Flutter of aircraft panels at low supersonic flight speeds

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Abstract

Flutter of panels can be of two possible types: single mode or coupled mode type of flutter. Coupled mode flutter has been thoroughly studied using piston theory, which represents air pressure acting on the plate. Single mode flutter cannot be studied using piston theory and requires potential flow theory or more complex aerodynamic theories. This type of flutter occurs at low supersonic Mach numbers and is studied insufficiently. In this paper a comprehensive numerical investigation of single mode flutter is conducted. Flutter boundaries and their transformations with change of the problem parameters are studied.

1. Introduction

Panel flutter is a phenomenon of self-exciting vibrations of skin panels of flight vehicle at high flight speeds. Such vibrations typically have high amplitude and cause fatigue damage of skin panels. From mathematical point of view, panel flutter problem can be studied through spectral problem for equation of plate motion in a gas flow:

$$\mathcal{D}\frac{\partial^4 w}{\partial x^4} + \rho_m h\frac{\partial^2 w}{\partial t^2} + p(x,t) = 0, \tag{1}$$

supplement with appropriate boundary conditions at $x = 0, \mathcal{L}$. Here $\mathcal{D}$, $\mathcal{L}$ and h are plate stiffness, length and thickness, ρ_m is plate material density, p is unsteady gas pressure caused by plate deflection. The pressure obtained from potential gas flow theory has form of a complex integro-differential operator of deflection w (see formula (5) below); however, in 1956 a simple approximation of the pressure as $M \to \infty$, called "piston theory", was derived:

$$p(x,t) = \frac{\rho u}{\sqrt{M^2-1}}\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x}\right), \tag{2}$$

where ρ, u and M are the flow density, speed and Mach number. The problem (1), (2) is a partial-differential equation, which, with appropriate boundary conditions, can be studied analytically or numerically [1–4]. A lot of studies conducted since piston theory was obtained, deal with different complications of the elastic part of the problem: nonlinear plate models, plates made of composite materials and shape memory alloys, chaotic vibrations of buckled and pre-stressed plates, etc (see detailed review [5]). However, aerodynamic part of the problem, piston theory (2), typically was not changed due to high complexity of the exact potential flow theory.

Initially piston theory was obtained as asymptotic expansion of the exact expression as $M \to \infty$, later it has been shown that it is valid starting from $M \approx 1.7$ [4]. Thus, it does not cover the range of low supersonic Mach number, $1 < M < 1.7$. Few papers among large amount of publications were devoted to study of panel flutter problem using potential flow theory or more complex theories: [6–11]. It was noticed that at low supersonic speeds single mode flutter (also know as single degree of freedom flutter) can occur, in contrast with coupled-mode type flutter (also referred as coalescence flutter) arising at higher M and studied through piston theory. Typical explanation of single mode flutter is "negative aerodynamic damping": expansion of pressure derived from potential flow theory at low frequencies yields

$$p(x,t) = \frac{\rho u}{\sqrt{M^2-1}}\left(\frac{M^2-2}{M^2-1}\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x}\right) + o(\omega^2). \tag{3}$$

Coefficient of $\partial w/\partial t$, which represents aerodynamic damping, is negative if $M < \sqrt{2}$, and in this case (3) always predicts single mode instability. However, this way to represent single mode flutter is simplistic and contradictory: (3)

predicts flutter in all the plate modes, for any (even very short) plates, for any in-plane plate loads. Detailed study of single mode flutter through potential flow theory or more complex theories has never been conducted, mainly because of high complexity of the mathematical problem and conviction of researchers that this type of flutter is weak and cannot occur in a real structure.

Nevertheless, over the last years in [12, 13] single mode flutter has been studied using asymptotic theory of global instability [14, 15], never used before in aeroelasticity. Asymptotic flutter criterion has been derived as plate length $\mathcal{L} \to \infty$, and nature of energy transfer mechanism between the plate and the flow has been studied. Later it has been proved that this theory works starting from $\mathcal{L} \approx 100h$, i.e. for quite short plates. Theoretical study of limit cycle amplitude of single mode flutter showed that increase of amplitude while entering flutter region in parameter space is much more rapid than for coupled-mode type of flutter. Recent experimental study [16] has confirmed that single mode type of flutter can occur in real structures.

In this paper we conduct comprehensive numerical study of 2-dimensional panel flutter problem using potential gas flow theory. From mathematical point of view, this problem is equivalent to those considered in [6, 9]. However, we are going to study several important features of single mode flutter not noticed before and discovered in [12], which require several modes taken in account and more variation of parameters. Among such features are: independence of single mode flutter boundaries on the gas density (while coupled mode flutter boundaries depend), loss of stability in several modes simultaneously, and existence of asymptotic flutter boundaries for very long plates.

2. Formulation of the problem

We consider linear stability of elastic plate in a uniform gas flow. Elastic properties of the plate are defined by parameters $\mathcal{D}$, $\mathcal{L}$, h, and ρ_m, defined above; flow parameters are: the flow speed u, speed of sound a, and the flow density ρ. Corresponding dimensionless parameters are:

$$D = \frac{\mathcal{D}}{a^2\rho_m h^3}, \quad L = \frac{\mathcal{L}}{h}, \quad M = \frac{u}{a}, \quad \mu = \frac{\rho}{\rho_m}$$

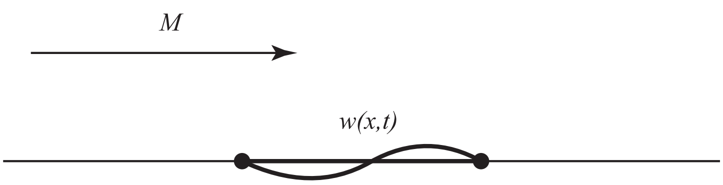

Figure 1: Plate in supersonic gas flow

The plate is mounted into an infinite absolutely rigid plane. At another side of the plane a constant pressure equal to undisturbed flow pressure, is applied (Fig. 1). Using the dimensionless variables, rewrite (1):

$$D\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} + p(x, t) = 0,$$

where dimensionless plate deflection is dimensional deflection rated to h.

Consider harmonic motion of the plate, $w(x, t) = W(x)\exp(-i\omega t)$, then we obtain:

$$D\frac{\partial^4 W}{\partial x^4} - \omega^2 W + p\{W, \omega\} = 0. \tag{4}$$

From potential gas flow theory [4], gas pressure has the form:

$$p\{W, \omega\} = \frac{\mu M}{\sqrt{M^2 - 1}}\left(-i\omega W(x) + M\frac{\partial W(x)}{\partial x}\right) + \\ + \frac{\mu\omega}{(M^2 - 1)^{3/2}}\int_0^x \left(-i\omega W(\xi) + M\frac{\partial W(\xi)}{\partial \xi}\right) \cdot \exp\left(\frac{iM\omega(x - \xi)}{M^2 - 1}\right)\left(iJ_0\left(\frac{-\omega(x - \xi)}{M^2 - 1}\right) + MJ_1\left(\frac{-\omega(x - \xi)}{M^2 - 1}\right)\right)d\xi. \tag{5}$$

We will consider plates simply supported at both edges, therefore:

$$W = \frac{\partial^2 W}{\partial x^2} = 0, \qquad x = 0, \quad x = L \tag{6}$$

The problem (4), (5), (6) is an eigenvalue problem of the plate in the gas flow. If some eigenfrequency ω_n has positive imaginary part, then the plate flutters; if $\operatorname{Im}\omega_n \leq 0$ for any n, then the plate is stable.

Piston theory (2) can be obtained by omitting intergal terms in (5):

$$p\{W, \omega\} = \frac{\mu M}{\sqrt{M^2 - 1}} \left(-i\omega W(x) + M \frac{\partial W(x)}{\partial x} \right). \tag{7}$$

We will use piston theory (7) for comparison with results obtained through potential flow theory (5).

3. Numerical method

3.1 Formulation of the numerical procedure

We will use Bubnov-Galerkin method for analysis of the problem. Basic functions are natural modes of the plate in vacuum:

$$W(x) = \sum_{n=1}^{N} C_n W_n(x), \qquad W_n(x) = \sin\left(\frac{n\pi x}{L}\right),$$

where C_n are unknown constants. Substitute this expression into (4), multiply by $W_m(x)$, $m = 1 \ldots N$, and integrate from 0 to L, we obtain a homogeneous system of algebraic equations with unknowns C_n. Frequency equation takes the form

$$\det \mathbf{A}(\omega) = \det\left(\mathbf{K} + \mathbf{P}(\omega) - \frac{L\omega^2}{2}\mathbf{I} \right) = 0, \tag{8}$$

where $\mathbf{K}$ is diagonal stiffness matrix, $\mathbf{P}$ is aerodynamic force matrix with coefficients

$$p_{jn}(\omega) = \int_0^L P\{W_n, \omega\} \cdot W_j \, dx, \tag{9}$$

$\mathbf{I}$ is the unit matrix. Matrix $\mathbf{P}(\omega)$ is complex and non-symmetric, with all coefficients non-zero. Therefore the problem is not self-adjointed, and all solutions of (8) are complex.

Iterative method is used for solving (8). Assume that we want to calculate n-th eigenfrequency ω_n. As initial condition, we use natural frequency of the plate in vacuum $\omega_n^0 = \sqrt{D}(n\pi/L)^2$. Next, let us have p-th approximation ω_n^p. Define matrix $\mathbf{A}_{p+1}(\omega_n^p, {\omega'}_n^{p+1})$ as follows, where ${\omega'}_n^{p+1}$ is "auxiliary" value used to calculate ω_n^{p+1} as defined below. All coefficients a_{jk}, except a_{nn}, are the same as of matrix $\mathbf{A}(\omega_n^p)$, while for a_{nn} we use the following formula:

$$a_{nn} = k_{nn} + p_{nn}(\omega_n^p) - \frac{L({\omega'}_n^{p+1})^2}{2} \tag{10}$$

where k_{nn} and p_{nn} are appropriate coefficients of matrixes $\mathbf{K}$ and $\mathbf{P}$.

Equation for ${\omega'}_n^{p+1}$ is

$$\det \mathbf{A}_{p+1}(\omega_n^p, {\omega'}_n^{p+1}) = 0.$$

It is linear with respect to $({\omega'}_n^{p+1})^2$; among two values of ${\omega'}_n^{p+1}$ we choose the one with positive real part: $\operatorname{Re}{\omega'}_n^{p+1} > 0$.

Finally we calculate $p+1$-th approximation of ω_n as follows:

$$\omega_n^{p+1} = (1 - \varkappa){\omega'}_n^{p+1} + \varkappa\omega_n^p,$$

where $\varkappa$ is relaxation coefficient, $0 \leq \varkappa < 1$.

Iterative procedure described typically converges (see Section 3.2 below for details). However, at high M and L, when coupled mode type of flutter is the primary type of instability, such a procedure can diverge. This happens when the first and the second eigenfrequencies are close to each other and almost coalesce, and the numerical solution at each iteration "jumps" from one value to another. In this case expression (10) was used for both a_{11} and a_{22}. Hence,

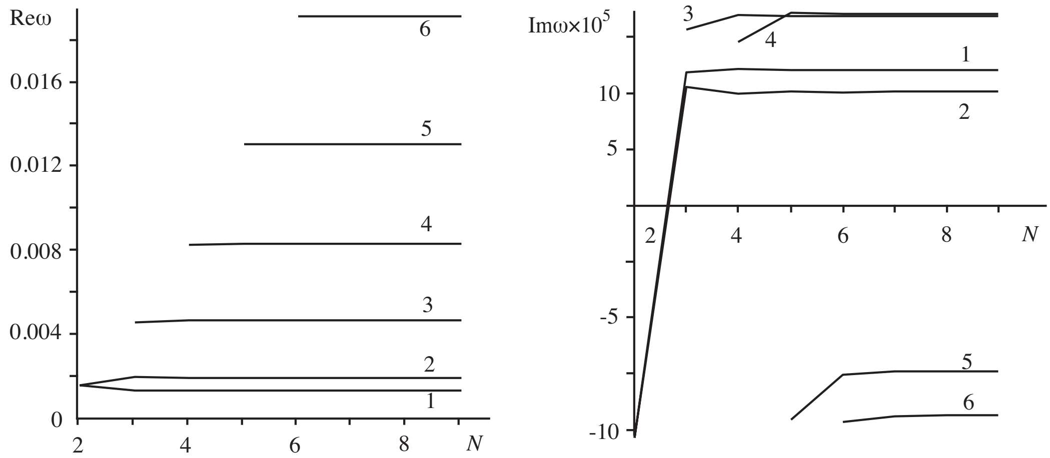

Figure 2: Convergence of ω_n iterations (n = 1...6) with increase of number of basic modes N for parameters D = 23.9, M = 1.2, $\mu = 12 \cdot 10^{-5}$, L = 300

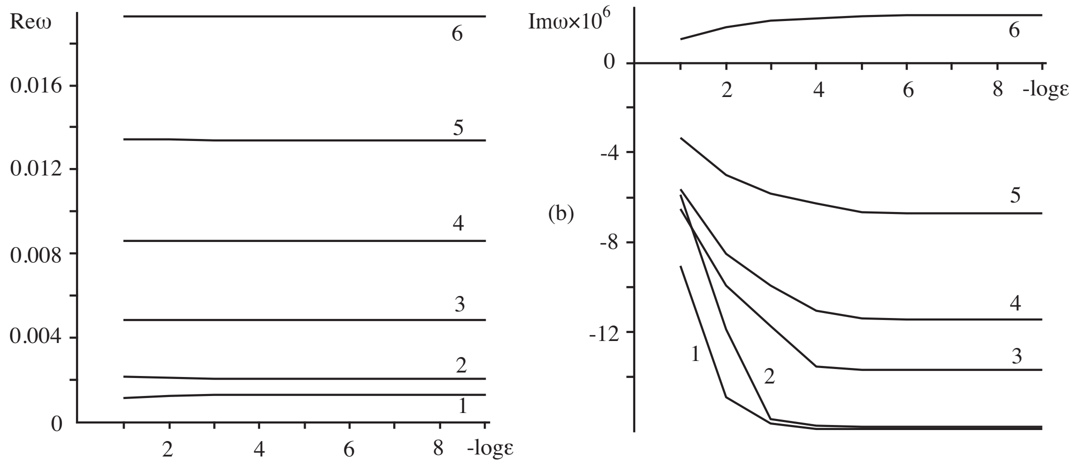

Figure 3: Convergence of ω_n iterations (n = 1...6) with decrease of $\varepsilon_1 = 10^{-1}...10^{-9}$ for parameters D = 23.9, M = 1.5, $\mu = 12 \cdot 10^{-5}$, L = 300.

equation for ${\omega'}_n^{p+1}$, $n = 1, 2$ is quadratic with respect to $({\omega'}_n^{p+1})^2$ and gives both first and second eigenvalues, which allows to easily distinguish them and follow the same eigenvalue at each iteration.

Iterations for ω_n continue until the following condition is satisfied:

$$\frac{\omega_n^p - \omega_n^{p-1}}{\omega_n^p} < \varepsilon_1$$

3.2 Convergence of the numerical procedure

Now we will study convergence of the numerical method described. First, let us consider number of basic functions, N, necessary to obtain accurate solution. In Fig. 2 shown are real and imaginary parts of the first six eigenfrequencies for D = 23.9, M = 1.2, $\mu = 12 \cdot 10^{-5}$, L = 300 (these parameters correspond to a steel plate in air flow at 3000 m above sea level). Here $\varepsilon_1 = 10^{-5}$ and relaxation coefficient $\varkappa = 0.5$ are used. One can see that $N = n + 1$ is enough to calculate ω_n with high accuracy, such that relative error of both $\mathrm{Re}\,\omega_n$ and $\mathrm{Im}\,\omega_n$ is less than 1%. In this paper we will study the first six modes; therefore, N = 7 is enough to get appropriate accuracy. This stays true for any medium-size plates,

$L < 600$. For higher L more modes are necessary. For the first six modes, $N = 9$ gives better solution for $L \approx 600$, $N = 11$ for $L \approx 700$, and $N = 13$ for $L \approx 800$.

In Fig. 3 calculated ω_n versus $\varepsilon_1 = 10^{-1}...10^{-9}$ are shown, $\varkappa = 0.5$. We can see that $\varepsilon_1 = 10^{-5}$ is enough to get relative error less than 1% for $n = 1...6$. We will use this value in all calculations.

4. Results

4.1 General situation

Analysis is conducted for a steel plate in air flow at 3000 m above sea level. Dimensionless parameters are

$$D = 23.9, \quad \mu = 12 \cdot 10^{-5} \tag{11}$$

Flutter boundaries obtained through exact potential flow theory are shown in Fig. 4 (dashed curves for parameters (11)). Each eigenmode has its own flutter region in $M - L$ plane. With increase of L, critical Mach numbers $M_n^*(L)$, $M_n^{**}(L)$ (upper and lower branches, respectively) decrease and tend to asymptotic values [12]: $M_n^* \to 1$, $M_n^{**} \to \sqrt{2}$ as $L \to \infty$. At certain L (thick dashed curve in Fig. 4) coalescence of the 1st and the 2nd modes and following coupled mode flutter occurs. This, however, does not affect 3rd and higher modes, so that their single mode flutter boundaries extend to higher L region.

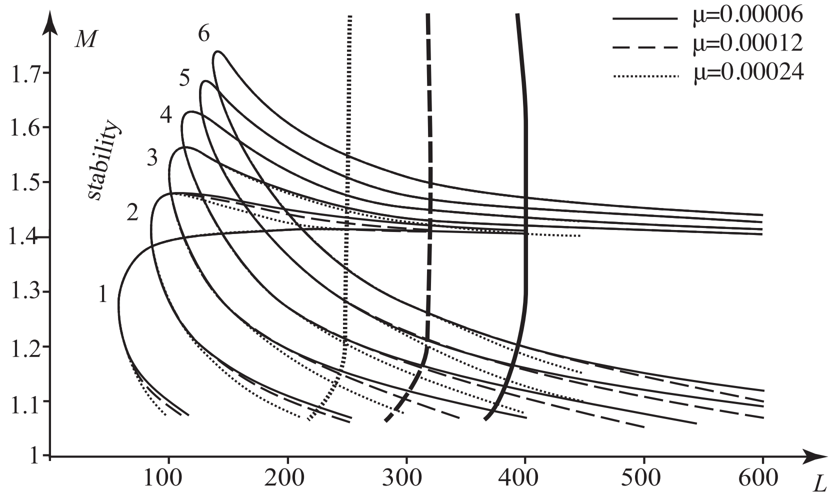

Figure 4: Stability boundaries in $M - L$ parameter plane for $D = 23.9$, $\mu = 6 \cdot 10^{-5}$, $12 \cdot 10^{-5}$, $24 \cdot 10^{-5}$. Thin curves: single mode flutter boundaries for modes 1–6, thick curves: coupled mode flutter boundaries.

In order to understand nature of single mode and coupled mode instabilities, let us consider motion of eigenfrequencies in ω-plane. For simplicity we will watch the first four eigenfrequencies; qualitative behaviour of the higher ones is the same. For $L = 250$, $M = 1.6$ all the first six eigenfrequencies lie in the lower half-plane (Fig. 5), and the corresponding eigenmodes are stable. Keeping L constant, we will decrease M from 1.6 to 1.05. At some M_n^{**} (upper branch in Fig. 4) the n-th eigenfrequency crosses the real ω axis and moves into the upper half-plane. The n-th eigenmode becomes unstable. Important feature of this transition to instability is that the eigenfrequencies do not move towards each other and do not affect each other, that is why we define this transition to instability the "single mode flutter". With further decrease of M, $\operatorname{Im}\omega_n$ get their maxima at decrease. At some $M = M_n^*$ (lower branch in Fig. 4) eigenfrequencies cross the real axis again and move into the lower half-plane; corresponding eigenmodes again become stable. Thus, n-th eigenmode, $n = 1, ..., 4$ is in single mode flutter for $M_n^* \leqslant M \leqslant M_n^{**}$.

Results of calculations using piston theory (7) at $1.05 < M < 1.6$, shown in Fig. 5 by dashed curves, are completely different from results of potential flow theory. There is no single mode transition to instability: all the eigenfrequecies lie in the lower half-plane for any M from the region $1.05 < M < 1.6$. Therefore, in accordance with [12], piston theory "does not see" single mode type of flutter and cannot predict it.

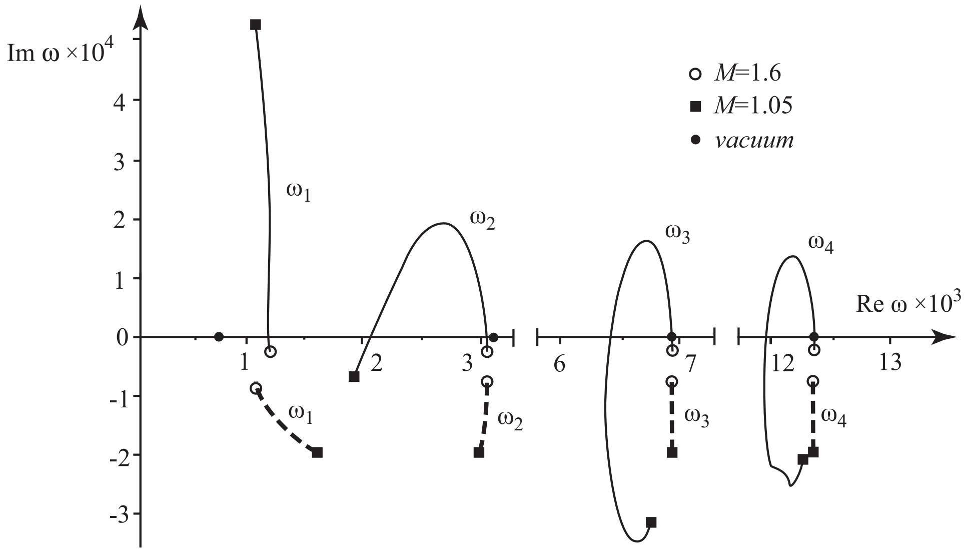

Figure 5: Trajectories of the first four eigenfrequencies in the complex ω plane for $D = 23.9$, $\mu = 12 \cdot 10^{-5}$, $L = 250$, $1.05 \le M \le 1.6$. Solid and dashed curves are results obtained through potential flow theory and piston theory, respectively.

Let us now consider transition from stability to single mode, and then to coupled mode type of flutter, with increase of the plate length. Take $M = 1.3$ and increase L from 60 to 400 (Fig. 6(a)). We observe, first, single mode instability, according to crossing of single mode flutter boundary in Fig. 4. Second, at $L = 320$ coalescence of the first and the second eigenfrequencies occurs (saying "coalescence" hereunder, we mean very close approach: eigenfrequencies do not merge exactly, but pass very close to each other). Immediately after coalescence $\operatorname{Im}\omega_1$ increases very rapidly: from $4.5 \cdot 10^{-5}$ at $L = 320$ to $4.77 \cdot 10^{-4}$ at $L = 400$, while $\operatorname{Im}\omega_2$ decreases: from $2.8 \cdot 10^{-5}$ at $L = 320$ to $-4.08 \cdot 10^{-4}$ at $L = 400$. We define such a behaviour the coupled mode type of flutter. As the coalescence occurs against single mode flutter background (indeed, according to Fig. 4 at $M = 1.3$ and increase of L we are entering coupled mode flutter region directly from single mode flutter region), coalescence occurs in the upper ω-half-plane. Higher eigenfrequencies stay in the upper ω-half-plane without any approach to each other, thus we observe both coupled mode type of flutter at the first two eigenmodes, and single mode type of flutter at higher eigenmodes.

In Fig. 6(b) shown are the same results calculated using piston theory. As before, one can see that there is no single mode instability, but there is coupled mode instability. At that, coupled mode instability occurs at plate length L which is very close to the one obtained through potential flow theory ($L = 325$ vs 320).

Results discussed above concern the first six eigenmodes. From theoretical point of view, we can add single mode flutter boundaries for more and more modes in Fig. 4: according to [12], at least for large L each mode has its own stability boundary $M_n^* \leqslant M \leqslant M_n^{**}$, at that $M_n^* \to \infty$, $M_n^{**} \to \infty$ as $n \to \infty$. However, in reality single mode flutter of very high modes is meaningless, because structural damping not considered in this paper increases unlimited as $n \to \infty$, while $\operatorname{Im}\omega_n$ is limited.

4.2 Influence of μ

As $L \to \infty$, asymptotic single mode flutter boundaries [12] do not depend on μ. As $\mu \to 0$, $\operatorname{Im}\omega_n$ tends to zero but keeps its sign. In other words, if for some parameters the plate is unstable due to single mode flutter, then for the same parameters but very small μ the plate stays unstable, but eigenmodes growth rate tends to zero. On the contrary, influence of μ on coupled mode flutter is essential. Indeed, coupled mode type of flutter occurs due to interaction of two eigenmodes through aerodynamic coupling. This interaction in its turn requires sufficiently high air density, such that air is "strong" enough to essentially move the plate natural frequencies in ω-plane. Thus, smaller μ should result in increase of critical Mach number and plate length.

Results of calculations fully confirm these considerations. In Fig. 4 shown are flutter boundaries for $\mu = 6 \cdot 10^{-5}$, $12 \cdot 10^{-5}$, $24 \cdot 10^{-5}$, variation of μ corresponds to the following change of the plate material and air densities (from the lowest to the largest μ): steel plate in the air flow at 10000 m above sea level, steel at 3000 m, titanium at 3000 m.

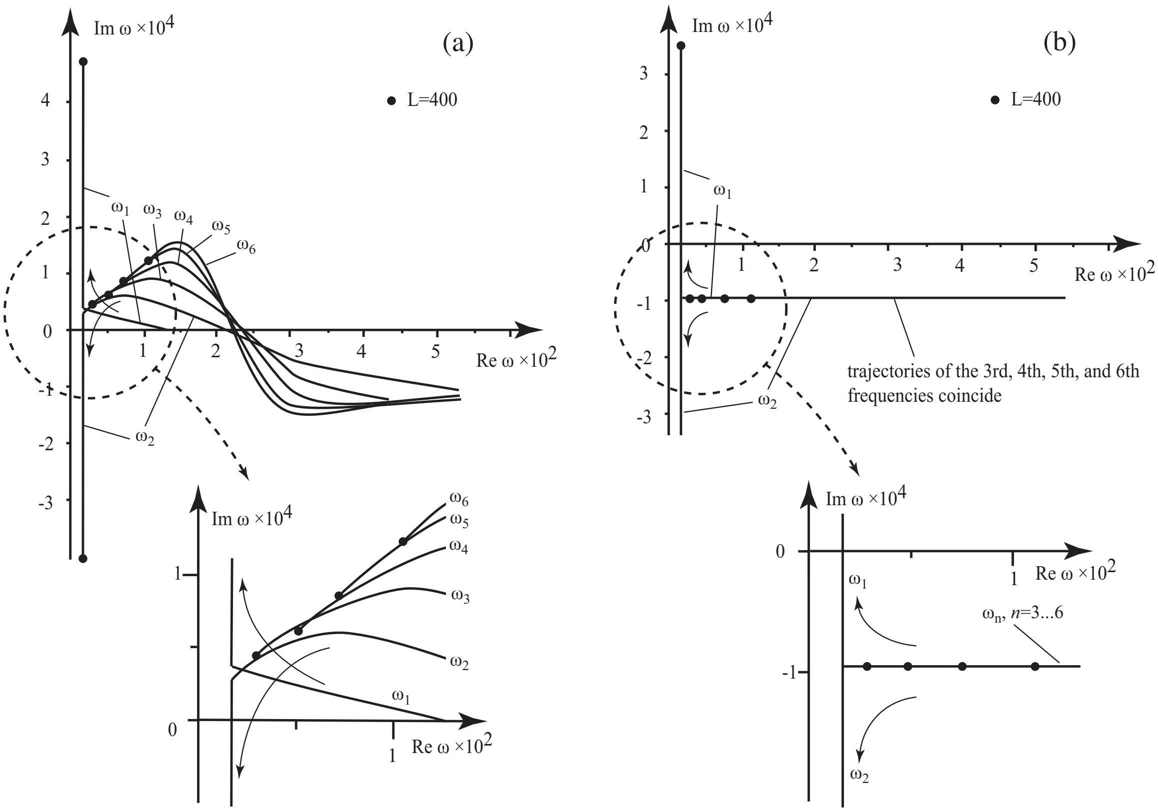

Figure 6: Trajectories of the first six eigenfrequencies in the complex ω plane for $D = 23.9$, $M = 1.3$, $\mu = 12 \cdot 10^{-5}$, $60 \leq L \leq 400$. (a) results obtained through potential flow theory; (b) results obtained through piston theory.

Comparing results for these μ in Fig. 4, one can see that difference in single mode flutter boundaries at $M > 1.25$ is negligible. At $M < 1.25$ weak influence of μ on lower branch of the flutter boundaries appears: lower M_n^* correspond to higher μ. However, even though μ affects flutter boundaries at $M < 1.25$, the difference is still minor.

For coupled mode type of flutter, μ is a significant parameter of the flutter boundary. In accordance with considerations above, lower μ for the same L gives less aerodynamic coupling of the modes, which results in higher L necessary for coupled mode type of flutter. For $\mu \to 0$ and fixed L, mode coupling disappears (actually moving to much higher M), and the plate becomes stable for any fixed M (with respect to coupled mode, but not always to single mode flutter).

5. Conclusions

In this paper flutter boundaries of simply supported plates have been studied using potential flow theory. Two types of flutter have been observed: coupled mode and single mode flutter. Coupled mode flutter boundaries are in a full agreement with known results obtained using piston theory for $M \gg 1$.

Another flutter type, single mode flutter, cannot be detected using piston theory. It has been shown that this type of flutter can occur in several modes simultaneously. At that, each mode has its own flutter region in the parameter space. For the 1st mode this region is in $1 < M < \sqrt{2}$ range, however for higher modes single mode flutter occurs at higher Mach numbers, up to $M = 1.73$ for the 6th mode for the parameters studied. Flutter regions of higher modes lie in higher Much number range. Theoretically, for any $M > 1$ one can find a mode of a plate of appropriate length, which flutters at this M, and occurrence of this flutter in reality is limited only by the structural damping of the plate, or its nonlinear properties.

From practical point of view, the most important property of single mode flutter is that it can occur for parameters, where coupled mode type of flutter is impossible: for lower Mach numbers and for shorter plates (Fig. 4). If single mode panel flutter occurs in a flight vehicle, which aeroelastic analysis was conducted through piston theory (which does not detect single mode flutter), it can result in unpredictable fatigue damage and destruction of skin panels.

6. Acknowledgement

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