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STATISTICAL ANALYSIS OF OCEAN ENVIRONMENTAL CONDITIONS WITH PTGEVD

Sheng Dong
College of Engineering
Ocean University of China
Qingdao 266071, China
Email: dongsh@mail.ouc.edu.cn

Xiaoli Hao
College of Engineering
Ocean University of China
Qingdao 266071, China
Email: haoxlly@yahoo.com.cn

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ABSTRACT

Poisson Trivariate Gumbel Extreme Value Distribution (PTGEVD), a multivariate from of the Compound Extreme Value Distribution, is presented to solve for the ocean environmental design criteria in this paper. The proposed model is combined with a discrete distribution of storm frequency and a continuous trivariate extreme value distribution of environmental conditions simultaneously occurred in storm processes. Different from traditional univariate design method, the proposed design method with PTGEVD can reflect the combined effect of multi-loads on offshore structures and result in reasonable reduction of the design criteria. Validated with the synchronically measured significant wave heights, wind speeds and current velocities of 20 typhoon processes, PTGEVD model shows that it is easy to be applied and has considerable economic potential in the exploitation of ocean oil and gas, especially for marginal field.

INTRODUCTION

The long-term prediction of extreme ocean environments is of importance in the design of marine structures. Ignoring the joint probability of concurrent environmental conditions, the traditional univariate method failed to consider the complexity, variation and randomness of the ocean environments and led to overestimation of the design criteria, which consequently caused unnecessary overspend in practice and had even been preventing the marginal oil fields from further exploit. In the last two decades, the combined effects of multivariable environmental loads on maritime structures have been receiving massive attention from academic circles and standard-drawn up societies[1-11]. Validated with hindcast data, this paper presents a Poisson Trivariate Gumbel Extreme Value Distribution, which is hereafter referred to PTGEVD, to account for the joint probability of concurrent significant wave height, wind speed and current velocity during typhoon processes. This model is developed on the basis of the theories of univariate and bivariate compound extreme value distributions. It accounts for the characteristics of typhoon processes as well as their occurrence frequencies. The design parameters obtained using different statistical models are compared through several test problems.

TRIVARIATE COMPOUND EXTREME VALUE DISTRIBUTION

GENERAL EXPRESSION OF PTGEVD

Let variable n denotes the occurrence frequency of a typhoon and Poisson distribution P_k describes its distribution. The values of typhoon-induced significant wave height and its associated wind speed and current velocity are denoted as (ξ_1, ξ_2, ξ_3), and (ζ_1, ζ_2, ζ_3) representing their corresponding values in non-typhoon year. Assuming (ξ_1, ξ_2, ξ_3) and (ζ_1, ζ_2, ζ_3) are both three-dimension continuous random vectors, their joint cumulative distributions can be respectively registered as $G(x_1, x_2, x_3)$ and $Q(x_1, x_2, x_3)$. The joint probability density function of (ξ_1, ξ_2, ξ_3) is $g(x_1, x_2, x_3)$ and $G_{x1}(x_1)$ is the distribution of ξ_1. It is convenient to assign (ξ_{1i}, ξ_{2i}, ξ_{3i}) as the ith independent observation value of (ξ_1, ξ_2, ξ_3). As we can see, n is a random non-negative integer independent from (ξ_1, ξ_2, ξ_3) and (ζ_1, ζ_2, ζ_3). Its distribution function can be written as

$$\begin{cases} P\{n=k\} = P_k, & k = 0, 1, \cdots \\ \sum P_k = 1 \end{cases} . \quad (1)$$

Defining a random vector (X_1, X_2, X_3) as

$$(X_1,X_2,X_3)=\begin{cases}(\zeta_1,\zeta_2,\zeta_3), & n=0\\ (\xi_{1j},\xi_{2j},\xi_{3j})\,|\,\xi_{1j}=\underset{1\le i\le n}{Max}\,\xi_{1i}, & n\ge 1\end{cases}, \quad (2)$$

we claim the function

$$\begin{aligned}&F(x_1,x_2,x_3)\\&=P_0+\sum_{k=1}^{\infty}P_k\cdot k\cdot\int_{-\infty}^{x_3}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}G_{x_1}{}^{k-1}(u_1)\cdot g(u_1,u_2,u_3)\,du_1du_2du_3\end{aligned} \quad (3)$$

as PTGEVD, which includes the discrete distribution of n and the continuous distribution of (ξ_1,ξ_2,ξ_3). The detailed derivation of PTGEVD is given as follows.

From Eq.(2), the joint distribution of (X_1, X_2, X_3) can be obtained, namely

$$\begin{aligned}&F(x_1,x_2,x_3)=P(X_1<x_1,X_2<x_2,X_3<x_3)\\&=P(\bigcup_{k=0}^{\infty}\{X_1<x_1,X_2<x_2,X_3<x_3\}\cap\{n=k\})\\&=\sum_{k=0}^{\infty}P(X_1<x_1,X_2<x_2,X_3<x_3\,|\,n=k)\cdot P(n=k)\\&=\sum_{k=0}^{\infty}P_kP(X_1<x_1,X_2<x_2,X_3<x_3\,|\,n=k)\\&=P_0\cdot Q(x_1,x_2,x_3)+\sum_{k=1}^{\infty}P_k\cdot P(X_1<x_1,X_2<x_2,X_3<x_3\,|\,n=k)\end{aligned} \quad (4)$$

where

$$\begin{aligned}&P(X_1<x_1,X_2<x_2,X_3<x_3\,|\,n=k)\\&=P(\bigcup_{i=1}^{k}\{X_1<x_1,X_2<x_2,X_3<x_3\}\cap\{\underset{1\le j\le k}{Max}\,\xi_{1j}=\xi_{1i}\}\,|\,n=k)\\&=\sum_{i=1}^{k}P(\{X_1<x_1,X_2<x_2,X_3<x_3\}\cap\{\underset{1\le j\le n}{Max}\,\xi_{1j}=\xi_{1i}\}\big|n=k)\\&=kP(\{X_1<x_1,X_2<x_2,X_3<x_3\}\cap\{\underset{1\le j\le n}{Max}\,\xi_{1j}=\xi_{11}\}\big|n=k)\\&=k\cdot P(\xi_{11}<x_1,\xi_{21}<x_2,\xi_{31}<x_3,\xi_{11}>\xi_{1j},j=2,3,\cdots\big|n=k)\\&=k\cdot\int_{-\infty}^{x_3}\int_{-\infty}^{x_2}\int_{-\infty}^{x}G_{x_1}(u_1)\cdot g(u_1,u_2,u_3)du_1du_2du_3\end{aligned} \quad . (5)$$

Substituting Eq.(5) into Eq.(4), we obtain

$$\begin{aligned}&F(x_1,x_2,x_3)=P_0\cdot Q(x_1,x_2,x_3)\\&+\sum_{k=1}^{\infty}P_k\cdot k\cdot\int_{-\infty}^{x_3}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}G_{x_1}{}^{k-1}(u_1)\cdot g(u_1,u_2,u_3)\,du_1du_2du_3\end{aligned} \quad . (6)$$

Let

$$\varepsilon(x_1,x_2,x_3)=P_0(1-Q(x_1,x_2,x_3)), \quad (7)$$

then

$$F_0(x_1,x_2,x_3)=F(x_1,x_2,x_3)+\varepsilon(x_1,x_2,x_3)\,. \quad (8)$$

Substituting Eq.(6) into Eq.(8), we have Eq.(3).

In typical typhoon area, non-typhoon factors generally produce mild sea states of insignificant wave height, wind speed and current velocity. The physical meaning of $(\zeta_1, \zeta_2, \zeta_3)$ indicates that there must exist $(\zeta_{10}, \zeta_{20}, \zeta_{30})$, which satisfies the condition $P\{(\zeta_1>\zeta_{10})\cup(\zeta_2>\zeta_{20})\cup(\zeta_3>\zeta_{30})\}=0$, namely $\varepsilon(x_1, x_2, x_3)=0$. In practical engineering, what we are mostly concerned is the situation when the trivariate cumulative probability $F(x_1, x_2, x_3)$ approaches very closely to 1. Therefore instead of $F(x_1, x_2, x_3)$, we can in turn calculate PTGEVD using $F_0(x_1, x_2, x_3)$ as $\varepsilon(x_1, x_2, x_3)=0$.

The expression of bivariate compound distribution can be derived from Eq.(3) if we integrate variable x_3 from $-\infty$ to $+\infty$

$$F(x_1,x_2)=P_0+\sum_{k=1}^{\infty}P_k\cdot k\cdot\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}G_{x_1}{}^{k-1}(u_1)\cdot g(u_1,u_2)\,du_1du_2 \quad (9)$$

which is developed by previous study[9-10].

In the similar way, Eq.(9) can be simplified in case of univariate distribution, namely

$$\begin{aligned}F(x)&=P_0+\sum_{k=1}^{\infty}P_k\cdot k\cdot\int_{-\infty}^{x}G_x{}^{k-1}(u)\cdot g(u)\,du\\&=P_0+\sum_{k=1}^{\infty}P_k\cdot G_x{}^{k}(u)\end{aligned}, \quad (10)$$

which is the formula of univariate compound extreme distribution proposed by former researches[12-14].

PTGEVD MODEL

In Eq.(3), if the occurrence frequency of typhoon n satisfies Poisson distribution and the continuous distribution $G(x_1, x_2, x_3)$ follows Trivariate Gumbel Extreme Value Distribution[15], then PTGEVD, one specific form of Multivariate Compound Extreme Distribution, can be derived.

As n follows Poisson distribution

$$P_k=\frac{e^{-\lambda}\lambda^k}{k!}, \quad (11)$$

the following form of Eq.(3) is then yielded:

$$\begin{aligned}&F(x_1,x_2,x_3)\\&=\mathrm{e}^{-\lambda}+\int_{-\infty}^{x_3}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}\sum_{k=1}^{\infty}\frac{\mathrm{e}^{-\lambda}\lambda^k}{k!}\cdot kG_{x_1}(u_1)^{k-1}g(u_1,u_2,u_3)du_1du_2du_3\end{aligned} \quad . (12)$$

Let $m=k$-1, then

$$F(x_1,x_2,x_3)=\mathrm{e}^{-\lambda}(1+\lambda\int_{-\infty}^{x_3}\int_{-\infty}^{x_2}\int_{-\infty}^{x_1}\mathrm{e}^{\lambda\cdot G_{x_1}(u_1)}g(u_1,u_2,u_3)du_1du_2du_3)\,. \quad (13)$$

Assuming that trivariate continuous distribution $G(x_1,x_2,x_3)$ follows Trivariate Gumbel distribution, the accumulative distribution can be expressed as

$$G(x_1,x_2,x_3)=\exp\left\{-\left[\sum_{i=1}^{3}\exp\left(-\frac{x_i-\mu_i}{\alpha\sigma_i}\right)\right]^{\alpha}\right\}, \quad (14)$$

where μ_i, σ_i, i=1,2,3 are respectively the location parameter and scale parameter of the marginal distribution of (ξ_1, ξ_2, ξ_3). The dependent parameter α can be estimated by[15]

$$\alpha = \frac{1}{3}\left(\sqrt{1-r_{12}} + \sqrt{1-r_{23}} + \sqrt{1-r_{31}}\right), \tag{15}$$

where r_{ij} is the correlation coefficient between each X_i and X_j. For notational convenience, put

$$\begin{cases} y_i = \exp\left(-\dfrac{x_i - \mu_i}{\sigma_i}\right), i = 1,2,3 \\ z = \left(\displaystyle\sum_{i=1}^{3} y_i^{\frac{1}{\alpha}}\right)^{\alpha} \end{cases}. \tag{16}$$

The joint probability density function is then simplified as

$$g(x_1, x_2, x_3) = \frac{y_1 y_2 y_3}{\sigma_1 \sigma_2 \sigma_3} z^{1-\frac{3}{\alpha}} \exp(-z) \cdot \left[z^2 + 2(\frac{1}{\alpha}-1)z + (\frac{1}{\alpha}-1)(\frac{2}{\alpha}-1)\right]. \tag{17}$$

Substitution of Eq.(17) into Eq.(13) finally results in the calculation of PTGEVD.

CASE STUDY OF PTGEVD

DISTRIBUTION OF TYPHOON OCCURRENCE FREQUENCY AND MARGINAL DISTRIBUTIONS OF H_S, W AND C

34-year of hindcast data was collected from South China sea from 1953 to 1986 and used to test PTGEVD model. The three series of extreme significant wave height (H_s), wind speed (W) and current velocity (C) contribute to the maximum overturning moment of the structure during every typhoon process. As aforementioned, the occurrence frequency of the typhoon process can be fitted with Poisson distribution, which is shown in Fig.1. This null hypothesis, regarding the comparison with the critical value in a Chi-Square test, is accepted at a level of significance level.

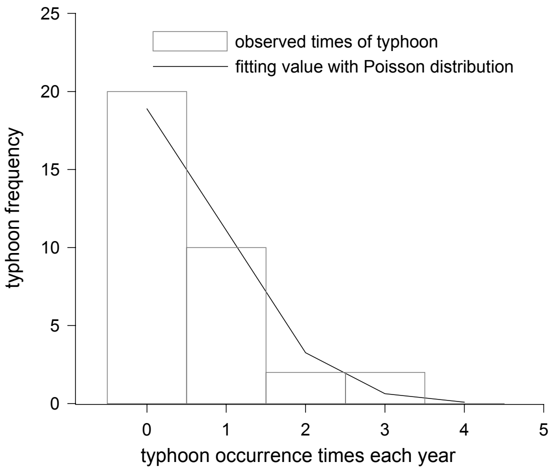
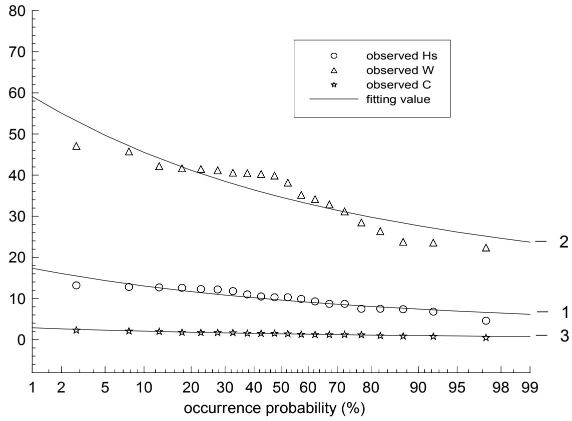


Fig.1 Poisson distribution fitting of typhoon occurrence

Similarly, K-S tests indicate that Gumbel distribution is eligible to define the marginal distributions of H_s, W and C them all, as depicted in Fig.2. The different return values of are estimated with Poisson Gumbel distribution and shown in Table 1.



1- H_s(m) 2- W(m/s) 3- C(m/s)

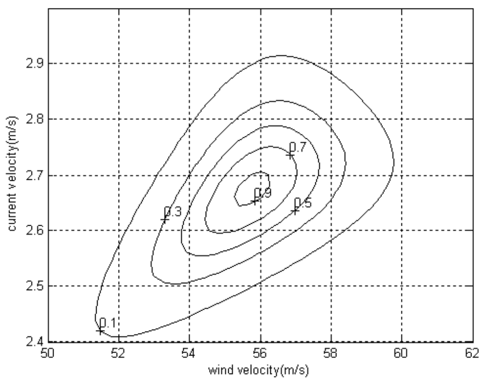
Fig.2 Marginal distribution fitting of H_s, W and C

Table 1 Return values of H_s, W and C

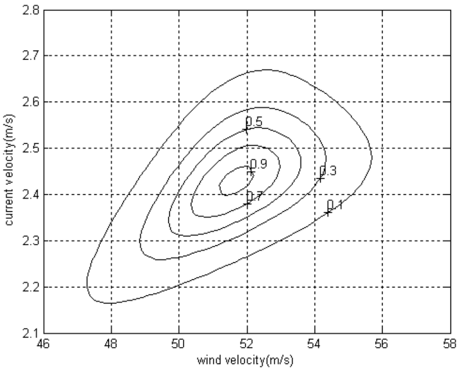
Return period (year)	Return values		
	H_s (m)	W (m/s)	C (m/s)
100	16.36	56.00	2.688
50	15.07	51.91	2.443
20	13.32	46.36	2.110
10	11.91	41.90	1.843
5	10.30	36.80	1.537

CONDITIONAL PROBABILITY DENSITY VALUES

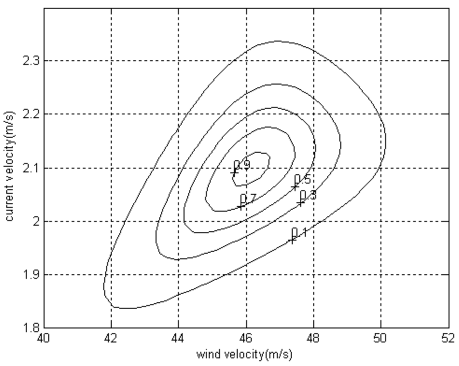
Registering 100-yr, 50-yr, 20-yr, 10-yr and 5-yr return period significant wave heights as $(H_s)_{1\%}$, $(H_s)_{2\%}$, $(H_s)_{5\%}$, $(H_s)_{10\%}$, and $(H_s)_{20\%}$ respectively and substituting them into Eq.17, the conditional PDF of W and C can be obtained and their corresponding contours are depicted in Fig.4.



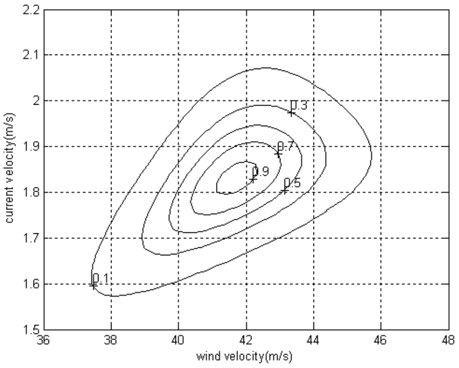
(a) $(H_s)_{1\%}$



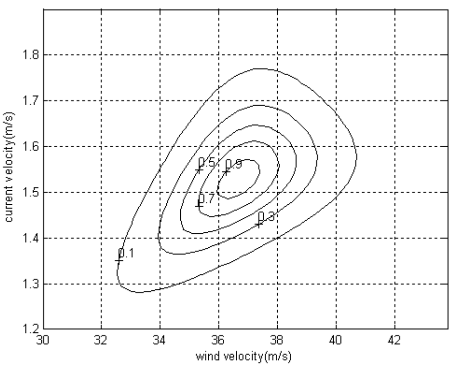
(b) $(H_s)_{2\%}$



(c) $(H_s)_{5\%}$



(d) $(H_s)_{10\%}$



(e) $(H_s)_{20\%}$

Fig.4 Conditional joint probability density contours of W and C concomitant with given H_s

The centers of contour lines are respectively the most likely values of W and C associated with different H_s, as shown in Table 2.

Table 2 Conditional joint probability of W and C associate with different return value of H_s

Different return value of H_s		Most likely values of W and C		Joint probability of H, W, and C (%)
H_s (m)	T (a)	W (m/s)	C (m/s)	
16.36	100	55.77	2.675	0.84
15.07	50	51.69	2.430	1.68
13.32	20	46.16	2.098	4.21
11.91	10	41.72	1.832	8.52
10.30	5	36.68	1.529	17.48

CALCULATION OF JOINT PROBABILITY WITH PTGEVD

For any given return period, the environmental design conditions are traditionally determined through univariate statistical analysis, such as the 100-yr wave height, 100-yr wind speed, and 100-yr current velocity. Apparently the return period of the concurrent state of all three extreme conditions will be larger than 100yr. The overestimation of the design parameters with univariate analysis results in massive increase of the construction investment. Figure 5 shows the joint probability of different load combinations, in which the univariate occurrence probabilities of H_s, W and C are the same. From Fig.5, we can see that the joint probability of given 100-yr H_s, 100-yr W and 100-yr C is about 0.82%, which is, though less than 1%, however much larger than 0.0001%. The similar results can be obtained when H_s, W and C are simultaneously given as 50-yr, 20-yr, 10-yr, and 5-yr return values respectively. This implies that univariate design method leads to much more conservative design criteria than multi-variate method.

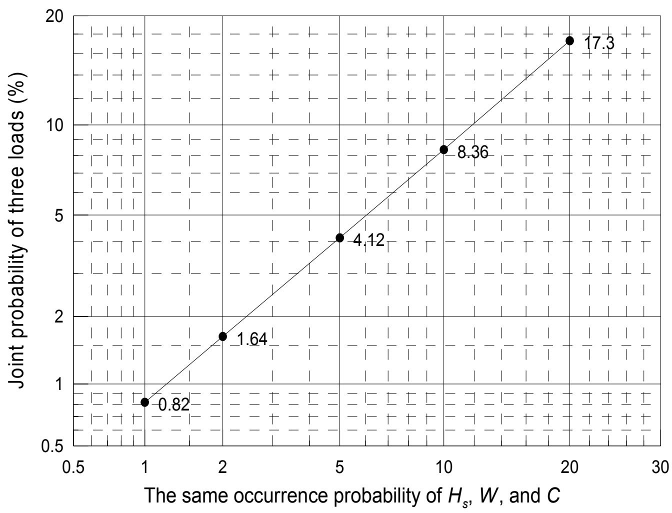


Fig.5 Joint probabilities of load combinations

RESULTS COMPARISON AND DISCUSSION

Comparisons are made among three methods to determine the design criteria with a 100-yr return level (see Table 3).

(1) Univariate design method. This statistical model uses univariate compound extreme value distribution. Sampling data of H_s, W and C as observed series, the 100-yr significant wave height combined with the 100-yr wind speed and the 100-yr current velocity can be estimated with statistical analysis. Actually the joint probability of these three loads is much less than 1%.

(2) The first method with the proposed PTGEVD. The most probably occurred combination of wind speed and current velocity associated with 100-yr return period wave height is shown in Table 3. Their joint occurrence probability is also less than 1%.

(3) The second method with the proposed PTGEVD. One possible combination by Eq.(13) is obtained through trial and error method. The joint probability of calculated H_s, W and C is 1%.

Table 3 Comparison of design parameters by 3 types of methods

Design method	Statistical model	Load combination			Joint probability (%)
		H_s (m)	W (m/s)	C (m/s)	
(1)	Poisson-Gumbel	16.36	56.00	2.688	0.82
(2)	PTGEVD	16.36	55.77	2.675	0.84
(3)	PTGEVD	16.10	55.20	2.100	1.00

CONCLUDING REMARKS

On the basis of measured data obtained during 34-year typhoon processes in the South China Sea, this paper presents the multi-variate formulation of the Compound Extreme Value Distribution to promote the statistical computation of extreme ocean environmental conditions. Taking account of the joint distribution of multi-dimensional extreme environment factors as well as the typhoon frequency, the new model, PTGEVD, describes the probability characteristics in more aspects and enriches the physical meaning of the extreme statistical modules as it. Compared with traditional univariate design method, the proposed design method with PTGEVD shows better ability of reflecting the combined effects of multi-loads on offshore structures and results in reasonable reduction of the existing design criteria, which may significantly lessen the unnecessary investments. Case study shows that this statistical model is ease to operate and has considerable potential economic benefits in the exploitation of ocean oil and gas.

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