This can be avoided by separating the torsional and vertical frequencies by a proper design of the cable. The results presented in this paper will be useful toward that end.

Figures 2 and 3 show the nondimensional fundamental torsional frequency (ratio of coupled fundamental torsional frequency to that of the uncoupled one) as a function of the axial-torsional coupling parameter B for the example line presented

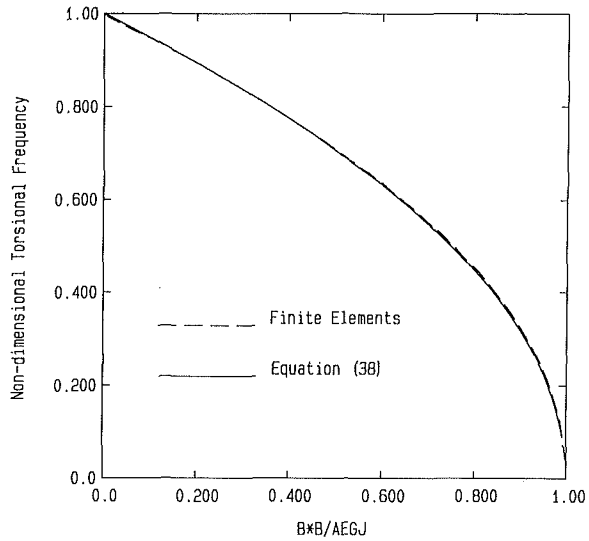

Fig. 3 Fundamental torsional natural frequency as a function of B^2/AEJG for "DRAKE ACSR" cable: sag-to-span ratio = 3 percent

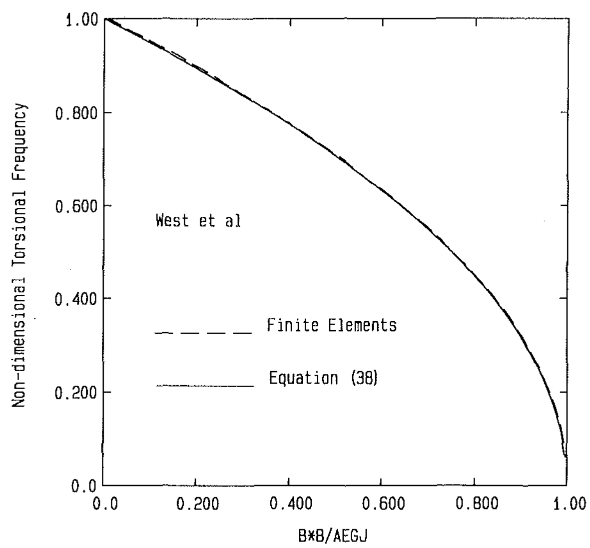

Fig. 4 Torsional frequency as a function of B

by McConnell and Chang (1986) for sag-to-spin ratios of 1 percent and 3 percent (values below and above the critical sag), respectively. The nondimensional frequencies are calculated using the Eq. (38) and also using the finite element program. Figure 4 shows the same type of graph for the example given by West et al. (1975). Since West et al. did not consider torsional oscillations, the value of GJ was not reported. Nor was the type of aerial cable specified. As a result, the torsional rigidity and the moment of inertia per unit length of the cable for the case of Fig. 4 are prorated from the data given by McConnell and Chang. The values are 98.21 N-m^2 and 9.41 $\times$ 10^{-4} Kg-m, respectively. The figures show agreement between finite element produced results and those obtained using Eq. (38) for the complete range of the coupling parameter B. The ratio, $B^2/AEGJ$, is less than unity for physical reality.

Conclusions

From this study, several conclusions can be made. These are:

(*1*) Axial-torsional coupling does not influence the frequency of vertical oscillation whereas it reduces the frequency of torsional oscillation and increases the frequency of oscillation in the span direction.

(*2*) Equation (38) may be used for finding the coupled torsional natural frequencies while designing electrical power transmission cables.

(*3*) Sag-to-span ratio does not have an appreciable effect on torsional frequencies as may be deduced from Eq. (38). The small changes in torsional frequencies for changes in sag-to-span ratio are due to corresponding small changes in free length.

(*4*) The assumption of negligible motion along the span direction is valid.

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