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Multiple Antenna Spectrum Sensing in Cognitive Radios

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***Abstract*—In this paper, we consider the problem of spectrum sensing by using multiple antenna in cognitive radios when the noise and the primary user signal are assumed as independent complex zero-mean Gaussian random signals. The optimal multiple antenna spectrum sensing detector needs to know the channel gains, noise variance, and primary user signal variance. In practice some or all of these parameters may be unknown, so we derive the Generalized Likelihood Ratio (GLR) detectors under these circumstances. The proposed GLR detector, in which all the parameters are unknown, is a blind and invariant detector with a low computational complexity. We also analytically compute the missed detection and false alarm probabilities for the proposed GLR detectors. The simulation results provide the available traded-off in using multiple antenna techniques for spectrum sensing and illustrates the robustness of the proposed GLR detectors compared to the traditional energy detector when there is some uncertainty in the given noise variance.**

***Index Terms*—Cognitive radio, Spectrum Sensing, Multiple antenna, Eigenvalue decomposition, Opportunity detection, GLR detector, Noise variance mismatch.**

I. Introduction

Recent measurements reveal that many portions of the licensed spectrum are not used during significant time periods [1]. Since the number of users and their data rates steadily increase, the traditional fixed spectrum policy is inefficient and is no longer a feasible approach. One proposal for alleviating the spectrum scarcity is allowing licence-exempted Secondary Users (SU) to exploit the unused spectrum holes over some frequency ranges by using Cognitive Radio (CR) technology [2]. One of the major challenges of implementing this technology is that the CRs must accurately monitor and be aware of the presence of the Primary Users (PUs) over a particular spectrum. To address this challenge, several efficient methods have been proposed [3]–[7]. In [7] the spectrum sensing in a wideband Orthogonal Frequency Division Multiplexing (OFDM) scenario, when the received power of PU is unknown and there are different amount of the priori knowledge about PU signal, has been investigated. The Energy Detector (ED) (a.k.a. radiometer) is a common method to detect an unknown signal in additive noise [8]. This method is optimal for white Gaussian noise if the noise variance is known. Unfortunately, the performance of the ED is susceptible to errors in the noise variance [9]. It has been shown that to achieve a desired probability of detection under uncertain noise variance, the Signal-to-Noise Ratio (SNR) has to be above a certain threshold [10]. For the case of unknown noise variance, the cyclostationarity property of communication signals is exploited in [5], [11], [12]. In contrast to noise, which is a wide-sense stationary random signal with impulse autocorrelation function, in general, the modulated signals have the periodical mean and autocorrelation function. These features can be used to distinguish the noise from the modulated signal. The drawbacks of this method are that the method requires a significantly long observation time and is highly computationally complex for practical implementation.

While there has been an intensive work on the spectrum sensing problem in the case of known noise variance, not enough attention has been made to the spectrum sensing under unknown noise variance except [10], [13]–[15].

Multiple antenna techniques currently are used in communications and their effectiveness have been shown in different aspects [16]. In the context of dynamic spectrum sharing, multiple antenna SU can be used for a reliable signal transmission and also spectrum sensing. In fact, using multiple antenna techniques in CRs is one of possible approaches for the spectrum sensing by exploiting available spatial domain observations and has been proposed in [17]–[19]. In [17], the authors have shown the efficiency of multiple antenna spectrum sensing in terms of required sensing time and hardware by using a two-stage sensing method. In [18], the ED has been proposed for spectrum sensing by using multiple antennas. The PU signal has been treated as an unknown deterministic signal and based on this model the performance of the energy detector has been evaluated in Rayleigh fading channels. In [19], it has been shown that a multiple antenna OFDM based CR scheme, when using the square law combing energy detector, has better performance than the single antenna scheme, even at low SNRs. In [20], a blind energy detector based on SNR maximization has been proposed and its performance has been evaluated in difference cases.

In this paper, we investigate the spectrum sensing problem by using multiple antennas when the PU signal can be well modeled as a complex Gaussian random signal in the presence of an Additive White Gaussian Noise (AWGN). We derive the optimum detector structure for spectrum sensing and investigate its performance. The optimal detector needs to know the noise and PU signal variances, and also the channel gains. In practice one or more of these parameters may be unknown, so in what follows we derive the Generalized Likelihood Ratio (GLR) detector when some or all of these parameters are

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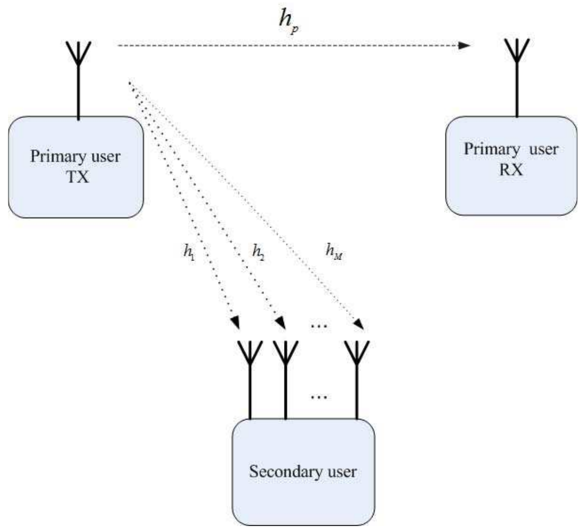

Fig. 1. SU with multiple antennas

unknown.

The remaining of the paper is organized as follows. In Section II, we describe the system model and the basic assumptions about the PU signal, noise, and channel gain vector. Conditioned on knowing the noise and PU variances and channel gain vector, we derive the optimal multiple antenna detection rule and evaluate its performance analytically. In Section III, we derive the GLR detectors for three cases, namely, unknown channel gain, unknown channel gain and the PU signal variance, and finally unknown all the aforementioned parameters. In this Section, we also show that the GLR detector when all of the parameters are unknown, is an invariant detector. In Section IV, we evaluate the performance of the proposed GLR detectors analytically. In Section V, we present some numerical results to evaluate the performance of the GLR detectors based on both analytical derivations and simulations, and investigate the available tarde-offs in the GLR detector performance. Also in order to evaluate the performance of the proposed GLR detectors in a practical scenario, we compare the performance of the GLR detectors with the cyclostionarity based detector and ED, when the PU signal is considered as a DTV signal in IEEE 802.22 standard. Finally, Section VI concludes the paper.

Throughout this paper, we use boldface letters for column vectors and boldface capital letters for matrices. We also denote $\mathbf{x}_k = [\mathbf{x}]_k$, $\mathbf{X}_{k,l} = [\mathbf{X}]_{k,l}$, and $\mathbf{X}_k = [\mathbf{X}]_k$, respectively, as the elements of a vector $\mathbf{x}$, the elements of a matrix $\mathbf{X}$, and the kth row of a matrix $\mathbf{X}$. We use the notation $\hat{x}$ to denote the estimation of unknown parameter x, which may be scalar, vector or matrix.

II. Basic Assumptions and Optimal Detector

Suppose that the SU has M receiving antennas and each antenna receives L samples as shown in Figure 1. We assume that the PU signal samples are independent zero-mean random variables with complex Gaussian distribution. This assumption, for instance, is valid for an OFDM signal in which each carrier is modulated by independent data streams. We denote the hypothesis of the PU signal being active and inactive (within the range of the CR) by $\mathcal{H}_0$, and $\mathcal{H}_1$, respectively. We assume that the additive noise samples at different antennas are independent zero-mean Gaussian random variables. Under $\mathcal{H}_1$, we assume that the PU signal and noise are independent. Let $\mathbf{Y} = [\mathbf{y}_1, \cdots, \mathbf{y}_L] \in \mathbb{C}^{M\times L}$ be a complex matrix containing the observed signals at M antennas. The multiple antenna PU detection problem can be expressed as the following binary hypothesis test:

$$\begin{cases} \mathcal{H}_0: & \mathbf{Y} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_M) \quad \text{if the PU is inactive,} \\ \mathcal{H}_1: & \mathbf{Y} \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2 \mathbf{h}\mathbf{h}^H + \sigma_n^2 \mathbf{I}_M) \quad \text{if the PU is active.} \end{cases} \tag{1}$$

where $\mathbf{h} \in \mathbb{C}^{M\times 1}$ denotes the channel gain vector between the PU and M antennas, and σ_n^2 and σ_s^2 are the variances of noise and PU signal, respectively. We assume that the channel gain vector, i.e., $\mathbf{h}$, is a constant parameter at each sensing time. For the optimal detection, the channel gain vector is assumed to be known, and for the other practical detections, we assume that the channel gain is unknown and we estimate this unknown parameter, as will be discussed in the following subsections.

A. Optimal Detector

For the optimal detector, the SU knows the channel gains, noise and PU signal variances. In this case, from (1), under $\mathcal{H}_0$ the Probability Density Function (PDF) of observation matrix, $\mathbf{Y}$, is as follows [21]:

$$\begin{aligned} f(\mathbf{Y}; \mathcal{H}_0, \sigma_n^2) &= \prod_{l=1}^{L} \frac{1}{(\pi\sigma_n^2)^M} \exp\left\{-\frac{1}{\sigma_n^2}\mathbf{y}_l^H \mathbf{y}_l\right\} \\ &= \frac{1}{(\pi\sigma_n^2)^{ML}} \exp\left\{-\frac{1}{\sigma_n^2}\sum_{l=1}^{L}\mathbf{y}_l^H \mathbf{y}_l\right\} \\ &= \frac{1}{(\pi\sigma_n^2)^{ML}} \exp\left\{\frac{-\mathrm{tr}(\mathbf{Y}\mathbf{Y}^H)}{\sigma_n^2}\right\}, \end{aligned} \tag{2}$$

where $\mathrm{tr}(.)$ denotes the trace of the matrix. By taking logarithm of PDF of observations under hypothesis $\mathcal{H}_0$, i.e., $\mathcal{L}_0(\mathbf{Y}) = \ln f(\mathbf{Y}; \mathcal{H}_0, \sigma_n^2)$, we will have:

$$\mathcal{L}_0(\mathbf{Y}) = -\frac{\mathrm{tr}(\mathbf{Y}\mathbf{Y}^H)}{\sigma_n^2} - ML\ln\pi - ML\ln\sigma_n^2. \tag{3}$$

Similarly, from (1) under the hypothesis $\mathcal{H}_1$, the PDF can be written as [21]:

$$\begin{aligned} f(\mathbf{Y}; \mathcal{H}_1, \mathbf{h}, \sigma_n^2, \sigma_s^2) &= \prod_{l=1}^{L} \frac{1}{\pi^M \det(\mathbf{R})} \exp\left\{-\frac{1}{\sigma_n^2}\mathbf{y}_l^H \mathbf{R}^{-1}\mathbf{y}_l\right\} \\ &= \frac{1}{\pi^{ML}\det(\mathbf{R})^L} \exp\left\{-\frac{1}{\sigma_n^2}\sum_{l=1}^{L}\mathbf{y}_l^H \mathbf{R}^{-1}\mathbf{y}_l\right\} \\ &= \frac{\exp\left\{-\mathrm{tr}(\mathbf{R}^{-1}\mathbf{Y}\mathbf{Y}^H)\right\}}{\pi^{ML}\det(\mathbf{R})^L}, \end{aligned} \tag{4}$$

where $\mathbf{R} \triangleq E[\mathbf{Y}\mathbf{Y}^H|\mathcal{H}_1] = \sigma_s^2\mathbf{h}\mathbf{h}^H + \sigma_n^2\mathbf{I}$ is the covariance matrix. We can easily show that $\det(\mathbf{R}) = (\sigma_s^2\|\mathbf{h}\|^2 + \sigma_n^2)(\sigma_n^2)^{(M-1)}$ and also by using the matrix inversion lemma [22], we obtain:

$$\mathbf{R}^{-1} = \sigma_n^{-2}\mathbf{I} - \sigma_n^{-2}\frac{\mathbf{h}\mathbf{h}^H}{\frac{\sigma_n^2}{\sigma_s^2} + \|\mathbf{h}\|^2}. \tag{5}$$

By taking the logarithm from (4) and denoting $\mathcal{L}_1(\mathbf{Y}) = \ln f(\mathbf{Y};\mathcal{H}_1,\mathbf{h},\sigma_n^2,\sigma_s^2)$, we obtain:

$$\begin{aligned}\mathcal{L}_1(\mathbf{Y}) &= -\mathrm{tr}(\mathbf{R}^{-1}\mathbf{Y}\mathbf{Y}^H) - ML\ln\pi \\ &\quad -L\ln\det(\sigma_s^2\|\mathbf{h}\|^2+\sigma_n^2) - L(M-1)\ln\sigma_n^2,\end{aligned} \tag{6}$$

which substituting (5) in (6) results in:

$$\begin{aligned}\mathcal{L}_1(\mathbf{Y}) &= -\frac{\mathrm{tr}(\mathbf{Y}\mathbf{Y}^H)}{\sigma_n^2} + \frac{\|\mathbf{h}^H\mathbf{Y}\|^2}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2} \\ &- ML\ln\pi - L\ln(\frac{\sigma_s^2}{\sigma_n^2}\|\mathbf{h}\|^2+1) - LM\ln\sigma_n^2.\end{aligned} \tag{7}$$

For optimal detector in Neyman-Pearson sense, we need to compare the Likelihood Ratio (LR) function or Logarithm of Likelihood Ratio (LLR) function with a threshold. From (6) and (3), the LLR function is equal to:

$$\begin{aligned}\mathrm{LLR} &= \ln\frac{f(\mathbf{Y};\mathcal{H}_1,\mathbf{h},\sigma_s^2,\sigma_n^2)}{f(\mathbf{Y};\mathcal{H}_0,\sigma_n^2)} \\ &= \mathcal{L}_1(\mathbf{Y}) - \mathcal{L}_0(\mathbf{Y}) \\ &= \frac{\|\mathbf{h}^H\mathbf{Y}\|^2}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2} - L\ln(\frac{\sigma_s^2}{\sigma_n^2}\|\mathbf{h}\|^2+1).\end{aligned} \tag{8}$$

Comparing the LLR function with a threshold results in the following optimal decision rule:

$$\frac{\|\mathbf{h}^H\mathbf{Y}\|^2}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2} - L\ln(\frac{\sigma_s^2}{\sigma_n^2}\|\mathbf{h}\|^2+1) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_1. \tag{9}$$

Where η_1 is the decision threshold. Considering that in optimal detector, the channel gains and noise and PU variance are known, with some straightforward simplifications, the optimal decision rule can be rewritten as follows:

$$T_{\mathrm{opt}} = \|\mathbf{h}^H\mathbf{Y}\|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta, \tag{10}$$

where $\eta \triangleq \frac{\eta_1 + L\ln(\frac{\sigma_s^2}{\sigma_n^2}\|\mathbf{h}\|^2+1)}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2}$ denotes the threshold of detection. In general, the detection threshold η is obtained by solving $F(\eta) = 1 - P_{\mathrm{fa}}$, where P_{fa} denotes the false alarm probability and $F(x)$ is the Cumulative Distribution Function (CDF) of the decision statistic. Also, it is possible to find this threshold by Monte-Carlo simulation method. From (10), the optimal detector is a maximum ratio combiner, which gives weights to the observations at the different antennas according to their corresponding channel gains, where the antenna with better reception has more contribution in the summation in (10).

In the context of dynamic spectrum sharing, the false alarm probability P_{fa} indicates the probability that a spectrum hole (a vacant band) is falsely detected as an occupied band, i.e., P_{fa} represents the percentage of the spectrum holes which are not used. Therefore, the SUs must reduce the false alarm probability P_{fa} as much as possible. On the other hand, the missed detection probability, i.e., $P_m = 1 - P_d$, determines the probability that an occupied band is mistakenly detected as a spectrum hole. Such a missed detection induces harmful interference for PU. Thus, the missed detection probability must be small enough to avoid perceptible performance loss for the PU.

B. *Performance of Optimal Detector*

To evaluate the performance of the optimal detector, we first compute the Complementary Cumulative Distribution Function (CCDF) of the decision statistic under $\mathcal{H}_0$ and $\mathcal{H}_1$. Under hypothesis $\mathcal{H}_0$, we notice that the elements of the observed matrix are i.i.d. Gaussian variables with zero mean and variance of σ_n^2, i.e. $\mathbf{Y} \sim \mathcal{CN}(\mathbf{0},\sigma_n^2\mathbf{I})$. As a result, conditioned on channel coefficients, the random vector $\mathbf{h}^H\mathbf{Y}$ has a Gaussian distribution,i.e., $\mathbf{h}^H\mathbf{Y} \sim \mathcal{CN}(\mathbf{0},\|\mathbf{h}\|^2\sigma_n^2 I_L)$. Then, from (10), the decision statistic under hypothesis $\mathcal{H}_0$ has the following distribution:

$$\frac{T_{\mathrm{opt}}(\mathbf{Y})}{\|\mathbf{h}\|^2\sigma_n^2} = \frac{\|\mathbf{h}^H\mathbf{Y}\|^2}{\|\mathbf{h}\|^2\sigma_n^2} \sim \chi_{2L}^2. \tag{11}$$

Therefore, the false alarm probability P_{fa} is easily obtained using CCDF of $T_{\mathrm{opt}}(\mathbf{Y})$ as follows [14]:

$$\begin{aligned}P_{\mathrm{fa}} &= P[T_{opt}(\mathbf{Y}) > \eta|\mathcal{H}_0] \\ &= \frac{\Gamma(L,\frac{\eta}{\|\mathbf{h}\|^2\sigma_n^2})}{\Gamma(L)},\end{aligned} \tag{12}$$

where $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$ and $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$ are the upper incomplete and complete gamma functions, respectively.

Similarly under hypothesis $\mathcal{H}_1$, we have $\mathbf{Y} \sim \mathcal{CN}(0,\sigma_s^2\mathbf{h}\mathbf{h}^H + \sigma_n^2\mathbf{I})$. Then, again conditioned on the channel coefficients, we easily conclude that

$$\mathbf{h}^H\mathbf{Y} \sim \mathcal{CN}\left(\mathbf{0},\|\mathbf{h}\|^2(\|\mathbf{h}\|^2\sigma_s^2+\sigma_n^2)\mathbf{I}_L\right), \tag{13}$$

and hence from (10)

$$\frac{T_{\mathrm{opt}}(\mathbf{Y})}{\|\mathbf{h}\|^2(\|\mathbf{h}\|^2\sigma_s^2+\sigma_n^2)} = \frac{\|\mathbf{h}^H\mathbf{Y}\|^2}{\|\mathbf{h}\|^2(\|\mathbf{h}\|^2\sigma_s^2+\sigma_n^2)} \sim \chi_{2L}^2. \tag{14}$$

Therefore, the detection probability P_d is easily evaluated as follows

$$\begin{aligned}P_d &= P[T_{opt}(\mathbf{Y}) > \eta|\mathcal{H}_1] \\ &= \frac{\Gamma\left(L,\frac{\eta}{\|\mathbf{h}\|^2(\|\mathbf{h}\|^2\sigma_s^2+\sigma_n^2)}\right)}{\Gamma(L)},\end{aligned} \tag{15}$$

which if we define the received SNR at the SU as $\gamma \triangleq \frac{\sigma_s^2\|\mathbf{h}\|^2}{\sigma_n^2}$, the detection probability can be written as:

$$P_d = \frac{\Gamma\left(L,\frac{\eta}{\|\mathbf{h}\|^2\sigma_n^2(1+\gamma)}\right)}{\Gamma(L)}. \tag{16}$$

III. GLR Detectors

The optimal detector needs to know the values of channel gains, noise and PU variances. In practice, we may have no knowledge about the values of some or all of these parameters. In these cases, we can use the GLR test to decide about the presence or absence of the PU. In this section, we derive the GLR test in the different cases, in which some or all of the parameters are unknown. Then, we investigate the invariancy of the derived GLR tests under several groups of transformations.

A. Case 1: Unknown Channel Gains (GLRD1)

In this part, we assume that the SU has knowledge about the noise and PU signal variances, but the channel gain vector $\mathbf{h}$ is unknown. In this case, since the variance of noise is known, the logarithm of PDF of observations $\mathbf{Y}$ under hypothesis $\mathcal{H}_0$ is computed as (3). Under hypothesis $\mathcal{H}_1$ the channel gains are unknown and in order to derive the GLR test, we first maximize (4) with respect to $\mathbf{h}$ to find the ML estimation of the channel gains. From (7), by setting $\frac{\partial}{\partial \mathbf{h}}\mathcal{L}_1(\mathbf{Y}) = 0$, we have:

$$\big(\alpha \mathbf{I} + \beta \mathbf{h}\mathbf{h}^H\big)\mathbf{Y}\mathbf{Y}^H\mathbf{h} = \nu \mathbf{h}, \tag{17}$$

where $\alpha = \frac{1}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2}$, $\beta = \frac{L}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2}$ and $\nu = \frac{1}{(\frac{\sigma_n^2}{\sigma_s^2}+\|\mathbf{h}\|^2)\sigma_n^2}$. If we define $\mathbf{A} \triangleq \big(\alpha\mathbf{I} + \beta \mathbf{h}\mathbf{h}^H\big)$, then since α and β are real positive numbers, the matrix $\mathbf{A} = \alpha \mathbf{I} + \beta \mathbf{h}\mathbf{h}^H$ is a full rank matrix and therefore it has an inverse. Using matrix inversion lemma and multiplying both sides of (17) by $\mathbf{A}^{-1} = \frac{1}{\alpha}(\mathbf{I} - \frac{\mathbf{h}\mathbf{h}^H}{\frac{\alpha}{\beta}+\|\mathbf{h}\|^2})$, we obtain

$$\begin{aligned} \frac{1}{L}\mathbf{Y}\mathbf{Y}^H\mathbf{h} &= \frac{\nu}{L}\mathbf{A}^{-1}\mathbf{h} \\ &= \frac{\nu}{L\alpha}(\mathbf{I} - \frac{\mathbf{h}\mathbf{h}^H}{\frac{\alpha}{\beta}+\|\mathbf{h}\|^2})\mathbf{h}. \end{aligned} \tag{18}$$

This obviously means that $\mathbf{h}$ is a eigenvector of $\widehat{\mathbf{R}} \triangleq \frac{1}{L}\mathbf{Y}\mathbf{Y}^H$, i.e.,

$$\widehat{\mathbf{R}}\mathbf{h} = \lambda \mathbf{h} \tag{19}$$

and $\lambda = \frac{\nu}{L}(\frac{\alpha}{\beta+\alpha\|\mathbf{h}\|^2})$ is a real number and the corresponding eigenvalue of sample covariance matrix $\widehat{\mathbf{R}} = \frac{1}{L}\mathbf{Y}\mathbf{Y}^H$. From (19), it is obvious that the ML estimation of the vector $\mathbf{h}$ is an eigenvector of $\widehat{\mathbf{R}}$. Now we must determine which eigenvectors of $\widehat{\mathbf{R}}$ maximizes the likelihood function. We first normalize the eigenvectors such that $\|\mathbf{h}\|^2 = 1$. From $\widehat{\mathbf{R}}\mathbf{h} = \lambda\mathbf{h}$ and the definition of $\widehat{\mathbf{R}}$, we obtain $\|\mathbf{h}^H\mathbf{Y}\|^2 = L\lambda\|\mathbf{h}\|^2 = L\lambda$. Replacing this equation in (6) we obtain

$$\begin{aligned} \mathcal{L}_1(\mathbf{Y}) &= -L\frac{\mathrm{tr}(\widehat{\mathbf{R}})}{\sigma_n^2} + \frac{L\lambda}{(\frac{\sigma_n^2}{\sigma_s^2}+1)\sigma_n^2} \\ &- ML\ln\pi - L\ln(\frac{\sigma_s^2}{\sigma_n^2}+1) - LM\ln\sigma_n^2. \end{aligned} \tag{20}$$

Since the above function is an increasing function with respect to λ, the ML estimation $\mathbf{h}$ is the eigenvector corresponding to the maximum eigenvalue of the matrix $\widehat{\mathbf{R}}$ which we denote this vector by $\widehat{\mathbf{h}}$, i.e.

$$\widehat{\mathbf{R}}\widehat{\mathbf{h}} = \lambda_{\max}\widehat{\mathbf{h}}, \tag{21}$$

where, by assumption, we have $\|\widehat{\mathbf{h}}\| = 1$. By replacing the above estimation in (20), we obtain:

$$\begin{aligned} \mathcal{L}_1(\mathbf{Y}) &= -L\frac{\mathrm{tr}(\widehat{\mathbf{R}})}{\sigma_n^2} + \frac{L\lambda_{\max}}{(\frac{\sigma_n^2}{\sigma_s^2}+1)\sigma_n^2} \\ &- ML\ln\pi - L\ln(\frac{\sigma_s^2}{\sigma_n^2}+1) - LM\ln\sigma_n^2. \end{aligned} \tag{22}$$

From (3) and (22), the LLR function is equal to:

$$\begin{aligned} \mathrm{LLR} &= \mathcal{L}_1(\mathbf{Y}) - \mathcal{L}_0(\mathbf{Y}) \\ &= \frac{L\lambda_{\max}}{(\frac{\sigma_n^2}{\sigma_s^2}+1)\sigma_n^2} - L\ln(\frac{\sigma_s^2}{\sigma_n^2}+1). \end{aligned} \tag{23}$$

For decision making, we must compare the LLR in (23) to a threshold which results in

$$\begin{aligned} &\frac{L\lambda_{\max}}{(\frac{\sigma_n^2}{\sigma_s^2}+1)\sigma_n^2} - L\ln(\frac{\sigma_s^2}{\sigma_n^2}+1) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_1 \Rightarrow \\ &T_{\mathrm{GLRD1}}(\mathbf{Y}) = \frac{\lambda_{\max}}{\sigma_n^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta, \end{aligned} \tag{24}$$

where $\eta = \frac{1}{L}[\eta_1 + L\ln(\frac{\sigma_s^2}{\sigma_n^2}+1)](\frac{\sigma_n^2}{\sigma_s^2}+1)$.

B. Case 2: Unknown Channel Gains and PU Variance (GLRD2)

In this part, in addition to unknown channel gains, we assume that the PU variance is also unknown. We maximize (22) with respect to σ_s^2 to find the ML estimation of PU variance σ_s^2. By setting $\frac{\partial}{\partial \sigma_s^2}\mathcal{L}_1(\mathbf{Y}) = 0$, we obtain

$$\widehat{\sigma_s^2} = \lambda_{\max} - \sigma_n^2. \tag{25}$$

We now replace (25) in (22), as

$$\begin{aligned} \mathcal{L}_1(\mathbf{Y}) &= -L\frac{\mathrm{tr}(\widehat{\mathbf{R}})}{\sigma_n^2} + \frac{L\lambda_{\max}}{\sigma_n^2} - L - ML\ln\pi \\ &\quad -L\ln(\frac{\lambda_{\max}}{\sigma_n^2}) - LM\ln\sigma_n^2. \end{aligned} \tag{26}$$

From (3) and (26), the LLR function is equal to:

$$\begin{aligned} \mathrm{LLR} &= \mathcal{L}_1(\mathbf{Y}) - \mathcal{L}_0(\mathbf{Y}) \\ &= \frac{L\lambda_{\max}}{\sigma_n^2} - L\ln(\frac{\lambda_{\max}}{\sigma_n^2}) - L. \end{aligned} \tag{27}$$

For decision making, the LLR function must be compared by a threshold. We easily obtain:

$$\frac{\lambda_{\max}}{\sigma_n^2} - \ln(\frac{\lambda_{\max}}{\sigma_n^2}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_1. \tag{28}$$

We notice that $g(x) = x - \ln(x)$ is an increasing function for $x \geq 1$. Under both $\mathcal{H}_0$ and $\mathcal{H}_1$ hypotheses, the ratio of $\frac{\lambda_{\max}}{\sigma_n^2}$ is greater than one with high probability [23] and hence we can simplify the GLR detector as the following form:

$$T_{\mathrm{GLRD2}}(\mathbf{Y}) = \frac{\lambda_{\max}}{\sigma_n^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta. \tag{29}$$

Interestingly, the derived GLR detector (GLRD1) has the same form as the detector given in (24), in which only the channel gains are unknown (GLRD1). So we can conclude that the knowledge about the PU variance, when the channel gains are unknown, does not lead to a better GLR detector. This is also intuitively expected as this case is equivalent to the case that the PU variance is one and the channel gain is $\sigma_s\mathbf{h}$.

C. Case 3: Unknown Channel Gains and PU and Noise Variance (GLRD3 or blind GLRD)

In the following, we derive the GLR detector when all of the mentioned parameters are unknown. Up to know, we have obtained the ML estimations of the channel gain vector $\mathbf{h}$ and σ_s^2. In order to derive the GLR test, we maximize the PDFs in (26) and (3) with respect to σ_n^2 and then form the LR function. By maximizing (3) with respect to σ_n^2, we obtain the ML estimation of noise variance under hypothesis $\mathcal{H}_0$ as follows:

$$\mathcal{H}_0 : \widehat{\sigma_n^2} \;=\; \frac{\mathrm{tr}(\mathbf{Y}\mathbf{Y}^H)}{ML} = \frac{\mathrm{tr}(\widehat{\mathbf{R}})}{M}, \tag{30}$$

and hence from (3) and (30), for PDF under hypothesis $\mathcal{H}_0$, we get:

$$\sup_{\sigma_n^2} f(\mathbf{Y}; \mathcal{H}_0, \sigma_n^2) = \frac{(ML)^{ML}}{(\pi e)^{ML}\left(\mathrm{tr}(\widehat{\mathbf{R}})\right)^{ML}}. \tag{31}$$

Similarly under hypothesis $\mathcal{H}_1$, from (26) by setting $\frac{\partial}{\partial \sigma_n^2}\mathcal{L}_1(\mathbf{Y}) = 0$, we obtain

$$\widehat{\sigma_n^2} \;=\; \frac{1}{M-1}\left(\mathrm{tr}(\widehat{\mathbf{R}}) - \lambda_{\max}\right). \tag{32}$$

From (21), (25) and (32), the ML estimations of unknown parameters under $\mathcal{H}_1$ are summarized as follows

$$\mathcal{H}_1 : \begin{cases} \widehat{\sigma_n^2} = \frac{1}{M-1}\left(\mathrm{tr}(\widehat{\mathbf{R}}) - \lambda_{\max}\right), \\ \widehat{\sigma_s^2} = \frac{1}{M-1}\left(M\lambda_{\max} - \mathrm{tr}(\widehat{\mathbf{R}})\right), \\ \widehat{\mathbf{h}} = \frac{\mathbf{v}_{\max}}{\|\mathbf{v}_{\max}\|} \end{cases} \tag{33}$$

where $\mathbf{v}_{\max}$ denotes the eigenvector corresponding to the largest eigenvalue. Using the ML estimations in (33) leads to the following PDF under hypothesis $\mathcal{H}_1$:

$$\sup_{\mathbf{h},\sigma_n^2,\sigma_s^2} f(\mathbf{Y}; \mathcal{H}_1, \mathbf{h}, \sigma_n^2, \sigma_s^2) = \tag{34}$$

$$\frac{(L(M-1))^{L(M-1)}L^L}{(\pi e)^{ML}(\lambda_{\max})^L(\mathrm{tr}(\widehat{\mathbf{R}}) - \lambda_{\max})^{L(M-1)}}.$$

From (31) and (34), the LR function is derived as:

$$\begin{aligned} \mathrm{LR}(\mathbf{Y}) \;&=\; \frac{\sup\limits_{(\mathbf{h},\sigma_n^2,\sigma_s^2)} f(\mathbf{Y}; \mathcal{H}_1, \mathbf{h}, \sigma_n^2, \sigma_s^2)}{\sup\limits_{\sigma_n^2} f(\mathbf{Y}; \mathcal{H}_0)} \\ &=\; \frac{\frac{(L(M-1))^{L(M-1)}L^L}{(\pi e)^{ML}(\lambda_{\max})^L(\mathrm{tr}(\widehat{\mathbf{R}})-\lambda_{\max})^{L(M-1)}}}{\frac{(ML)^{ML}}{(\pi e)^{ML}(\mathrm{tr}(\widehat{\mathbf{R}}))^{ML}}} \\ &=\; \frac{(M-1)^{(M-1)L}}{M^{ML}} \frac{(\mathrm{tr}(\widehat{\mathbf{R}}))^{ML}}{(\lambda_{\max})^L(\mathrm{tr}(\widehat{\mathbf{R}}) - \lambda_{\max})^{L(M-1)}}. \end{aligned} \tag{35}$$

If we define $\mu \overset{\mathrm{def}}{=} \frac{\lambda_{\max}}{\mathrm{tr}(\widehat{\mathbf{R}})}$ then above LR function can be written as:

$$\mathrm{LR}(\mathbf{Y}) \;=\; \frac{(M-1)^{(M-1)L}}{M^{ML}}\left(\frac{1}{\mu(1-\mu)^{M-1}}\right)^L. \tag{36}$$

For decision making, (36) must be compared with a threshold, i.e.,

$$\begin{aligned} &\frac{(M-1)^{(M-1)L}}{M^{ML}}\left(\frac{1}{\mu(1-\mu)^{M-1}}\right)^L \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau_1 \\ &\Longrightarrow \frac{1}{\mu(1-\mu)^{M-1}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau_2, \end{aligned} \tag{37}$$

where $\tau_2 = \left(\frac{M^{ML}}{(M-1)^{(M-1)L}}\tau_1\right)^{\frac{1}{L}}$.

Let $\lambda_1 = \lambda_{\max} \geq \lambda_2 \geq \cdots \geq \lambda_M$ denote the eigenvalues of matrix $\widehat{\mathbf{R}}$ in descending order. It can be easily observed that the supremum and infinimum of the expression $\mu = \frac{\lambda_{\max}}{\mathrm{tr}(\widehat{\mathbf{R}})} = \frac{\lambda_1}{\sum_{i=1}^M \lambda_i}$ are respectively 1 and $\frac{1}{M}$, i.e., $\frac{1}{M} < \mu < 1$. Now since the above LR is an increasing function of μ in the interval $(\frac{1}{M}, 1)$, the GLR test in (37) can be simplified as:

$$\mu = \frac{\lambda_1}{\sum_{i=1}^M \lambda_i} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau, \tag{38}$$

where we have used the fact that $\mathrm{tr}(\widehat{\mathbf{R}}) = \sum_{i=1}^M \lambda_i$. As another form, by manipulation of the above detector, we can express the GLR detector as the following equivalent form:

$$T_{\mathrm{GLRD3}}(\mathbf{Y}) = \frac{\lambda_1}{\sum_{i=2}^M \lambda_i} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta, \tag{39}$$

where $\eta = \frac{\tau}{1-\tau}$.

D. Computational Complexity

The computational complexity of the proposed detectors comes from two major operations: computation of sample covariance matrix and eigenvalue decomposition of the covariance matrix. Here we consider the computational complexity of a trivial approach (which may be numerically non-efficient) in which the largest root of the sample covariance matrix is calculated. For the first part, since the covariance matrix is a block Toeplitz and Hermitian, we need only to compute its first block row. Hence M^2L multiplications and $M^2(L-1)$ additions are required. For the second part, in general at most $\mathcal{O}(M^3)$ multiplications and additions are needed. Thus the total computational complexity is as follows [1]:

$$M^2(2L-1) + \mathcal{O}(M^3) \tag{40}$$

In practice the number of temporal samples L is usually much larger than the number of antennas M and the dominant term is the first term. On the other hand, the ED needs ML multiplications and $M(L-1)$ additions and thus the computational complexity of the proposed detectors is about M times that of the ED.

[1]Please note that the considered approaches for the calculations of the above parameters are not necessarily the best ones, from computational complexity aspect. For instance, the computational complexity can be reduced by determining the dominant singular value of the observation matrix $\mathbf{Y}$, instead of calculating the the largest root of the sample covariance matrix.

E. Invariancy of the GLR Detectors

In the following, we investigate the invariancy of the GLR detectors derived in (39), (29) and (24) under two groups of transformations namely orthogonal transformation and scale:

$$\begin{aligned} G_Q &= \left\{ g_Q | g_Q(\mathbf{Y}) = \mathbf{QY}, \forall \mathbf{Q} \in \mathbb{C}^{M\times M}, \mathbf{Q}^H\mathbf{Q} = \mathbf{I}_M \right\} \\ G_d &= \left\{ g_d | g_d(\mathbf{Y}) = d\mathbf{Y}, \forall d > 0 \right\}. \end{aligned} \tag{41}$$

The above transformations are groups, since they are closed, associative, and contain the identity and inverse elements.

The invariancy of the GLR detector under orthogonal transformation indicates that the proposed GLR detector has the same form if we use the frequency samples instead of temporal samples. The reason is that taking FFT from the temporal samples is equal to applying a unitary matrix transformation to the temporal samples which is an orthogonal transformation. Also invariancy under scale implies that the received samples at the antennas can be amplified or attenuated during sensing process.

In the following, we prove that for blind GLRD, the distribution of the observations and the parameter spaces remain invariant under any compositions of the transformation groups in (41), and the GLRD1 is invariant only under orthogonal transformation.

1) *Orthogonal Transformation*: under $\mathcal{H}_0$, from $\mathbf{Y} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I}_M)$, we get

$$g_Q(\mathbf{Y}) = \mathbf{QY} \sim \mathcal{CN}(\mathbf{0}, \mathbf{QQ}^H\sigma_n^2\mathbf{I}_M) = \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I}_M). \tag{42}$$

under $\mathcal{H}_1$, $\mathbf{Y} \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2\mathbf{hh}^H + \sigma_n^2\mathbf{I}_M)$ and thus we get

$$\begin{aligned} g_Q(\mathbf{Y}) &= \mathbf{QY} \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2 Q\mathbf{hh}^H\mathbf{Q}^H + \mathbf{QQ}^H\sigma_n^2\mathbf{I}_M) \\ &= \mathcal{CN}(\mathbf{0}, \sigma_s^2\mathbf{h}'\mathbf{h}'^H + \sigma_n^2\mathbf{I}_M). \end{aligned} \tag{43}$$

where $\mathbf{h}' = \mathbf{Qh}$. Since the channel gain $\mathbf{h}$ is unknown, the transformed channel gain $\mathbf{h}'$ is also unknown with the same unity norm $\|\mathbf{h}'\|^2 = \|\mathbf{Qh}\|^2 = \mathbf{h}^H\mathbf{Q}^H\mathbf{Qh} = \mathbf{h}^H\mathbf{h} = \|\mathbf{h}\|^2$. Hence the distribution family of the transformed signal is unchanged. The above discussion is valid for both GLRD1 and blind GLRD. As another approach, we know from linear algebra theory that the orthogonal transformation does not change the eigenvalues of a given matrix. Thus, under orthogonal transformation, the decision statistics of both GLR detectors remain unchanged and they are invariant under orthogonal transformation.

2) *Scale*: For blind GLR under $\mathcal{H}_1$, from $\mathbf{Y} \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2\mathbf{hh}^H + \sigma_n^2\mathbf{I}_M)$ we obtain

$$g_d(\mathbf{Y}) = d\mathbf{Y} \sim \mathcal{CN}(\mathbf{0}, d^2\sigma_s^2\mathbf{hh}^H + d^2\sigma_n^2\mathbf{I}_M). \tag{44}$$

Since σ_n^2 and σ_s^2 are unknown, the distribution family of the transformed signal is not changed. Under the null hypothesis, the proof is similar. It is obvious that the GLRD1 is not invariant under scale transformation. In fact, by scaling the observation matrix, all of the eigenvalues are scaled and hence their ratio will be constant for blind GLRD.

IV. Analytical Performance Evaluation

In this section we evaluate the performance of the GLR detectors in terms of detection probability, P_d, and false alarm probability, P_{fa}. For computing the detection probability, we need to compute the statistics of the principal components when the PU signal is present. Also for the false alarm probability, we need to determine the behavior of eigenvalues under null hypothesis, i.e., $\mathcal{H}_0$. We evaluate these probabilities in the following parts.

A. False Alarm Probability

In this part, we first introduce a statistical result for the eigenvalues of the sample correlation matrix , i.e., $\widehat{\mathbf{R}}$, under hypothesis $\mathcal{H}_0$. We then use the result for the calculation of P_{fa}.

Lemma 1: The normalized largest eigenvalue of a complex correlation matrix, i.e., λ_1/σ_n^2, in null case is distributed as Tracy-Widom distribution of order 2 [24], i.e,

$$\frac{\lambda_1/\sigma_n^2 - \mu_{LM}}{\sigma_{LM}} \to W_2 \sim TW2, \tag{45}$$

where the limit is in distribution, and

$$\mu_{LM} = \left(1 + \sqrt{\frac{M}{L}}\right)^2 \tag{46}$$

$$\sigma_{LM} = \frac{1}{\sqrt{L}}\left(1 + \sqrt{\frac{M}{L}}\right)\left(\frac{1}{\sqrt{L}} + \frac{1}{\sqrt{M}}\right)^{1/3}. \tag{47}$$

For the analytical formula of TW_2 refer to [24], and for the tables of its CDF refer to [25].

In the following parts, we assume that the number of antennas, i.e., M, and received sample, i.e., L, are large enough and using (45), we compute the false alarm probability P_{fa} for different GLR detectors derived in parts IV-B1 and IV-B2.

We note that the detector derived in (29) is the same as (24) and thus they have the same performance.

1) GLRD1 (or GLRD2): In this case from (24) and (45), we have:

$$\begin{aligned} P_{\text{fa}} &= P[T_{\text{GLRD1}}(\mathbf{Y}) > \eta | \mathcal{H}_0] \\ &= P[\lambda_1/\sigma_n^2 > \eta | \mathcal{H}_0] \\ &= P[\frac{\lambda_1/\sigma_n^2 - \mu_{LM}}{\sigma_{LM}} > \frac{\eta - \mu_{LM}}{\sigma_{LM}} | \mathcal{H}_0] \\ &= 1 - F_{TW2}(\frac{\eta - \mu_{LM}}{\sigma_{LM}}), \end{aligned} \tag{48}$$

where $F_{TW2}(x)$ is the CDF of Tracy-Widom distribution of order 2. Now for a given P_{fa}, the threshold can be easily obtained as:

$$\eta = \left(1 + \sqrt{\frac{M}{L}}\right)^2 + \frac{1}{L}\frac{(\sqrt{L} + \sqrt{M})^{4/3}}{(\sqrt{LM})^{1/3}} F_{TW2}^{-1}(1 - P_{\text{fa}}), \tag{49}$$

where $F_{TW2}^{-1}(.)$ denotes the inverse CDF of Tracy-Widom distribution of order 2. We note that, unlike the optimum detector, the threshold does not depend on noise variance and channel gains, and can be pre-computed based only on M, L and P_{fa}.

2) GLRD3 (Blind GLRD): In this case, since the distribution of largest eigenvalue is known, from (39) we must compute the distribution of the summation of other eigenvalues under hypothesis $\mathcal{H}_0$. Under $\mathcal{H}_0$, since we have assumed that the number of antennas and samples are large enough, the summation $\frac{1}{M-1}\sum_{i=2}^{M}\lambda_i$ is approximately constant and equals to the variance of noise [26], i.e.,

$$\mathcal{H}_0 : \frac{1}{M-1}\sum_{i=2}^{M}\lambda_i \approx \sigma_n^2, \tag{50}$$

and we conclude that $\sum_{i=2}^{M}\lambda_i \approx (M-1)\sigma_n^2$. Therefore, under hypothesis $\mathcal{H}_0$ the decision statistics can be written as the following form:

$$\mathcal{H}_0 : \frac{\lambda_1}{\sum_{i=2}^{M}\lambda_i} \approx \frac{1}{M-1}\frac{\lambda_1}{\sigma_n^2}. \tag{51}$$

Thus from (45), the false alarm probability can be easily calculated as:

$$P_{\text{fa}} = 1 - F_{TW2}\left(\frac{\eta - \frac{\mu_{LM}}{M-1}}{\frac{\sigma_{LM}}{M-1}}\right), \tag{52}$$

and for a given P_{fa}, the threshold can be easily obtained as:

$$\begin{aligned}\eta &= \frac{\left(1+\sqrt{\frac{M}{L}}\right)^2}{M-1} \\ &+ \frac{1}{L(M-1)}\frac{(\sqrt{L}+\sqrt{M})^{4/3}}{(\sqrt{LM})^{1/3}}F_{TW2}^{-1}(1-P_{\text{fa}}).\end{aligned} \tag{53}$$

Again, we note that, the threshold does not depends on noise variance and channel gains and can be pre-computed based only on M, L and P_{fa}.

B. Detection Probability

Let $l_1 \geq l_2 \geq \cdots \geq l_M$ and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ denote the eigenvalues, in the descending order, of the actual covariance matrix $\mathbf{R}$ and the sample covariance matrix $\widehat{\mathbf{R}}$ defined in (4) and (18), respectively. Under hypothesis $\mathcal{H}_1$, we have the following lemma for the distribution of the largest eigenvalue:

Lemma 2: The largest eigenvalue of a sample matrix under hypothesis $\mathcal{H}_1$, i.e., λ_1, has the normal distribution as follows [23]:

$$\lambda_1 \sim \mathcal{N}\left(l_1 + \frac{(M-1)l_1\sigma_n^2}{L(l_1-\sigma_n^2)}, \frac{l_1^2}{L}\right). \tag{54}$$

Under hypothesis $\mathcal{H}_1$, from (4), we have $\mathbf{R} = \sigma_s^2\mathbf{h}\mathbf{h}^H + \sigma_n^2\mathbf{I}$ and $\det(\mathbf{R}) = (\sigma_s^2\|\mathbf{h}\|^2 + \sigma_n^2)(\sigma_n^2)^{(M-1)}$. Thus, we can conclude that $l_1 = \sigma_s^2\|\mathbf{h}\|^2 + \sigma_n^2$ and $l_2 = l_3 = \cdots = l_M = \sigma_n^2$. By using the received SNR definition, i.e., γ, in (16), (54) can be rewritten as:

$$\lambda_1/\sigma_n^2 \sim \mathcal{N}\left((1+\gamma)\left(1+\frac{M-1}{L\gamma}\right), \frac{(1+\gamma)^2}{L}\right) \tag{55}$$

In the following, by using (55), we compute the detection probability, i.e., P_d for different scenarios.

1) GLRD1 (or GLRD2): In this case, from (24) and (55), the probability of detection can be computed as follows:

$$\begin{aligned}P_d &= P[T_{\text{GLRD1}}(\mathbf{Y}) > \eta|\mathcal{H}_1] \\ &= Q\left(\frac{\eta - (1+\gamma)\left(1+\frac{M-1}{L\gamma}\right)}{\frac{1+\gamma}{\sqrt{L}}}\right) \\ &= Q\left(\frac{\sqrt{L}\eta}{1+\gamma} - \frac{M-1}{\sqrt{L}\gamma} - \sqrt{L}\right).\end{aligned} \tag{56}$$

The above detection probability is conditional on instantaneous SNR, i.e., γ. In the fading channel, the average probability of detection can be computed by averaging over γ distribution, i.e., $f_\gamma(x)$, as follows:

$$\begin{aligned}\overline{P_d} &= \int_0^\infty P_d(x)f_\gamma(x)dx \\ &= \int_0^\infty Q\left(\frac{\sqrt{L}\eta}{1+x} - \frac{M-1}{\sqrt{L}x} - \sqrt{L}\right)f_\gamma(x)dx,\end{aligned} \tag{57}$$

For instance, in Rayleigh fading channel, $f_\gamma(x) = \frac{1}{\bar{\gamma}}e^{-\frac{x}{\bar{\gamma}}}$ and the average probability can be computed by numerical integration.

2) GLRD3 (Blind GLRD): Having the largest eigenvalue distribution, from (39), we must compute the distribution of the summation of the other eigenvalues. We have the following result on the summation of the other eigenvalues:

Lemma 3: Under hypothesis $\mathcal{H}_1$, for the averaged summation of eigenvalues except the largest one, we have [26]:

$$\mathcal{H}_1 : \frac{1}{M-1}\sum_{i=2}^{M}\lambda_i \approx \sigma_n^2 - \frac{\sigma_n^2 l_1}{L(l_1-\sigma_n^2)}. \tag{58}$$

Then, we obtain:

$$\sum_{i=2}^{M}\lambda_i \approx (M-1)(1-\frac{1+\gamma}{L\gamma})\sigma_n^2, \tag{59}$$

where γ is defined in (16). From (55) and (59), the decision statistic will have a Gaussian distribution as follows:

$$\begin{aligned}&\mathcal{H}_1 : \frac{\lambda_1}{\sum_{i=2}^{M}\lambda_i} \sim \\ &\mathcal{N}\left(\frac{(1+\gamma)(1+\frac{M-1}{L\gamma})}{(M-1)(1-\frac{1+\gamma}{L\gamma})}, \frac{(1+\gamma)^2}{L(M-1)^2(1-\frac{1+\gamma}{L\gamma})^2}\right).\end{aligned} \tag{60}$$

So, the detection probability can be calculated as:

$$P_d = Q\left(\frac{\eta\sqrt{L}(M-1)(1-\frac{1+\gamma}{L\gamma})}{1+\gamma} - \frac{\sqrt{L}}{1+\frac{M-1}{L\gamma}}\right), \tag{61}$$

and in fading channel, the average detection probability can be computed by averaging the above probability over SNR, i.e., γ.

$$\begin{aligned}&\overline{P_d} = \\ &\int_0^\infty Q\left(\frac{\eta\sqrt{L}(M-1)(1-\frac{1+x}{Lx})}{1+x} - \frac{\sqrt{L}}{1+\frac{M-1}{Lx}}\right)f_\gamma(x)dx.\end{aligned} \tag{62}$$

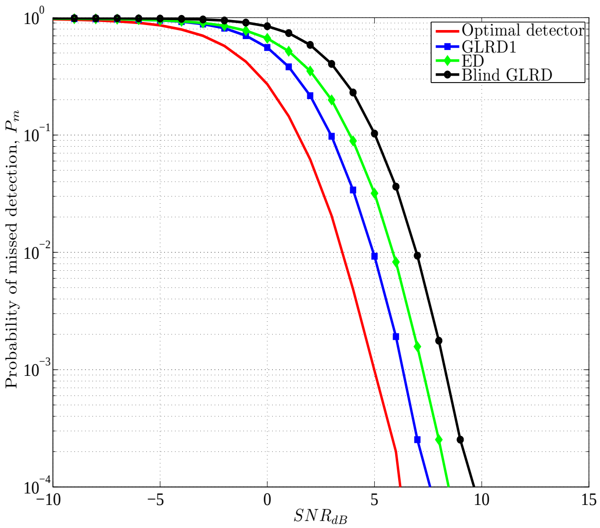

Fig. 2. The probability of missed detection of the GLR detectors, ED and optimal detector versus SNR for $P_{\text{fa}} = 10^{-2}$, $M = 4$ and $L = 16$.

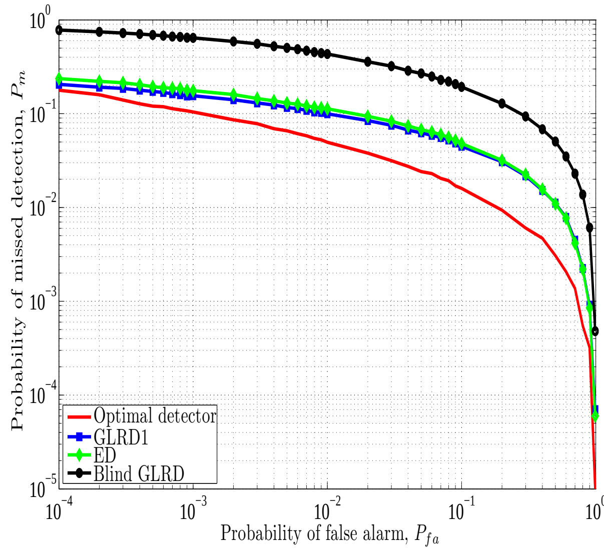

Fig. 3. The complementary ROC (P_m vs. P_{fa}) of different detectors in AWGN channel, for $SNR = 5$ dB, $M = 2$ and $L = 8$.

V. Simulation Results and Discussion

In this section, we present some numerical results to e ate the performance of the proposed detectors. Figure 2 d the probability of missed detection P_m of the optimal det the proposed GLR detectors and the energy detector v SNR at a false alarm rate of $P_{\text{fa}} = 10^{-2}$, $L = 1$ $M = 4$. In order to determine the threshold for a false alarm probability, we have generated the decision sta randomly according to its distributions for 10^6 indepe trials (in absence of PU signal) and chosen the dete threshold as $100P_{\text{fa}}$ percentile of the generated data, i.e $P_{\text{fa}} = 10^{-3}$, $100 \times 10^{-3} = 0.1\%$ of the generated de statistic (out of 10^6) are above the determined threshol can be observed, by increasing the SNR the performance detectors improves and the ED which knows the exact of noise variance performs only better than the blind G Also, the performance of the GLRD1 [2] is better than th and blind GLRD.

Figure 3 show the complementary ROC (Receiver Operating characteristics) or the probability of missed detection, i.e. P_m, versus probability of the false alarm, i.e., P_{fa}, for different detectors in AWGN channel for SNR = 5 dB, $M = 2$, and $L = 8$. Also, Figures 4, 5, and 6 show these curves for average SNR, $\overline{\gamma} = 5$ dB and $L = 8$, respectively, for $M = 2$, $M = 4$ and $M = 6$ in Rayleigh fading channel. As can be observed, the performance of all detectors degrades slightly in fading channel compared with AWGN channel. One approach to improve the performance in the fading channels, is to use collaborative spectrum sensing [3], [4], [27]. By collaboration among the SUs, the deleterious effect of fading can be mitigated and a more reliable spectrum sensing can be achieved. In fact, in collaborative spectrum sensing, the SUs use the available spatial diversity to improve their

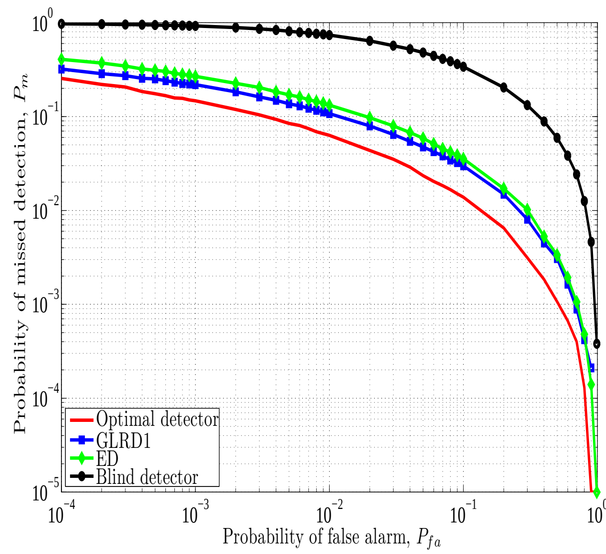

Fig. 4. The complementary ROC of different detectors in Rayleigh fading channel, for average $SNR = 5$ dB, $M = 2$ and $L = 8$.

performance.

As can be seen from Figures 4,5,6 by using more antennas, like in a collaborative spectrum sensing, the performance improves due to the spatial diversity provided.

Our further simulation results, which have not provided here, indicates that by increasing the number of samples, i.e., L, the performance improves[3]. However as expected, the simulation results indicate that increasing the number of antennas, i.e., M, compared to increasing the number of samples, i.e., L, has more substantial effect on the performance

[2]Please note that the GLRD1 and GLRD2 derived in Sections III-A and III-B are identical.

[3]Note that, we can not increase L arbitrarily since L determines the acquisition time (the waiting time-lag before a decision can be made). Thus in practice, we have to make a trade-off between P_{fa} (the spectrum usage efficiency), P_{m} (PU interference protection level) and L (the acquisition time).

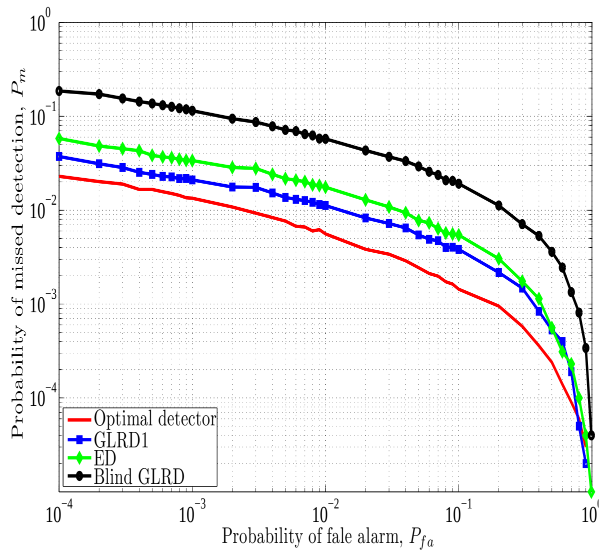

Fig. 5. The complementary ROC of different detectors in Rayleigh fading channel, for average $SNR = 4$ dB, $M = 4$ and $L = 8$.

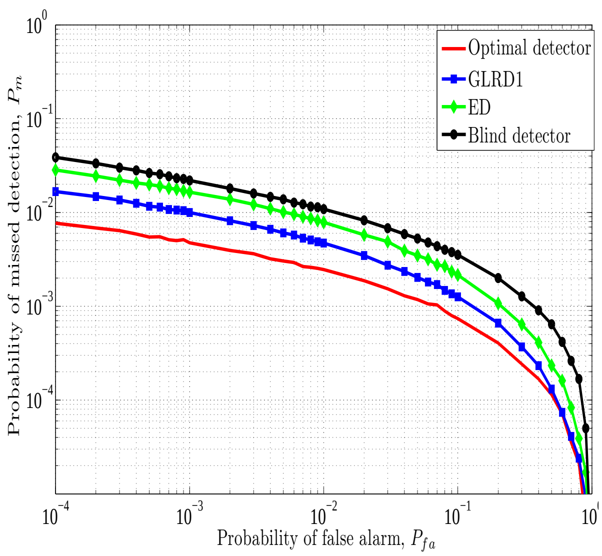

Fig. 6. The complementary ROC of different detectors in Rayleigh fading channel, for average $SNR = 5$ dB, $M = 6$ and $L = 8$.

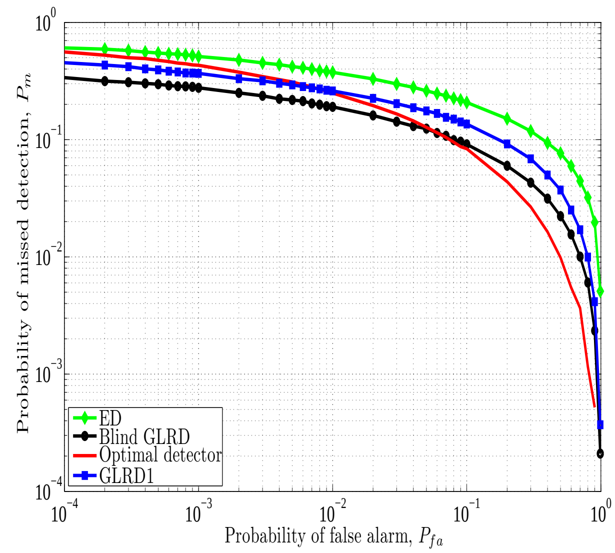

Fig. 7. The effect of noise variance mismatch on the performance of the ED and GLR and optimal detectors, for α_{dB}=0.5, $SNR = 3$dB, $L = 16$ and $M = 4$.

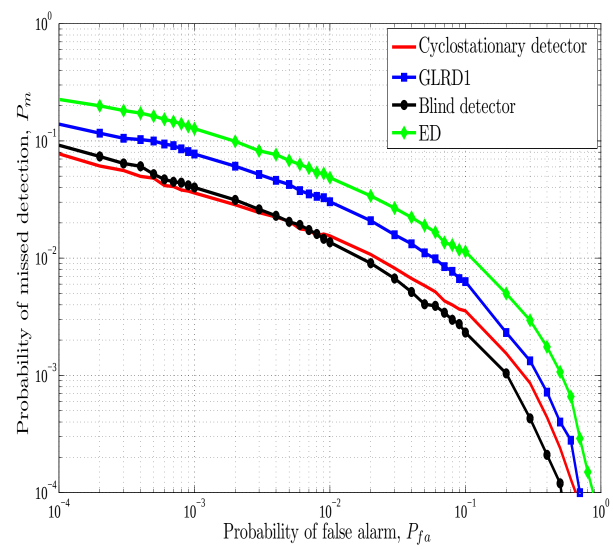

Fig. 8. The Performance of Blind detector, GLRD1, Energy detector (ED) and cyclostationarity based detector, for α_{dB}=0.5, $SNR = -10$dB, $L = 4096$, and $M = 4$.

improvement of the different detectors in fading channels.

In Figure 7, we compare the proposed GLR detectors with the optimal detector and ED under noise variance mismatch of 0.5 dB. For noise mismatch, it is assumed that $|10\log_{10}(\frac{\tilde{\sigma}_n^2}{\sigma_n^2})| = \alpha_{\mathrm{dB}}$, where $\tilde{\sigma}_n^2$ is the actual noise variance, and α_{dB}, defined as noise uncertainty factor, is considered as a uniform distribution variable in the interval $\alpha_{dB} \sim U[-0.5, 0.5]$. In practice, the noise uncertainty factor, i.e., α_{dB}, in receiver is normally 1-2 dB which due to the existence of interference can be much higher [10], [15]. As can be realized, the GLR detectors are more robust to the noise uncertainty than the optimal detector and ED, and in fact under noise variance mismatch, the optimal detector performs similar to the GLRD1. Also, the blind GLRD performs slightly better than the GLRD1 in which only the variance of noise is known. Under a greater noise uncertainty factor, the performance of ED and optimal detectors degrades more substantially and the GLR detectors present a better performance. From this figure and our further simulations, we conclude that the optimal detector can outperform the GLR detectors provided that the optimal detector knows the noise variance accurately enough. However in practice, there are uncertainty about the noise variance which under these unavoidable circumstances, the blind GLRD outperforms the optimal detector and ED. In Figure 8, we evaluate the performance of the proposed detectors in a typical practical applications. In IEEE 802.22 standard (WRAN), the CRs need to detect the presence or absence of

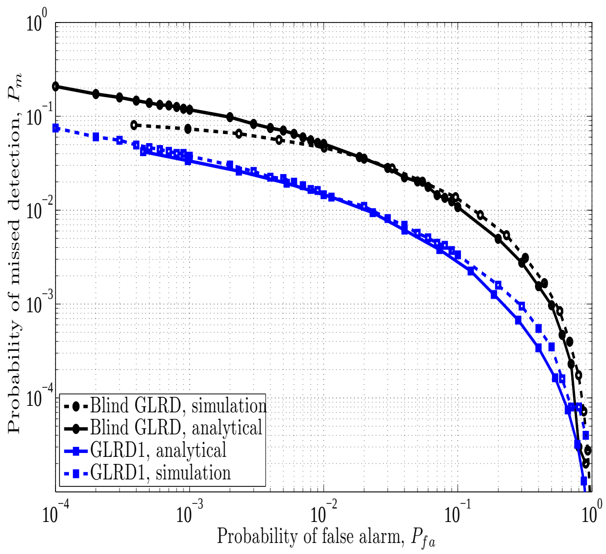

Fig. 9. Comparison between simulation and analytical performance of GLR detectors, for SNR= 3 dB, $M = 8$ and $L = 32$.

wireless microphone, and digital and analog TV signals which in north America these signals are respectively FM, NTSC, and ATSC signals [28]. This figure illustrates the performance of blind GLR, GLRD1, cyclostationarity based detecor, and ED, when the PU signals are considered as captured DTV signals and the parameters are set as $\alpha_{\rm dB} = 0.5$, $SNR = -10dB$, $L = 4096$, and $M = 4$. For simulation, the captured DTV signal samples have been taken from [29]. As can be seen, in this case even thought the PU signal is not a Gaussian signal, the performance of the proposed detectors, i.e., blind detector and GLRD1, are acceptable, and the blind detector performs like and even slightly better that the cyclostationarity based detector. We note that the cyclostionarity based detector uses the available information of DTV signal such as time duration and waveform and cyclic frequencies, and is highly computationally complex for practical implementation. On the other hand, as mentioned before, the blind detector can be implemented easily and does not use any information of PU signal. Our further simulations indicate similar behaviors for the other considered IEEE 802.22 potential signals, i.e., for FM wireless microphone and analog TV signals. In Figure 9, we have presented the performance evaluation of the GLR detectors based on both analytical and simulations, for SNR = 3 dB, $M = 8$, and $L = 32$. As can be observed, the simulation results well confirm the analytical derivations. It is notable that because of asymptotic approximations used in deriving analytical results, the simulation and analytical results will well coincide provided that the number of samples and antennas are large enough. In fact, the available gap between the analytical and the simulation results will decrease by increasing the number of samples or the number of antennas.

VI. Conclusion

In this paper, we considered the spectrum sensing for the CRs equipped with multiple antenna receivers. We derived the optimal detector which needs to know the variances of the PU signal and noise as well as the channel gains. We also presented the GLR detectors in which some or all of these parameters are unknown. We evaluated the performance of the proposed detectors in terms of false alarm and detection probabilities. The simulation results revealed that the proposed GLR detectors perform better than the ED and almost identical to the optimal detector under noise variance mismatch.

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