A NOTE ON A PERIODIC REVIEW INVENTORY MODEL WITH UNCERTAIN DEMAND IN A RANDOM ENVIRONMENT

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Received September 18, 2003

Abstract. This paper is on the analysis of a single product, periodic review inventory model, where the distributions of demands vary with the state of the environment variable. The state of the environment is assumed to follow a discrete-time Markov chain. The optimal inventory policy to minimize the total discounted expected cost is derived via dynamic programming. For the finite-horizon model, we show that an environmental-dependent base-stock policy is optimal, and derive some characteristics of the optimal policy. Under additional conditions, we further derive the monotonicity of the optimal policy.

1 Introduction The inventory control has long focused on managing certain specific types of probability in the demand for the products. But, on the other hand, consumer's liking becomes variously in the real-life. The demand is fluctuated by the economic climate, weather condition, trend of public opinion, and so forth. Mere including a purely random component in the demand process will be impossible to express such situations.

So, in this paper, under the assumption that the environmental process follows a discrete-time Markov chain, we model a single product inventory system of which the distributions of demands depend on environmental fluctuations, and discuss the management policy. We further investigate the effect of the environmental fluctuations on the optimal policy. The main advantage of the Markov chain approach is that it provides a national and flexible framework for formulating various changes described above.

The effect of a randomly changing environment in inventory model received only limited attention in the earlier paper. Kalymon[12] studies a multiple-period inventory model in which the unit cost of the product is determined by a Markov process, and the distribution of demand in each period depends on the current cost. Feldman[8] models the demand environment as a continuous-time Markov chain. The demand is modulated by a compound Poisson process where the parameters are determined by the state of the environment. But he studies only the steady-state distribution of the inventory position. Song and Zipkin[18] present a continuous-review inventory model where the demand process is a Markov modulated Poisson process, and they derive some basic characteristics of the optimal policy and algorithms for computing the optimal policy. In recent articles, Özekici and Parlar[15] develop an infinite-horizon periodic-review inventory model with unreliable suppliers where the demand, supply and cost parameters are influenced by a random environment. Cheng and Sethi[2] analyze the joint promotion-inventory management problem for a single item in the context of Markov decision processes.

The purpose of this paper is to show that the environmental-dependent base-stock policy is optimal, and that the optimal policy have the monotonicity for review periods by analyzing finite-horizon periodic-review inventory model where the demand distribution depend

2000 *Mathematics Subject Classification.* 49J15; 60J10; 90B05; 90C39.
Key words and phrases. Inventory; Dynamic programming; Periodic review models.

on a Markov environmental process. In addition, we derive the monotonicity of the optimal policy for the environmental states by ordering them. In this paper, we focus on the finite-horizon analysis, since it gives us concrete and realistic insights.

This paper is organized as follows; Section 2 presents the formulation of the general problem as a dynamic programming model. Section 3 provides analysis for finite-horizon problem. Section 4 investigates the effect of the environmental fluctuations on the optimal policy. The paper concludes with some final remarks in Section 5.

2 Assumption and Notation In this section, we introduce notations and basic assumptions used throughout the paper:

n : number of periods remaining in the finite-horizon problem;
$E = \{1, 2, \ldots, R\}$: a finite set of possible environmental state;
I_n : the state of the environment observed at the beginning of period n;
$I = \{I_n; n \geq 0\}$: a Markov chain on E;
$P(i,j)$: the transition probability that the environmental state changes from i to j in one period, $i, j \in E$;
D_n : the total demand during period n;
$A_i(z) = P[D_n \leq z | I_n = i]$: the conditional distribution function of D_n when $I_n = i$;
$a_i(z)$: the probability density function corresponding to $A_i(z)$;
X_n : the inventory level observed at the beginning of period n;
$Y_n(i, x_n)$: the order-up-to level if the environmental state is i and the inventory level is x_n at the beginning of period n;
c : a unit ordering cost;
c^0 : a unit ordering cost in period 0;
h : a unit holding cost incurred at the end of period;
p : a unit shortage cost incurred at the end of period;

We assume $A_i(0) = 0,\ a_i(\cdot) > 0$.
To motivate ordering, we assume that $p > c$ as in standard models. Also, we assume that unsatisfied demands are fully backlogged.

Remark 1 *The basic assumption of this model is that the demand distribution at any period depends on the state of the environment at the beginning of that period. Therefore, the decision maker observes both the inventory level and the environmental state to decide on the optimal order quantity which is delivered immediately.*

Remark 2 *The admissibility condition requires that $Y_n(i, x_n) \geq x_n$ since we do not allow for disposing of any inventory without satisfying demand. For any y_n, it is noted that the inventory level X_n is a Markov chain, where*

$$X_{n-1} = x_n + [y_n(i, x_n) - x_n]^+ - D_n,\ n \geq 0.$$

Figure 1 illustrates the behavior of the inventory level.

Now, let $V_i^n(x)$ be the minimum expected total discount cost of operating for n-period with the state of the environment i and the initial inventory level x, under the best ordering decision is used at period n through period 1. Then, a dynamic programming equation

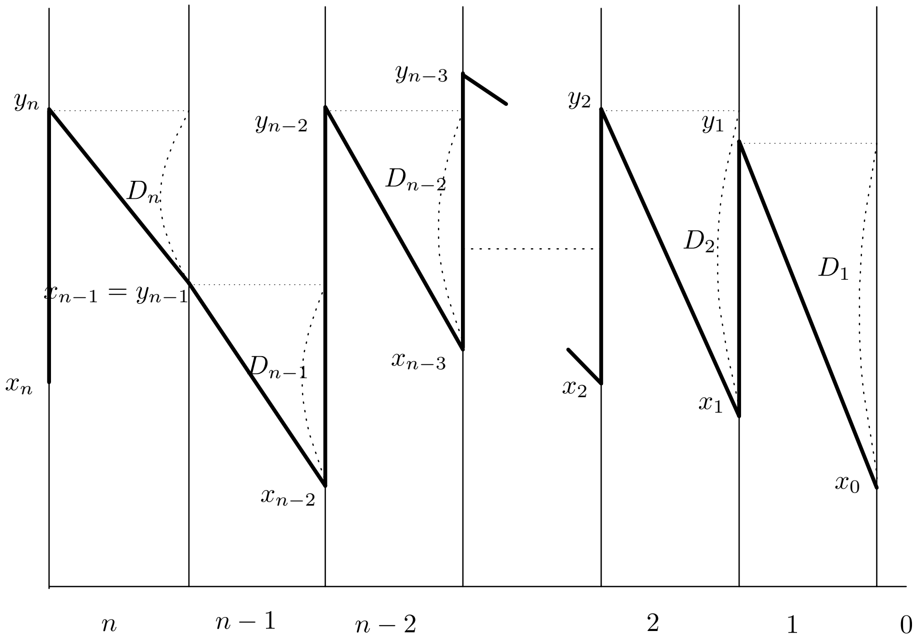

Figure 1: The behavior of the inventory level

(DPE) for the problem can be given by

$$
\begin{aligned}
&(1) \qquad V_i^0(x) = 0, \\
&(2) \qquad V_i^n(x) = \min_{y \geq x}\Big\{cy + L_i(y) + \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty V_j^{n-1}(y-z)dA_i(z) - cx\Big\}, \\
&\qquad\qquad n > 0, i \in E,
\end{aligned}
$$

where y is the inventory level after the order is delivered.

$$L_i(y) = h\int_0^y (y-z)dA_i(z) + p\int_y^\infty (z-y)dA_i(z),\ n > 0, i \in E$$

is the expected one-period holding and shortage cost function, and α is the discount factor per period. The first and second derivatives of $L_i(y)$ are

$$L_i'(y) = (h+p)A_i(y) - p,\ L_i''(y) = (h+p)a_i(y) > 0.$$

To simplify our analysis, by using the relation

$$W_i^n(x) = V_i^n(x) + cx,\ n \geq 0,$$

we change (1) and (2) to following DPE.

$$
(3) \qquad
\begin{aligned}
W_i^0(x) &= c^0 x, \\
W_i^n(x) &= \min_{y \geq x}\big\{G_i^n(y)\big\},
\end{aligned}
$$

where

$$
\begin{aligned}
G_i^n(y) &= cy(1-\alpha) + c\alpha \int_0^\infty z dA_i(z) + L_i(y) \\
&\quad + \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j^{n-1}(y-z) dA_i(z). \qquad (4)
\end{aligned}
$$

Furthermore, in this paper, we assume that no action is taken in period 0. So, $c^0 = 0$. Thus,

$$W_i^0(x) = 0.$$

The decision variable in this model is y, so (4) which is a function of y plays a central role in getting the optimal value y^*.

We assume that all parameters and costs are nonnegative, and that all relevant functions are differentiable.

3 Finite-Horizon Analysis In this section, we analyze the finite-horizon problem for the model introduced in the last section.

When $n = 2$, from (3) and (4),

$$
\begin{aligned}
W_i^2(x) &= \min_{y \geq x} \{G_i^2(y)\}, \\
G_i^2(y) &= cy(1-\alpha) + c\alpha \int_0^\infty z dA_i(z) + L_i(y) \\
&\quad + \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j^1(y-z) dA_i(z).
\end{aligned}
$$

We obtain the first two derivatives of $G_i^2(y)$ as follows:

$$
\begin{aligned}
G_i'^2(y) &= c(1-\alpha) + L_i'(y) + \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^1(y-z) dA_i(z), \\
G_i''^2(y) &= L_i''(y) + \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j''^1(y-z) dA_i(z).
\end{aligned}
$$

For $n = 1$,

$$
\begin{aligned}
&W_i^1(x) = \min_{y \geq x} \{G_i^1(y)\}, \\
&G_i^1(y) = cy(1-\alpha) + c\alpha \int_0^\infty z dA_i(z) + L_i(y), \\
&G_i'^1(y) = c(1-\alpha) + L_i'(y), \ G_i''^1(y) = L_i''(y) > 0.
\end{aligned}
$$

Then,

$$\lim_{y \to \infty} G_i'^1(y) = c(1-\alpha) + h > 0, \ \lim_{y \to 0} G_i'^1(y) = c(1-\alpha) - p < 0.$$

So, there exists a unique S_i^1 such that $G_i'^1(S_i^1) = 0$, i.e.,

$$S_i^1 = A_i^{-1}\Big[\frac{p - c(1-\alpha)}{h+p}\Big].$$

S_i^1 is nonnegative and finite because $p - c(1-\alpha) > 0$ and $p - c(1-\alpha) < h + p$. Hence, the optimal policy for period 1 is the environmental-dependent base-stock policy defined by

$$Y_1^*(i,x) = \begin{cases} S_i^1 & (x \le S_i^1), \\ x & (x > S_i^1), \end{cases}$$

and the optimal cost incurred by this policy is

$$W_i^1(x) = \begin{cases} G_i^1(S_i^1) & (x \le S_i^1), \\ G_i^1(x) & (x > S_i^1). \end{cases}$$

Furthermore, for $x > S_i^1$,

$$W_i'^1(x) = G_i'^1(x) > 0,\ W_i''^1(x) = G_i''^1(x) > 0.$$

Then,

$$\begin{aligned} &G_i''^2(y) > 0, \\ &\lim_{y\to\infty} G_i'^2(y) = c(1-\alpha^2) + h(1+\alpha) > 0,\ \lim_{y\to 0} G_i'^2(y) = c(1-\alpha) - p < 0. \end{aligned}$$

So, there exists a unique S_i^2 such that $G_i'^2(S_i^2) = 0$.
Hence,

$$\begin{aligned} Y_2^*(i,x) &= \begin{cases} S_i^2 & (x \le S_i^2), \\ x & (x > S_i^2), \end{cases} \\ W_i^2(x) &= \begin{cases} G_i^2(S_i^2) & (x \le S_i^2), \\ G_i^2(x) & (x > S_i^2). \end{cases} \end{aligned}$$

Furthermore, for $x > S_i^2$,

$$W_i'^2(x) = G_i'^2(x) > 0,\ W_i''^2(x) = G_i''^2(x) > 0.$$

On the other hand,

$$G_i'^2(y) - G_i'^1(y) = \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^1(y-z)dA_i(z) \ge 0.$$

So,

$$\begin{aligned} S_i^2 &\le S_i^1, \\ W_i'^2(x) &\ge W_i'^1(x). \end{aligned}$$

Moreover,

$$G_i^2(y) - G_i^1(y) = \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j^1(y-z)dA_i(z) \ge 0.$$

So,

$$W_i^2(x) - W_i^1(x) \ge \min_{y\ge x}\{G_i^2(y) - G_i^1(y)\} \ge 0.$$

For an n-period problem,

$$\begin{aligned}
W_i^n(x) &= \min_{y \geq x}\{G_i^n(y)\}, \\
G_i^n(y) &= cy(1-\alpha) + c\alpha \int_0^\infty z dA_i(z) + L_i(y) \\
&\quad + \alpha \sum_{j=1}^R P(i,j) \int_0^\infty W_j^{n-1}(y-z) dA_i(z), \\
G_i'^n(y) &= c(1-\alpha) + L_i'(y) + \alpha \sum_{j=1}^R P(i,j) \int_0^\infty W_j'^{n-1}(y-z) dA_i(z), \\
G_i''^n(y) &= L_i''(y) + \alpha \sum_{j=1}^R P(i,j) \int_0^\infty W_j''^{n-1}(y-z) dA_i(z).
\end{aligned}$$

To use induction, we assume that the following properties hold for the $(n-1)$-period problem where the state of the environment is $j \in E$.

$$\begin{aligned}
& G_j'^{n-1}(S_j^{n-1}) = 0,\ G_j''^{n-1}(y) > 0, \\
& \lim_{y \to \infty} G_j'^{n-1}(y) = c(1-\alpha^{n-1}) + h \sum_{k=0}^{n-2} \alpha^k > 0, \\
& \lim_{y \to 0} G_j'^{n-1}(y) = c(1-\alpha) - p < 0, \\
& W_j^{n-1}(x) = \begin{cases} G_j^{n-1}(S_j^{n-1}) & (x \leq S_j^{n-1}), \\ G_j^{n-1}(x) & (x > S_j^{n-1}), \end{cases} \\
& W_j'^{n-1}(x) = \begin{cases} 0 & (x \leq S_j^{n-1}), \\ G_j'^{n-1}(x) & (x > S_j^{n-1}), \end{cases} \geq 0, \\
& W_j''^{n-1}(x) = \begin{cases} 0 & (x \leq S_j^{n-1}), \\ G_j''^{n-1}(x) & (x > S_j^{n-1}), \end{cases} \geq 0, \\
& W_j^{n-1}(x) \geq W_j^{n-2}(x),\ W_j'^{n-1}(x) \geq W_j'^{n-2}(x).
\end{aligned}$$

Then,

$$\begin{aligned}
& G_i''^n(y) > 0, \\
& \lim_{y \to \infty} G_i'^n(y) = c(1-\alpha^n) + h \sum_{k=0}^{n-1} \alpha^k > 0,\ \lim_{y \to 0} G_i'^n(y) = c(1-\alpha) - p < 0.
\end{aligned}$$

So, there exists a unique S_i^n such that $G_i'^n(S_i^n) = 0$.
Hence,

$$\begin{aligned}
Y_n^*(i,x) &= \begin{cases} S_i^n & (x \leq S_i^n), \\ x & (x > S_i^n), \end{cases} \\
W_i^n(x) &= \begin{cases} G_i^n(S_i^n) & (x \leq S_i^n), \\ G_i^n(x) & (x > S_i^n). \end{cases}
\end{aligned}$$

Furthermore, for $x > S_i^n$,

$$W_i'^n(x) = G_i'^n(x) > 0,\ W_i''^n(x) = G_i''^n(x) > 0.$$

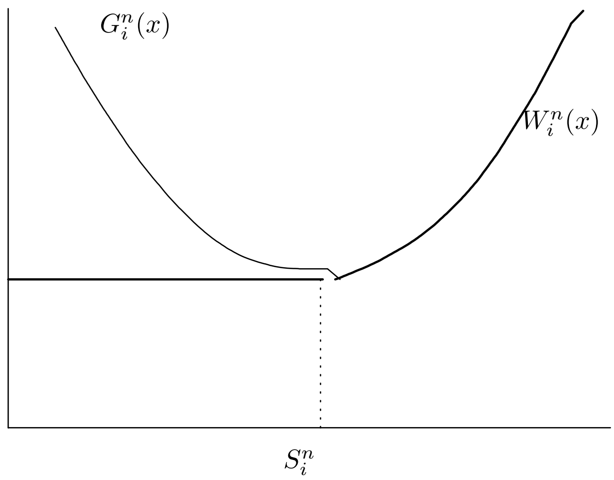

Figure 2: The behavior of $W_i^n(x)$ and $G_i^n(x)$

The behavior of $W_i^n(x)$ and $G_i^n(x)$ is shown graphically in Figure 2. On the other hand,

$$\begin{aligned} &G_i'^n(y) - G_i'^{n-1}(y) \\ &\quad = \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty \{W_j'^{n-1}(y-z) - W_j'^{n-2}(y-z)\} dA_i(z) \geq 0. \end{aligned}$$

So,

$$\begin{aligned} S_i^n &\leq S_i^{n-1}, \\ W_i'^n(x) &\geq W_i'^{n-1}(x). \end{aligned}$$

Moreover,

$$\begin{aligned} &G_i^n(y) - G_i^{n-1}(y) \\ &\quad = \alpha \sum_{j=1}^{R} P(i,j) \int_0^\infty \{W_j^{n-1}(y-z) - W_j^{n-2}(y-z)\} dA_i(z) \geq 0. \end{aligned}$$

Hence,

$$W_i^n(x) - W_i^{n-1}(x) \geq \min_{y \geq x} \{G_i^n(y) - G_i^{n-1}(y)\} \geq 0.$$

4 The monotonicity of the optimal policy for the environmental states In this section, we investigate the effect of the environmental fluctuations on the optimal policy.

To simplify our analysis, we introduce the concept of "stochastic ordering", and set some additional assumptions.

Definition 1 *Let $F(x) = P(X \leq x)$ and $G(y) = P(Y \leq y)$ denote cumulative distributions of the one-dimension random variables, X and Y, respectively. We define that "Y is stochastically larger than X" or that "G is stochastically larger than F", as follows:*

$$F(t) \geq G(t), \; -\infty < t < \infty.$$

Then, it is represented by $F \leq_{SL} G$ or $X \leq_{SL} Y$.

Theorem 1 *The following condition is necessary and sufficient to be $X \leq_{SL} Y$*

$$E\Phi(X) \leq E\Phi(Y)$$

where Φ is any non-decreasing function for which expectations exist.

We assume that the following relation holds on the demand distributions in respective states.

Assumption 1 $A_1 \leq_{SL} A_2 \leq_{SL} \cdots \leq_{SL} A_R$*, i.e.,* $A_1(y) \geq A_2(y) \geq \cdots \geq A_R(y)$*.*

We further make an assumption on the transition probability.

Assumption 2 *The transition probability $P(i,j)$ is assumed to be "totally positively 2 (TP2)".*

Now, when Assumption 1 and 2 hold, we prove the following (1) $\sim$ (5).

For any states i and i' $(1 \leq i < i' \leq R)$,
(1)$W_i^n(x) \geq W_{i'}^n(x)$,
(2)$G_i^n(y) \geq G_{i'}^n(y)$,
(3)$W_i'^n(x) \geq W_{i'}'^n(x)$,
(4)$G_i'^n(y) \geq G_{i'}'^n(y)$,
(5)$S_i^n \leq S_{i'}^n$.

We use induction to prove them.
For $n = 1$, they are apparent from the last section. For the $(n-1)$-period problem where the state of the environment is $j \in E$, we assume that $W_j^{n-1}(x)$ is non-increasing in j. Then, since $\sum_{j=r}^R P(i,j)$ is non-decreasing in i from Assumption 2, we have

$$\sum_{j=1}^R P(i,j) \int_0^\infty W_j^{n-1}(y-z)dA_{i'}(z) \geq \sum_{j=1}^R P(i',j) \int_0^\infty W_j^{n-1}(y-z)dA_{i'}(z).$$

Since $W_j^{n-1}(x)$ is non-decreaing in x, we obtain from Theorem 1

$$\sum_{j=1}^R P(i,j) \int_0^\infty W_j^{n-1}(y-z)dA_i(z) \geq \sum_{j=1}^R P(i,j) \int_0^\infty W_j^{n-1}(y-z)dA_{i'}(z).$$

Hence,

$$\sum_{j=1}^R P(i,j) \int_0^\infty W_j^{n-1}(y-z)dA_i(z) \geq \sum_{j=1}^R P(i',j) \int_0^\infty W_j^{n-1}(y-z)dA_{i'}(z).$$

So,

$$G_i^n(y) \geq G_{i'}^n(y).$$

Moreover,

$$W_i^n(x) \geq W_{i'}^n(x).$$

Next, for $(n-1)$-period problem where the state of the environment is $j \in E$, we assume that $W_j'^{n-1}(x)$ is non-increasing in j. Then, from Assumption 2, we have

$$\sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^{n-1}(y-z)dA_{i'}(z) \geq \sum_{j=1}^{R} P(i',j) \int_0^\infty W_j'^{n-1}(y-z)dA_{i'}(z).$$

Since $W_j'^{n-1}(x)$ is non-decreasing in x, we obtain from Theorem 1

$$\sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^{n-1}(y-z)dA_i(z) \geq \sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^{n-1}(y-z)dA_{i'}(z).$$

Hence,

$$\sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^{n-1}(y-z)dA_i(z) \geq \sum_{j=1}^{R} P(i',j) \int_0^\infty W_j'^{n-1}(y-z)dA_{i'}(z).$$

So,

$$\begin{aligned} G_i'^n(y) &\geq G_{i'}'^n(y), \\ S_i^n &\leq S_{i'}^n. \end{aligned}$$

Moreover,

$$W_i'^n(x) \geq W_{i'}'^n(x).$$

We further introduce a concept stronger than Definition 1, and set a new assumption.

Definition 2 *Let X and Y be the one-dimension random variables with cumulative distributions $F(x) = P(X \leq x)$, $G(y) = P(Y \leq y)$, and probability densities $f(x) = F'(x)$, $g(y) = G'(y)$, respectively. We define that "Y is larger than X in the sense of first moment ordering" or that "g is larger than f in the sense of first moment ordering", as follows.*

$$g(t) \quad = \quad f(t+C), \ \forall t \geq 0$$

where C is an any positive constant. Then, it is represented by $f \leq_{FM} g$ or $X \leq_{FM} Y$.

Assumption 3 $a_1 \leq_{FM} a_2 \leq_{FM} \cdots \leq_{FM} a_R$.

When Assumption 2 and 3 hold, we prove that S_i^n is constant in n. First, to simplify our analysis, we introduce a new notation. Let ν_i be a mean of the demand in an any state i, i.e., $\nu_i = \int_0^\infty z dA_i(z), \ \forall i$.
Then, from Assumption 3,

$$a_1(y-\nu_1) = a_2(y-\nu_2) = \cdots = a_R(y-\nu_R).$$

Since the demand should not be negative, $a_1(y) = 0, \ \forall y \leq 0$. Hence,

$$a_i(y) = 0, \ \forall y \leq \nu_i - \nu_1, \ i = 1, \ldots R. \tag{5}$$

¿From $S_i^1 = A_i^{-1}\left[\frac{p-c(1-\alpha)}{h+p}\right]$,

$$S_1^1 - \nu_1 = S_2^1 - \nu_2 = \cdots = S_R^1 - \nu_R. \tag{6}$$

Now, for the $(n-1)$-period problem where the state of the environment is $j \in E$, we assume that $S_j^{n-1} = S_j^1$. Then, from the last section,

$$W_j'^{n-1}(x) \begin{cases} = 0 & (x \leq S_j^{n-1}), \\ \geq 0 & (x > S_j^{n-1}). \end{cases}$$

Since $S_1^{n-1} \leq S_2^{n-1} \leq \cdots \leq S_R^{n-1}$ from Assumption 2,

$$\sum_{j=1}^{R} P(i,j) W_j'^{n-1}(x) \begin{cases} = 0 & (x \leq S_1^{n-1} = S_1^1), \\ \geq 0 & (x > S_1^{n-1} = S_1^1). \end{cases}$$

Moreover, from (5) and (6),

$$\sum_{j=1}^{R} P(i,j) \int_0^\infty W_j'^{n-1}(y-z) dA_i(z) \begin{cases} = 0 & (y \leq S_1^1 + \nu_i - \nu_1 = S_i^1), \\ \geq 0 & (y > S_1^1 + \nu_i - \nu_1 = S_i^1). \end{cases}$$

Hence,

$$S_i^n = S_i^1.$$

Therefore, when Assumption 2 and 3 hold, S_i^n is constant in n.

5 Concluding Remarks In this paper, first, we show the existence of the environmental-dependent optimal base-stock policy and its monotonicity for review periods by analyzing finite-horizon periodic-review inventory model where the demand distributions depend on a Markov environment process. We further show that the optimal policy have the monotonicity for the environmental states by ordering them.

When the cost-parameters depend on a Markov environment process, we can prove the existence of the environmental-dependent optimal base-stock policy. Also, in the presence of a fixed ordering cost, we can show the existence of the environmental-dependent optimal (s, S) policy. Furthermore, we can derive the optimal policy in the model that incorporates variable capacity and stochastically proportional yield. But, in above cases, it is difficult to clarify the effect of the environmental fluctuations on the optimal policy. It is a future direction to investigate the effect of the environmental fluctuations on the optimal policy in more complex model.

In our model, we analyze the inventory system that depends on the exogenous factors. So, it is also one of the possible extensions to present the inventory system that depends on the endogenous factors where the demand is influenced by the order quantity, price discount, marketing activity, and so on.

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