Estimating aquifer hydraulic properties from the inversion of surface Streaming Potential (SP) anomalies

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[1] Electrokinetic effects of water flow during pumping tests have been shown to generate surface Streaming Potential (SP) anomalies of several tens of mV that are well correlated with the geometry of the water table. It follows that SP measurements can be used to estimate aquifer hydraulic properties. We have developed an inversion scheme for surface SP data generated by flow pumping and found that we are able to estimate the hydraulic conductivity and the depth and the thickness of the aquifer. We applied our inversion scheme to the data from *Bogoslovsky and Ogilvy* [1973] and found a hydraulic conductivity of 10^{-5} m.s^{-1}, an aquifer thickness of roughly 28 m and an electrokinetic coupling coefficient of -1 mV.m^{-1}. These values are in the range of what is expected for this kind of environment.

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1. Introduction

[2] Field estimates of aquifer hydraulic properties require the use of several expensive observation wells. An alternative is to use minimally invasive geophysical methods to estimate groundwater fluxes. Standard methods (e.g., electrical sounding, ground-penetrating radar, time-domain electromagnetic methods) detect the presence of water by changes of ground physical properties (e.g., electrical conductivity, dielectric permittivity) but none of them are sensitive to actual water flow, except the Streaming Potential (SP) method. Indeed, the SP method measures with electrodes the natural electric potential variations that include those generated by the electrokinetic effect of underground fluid flow. Thus, the distribution of the electric potentials allows us to map groundwater flow features [e.g., *Corwin and Hoover*, 1979].

[3] In the case of water pumping, the electrokinetic effect generates surface SP anomalies of several tens of mV that are very well correlated to the geometry of the water table [*Bogoslovsky and Ogilvy*, 1973, *Kelly and Mares*, 1993]. Therefore, can we use the SP technique for estimating hydraulic properties of the aquifer?

[4] To investigate this matter, several potential-field inversion techniques are available. All these techniques describe either the shape of the water table [*Fournier*, 1989; *Birch*, 1998] or the location and/or the shape of the electrokinetic sources [*Patella*, 1997; *Sailhac and Marquis*, 2001; *Revil et al.*, 2003] but none of these describes completely the geometry of the aquifer, especially its thickness. We have therefore developed a new inversion scheme of surface SP data obtained during flow pumping to estimate the aquifer hydraulic properties (thickness, depth and hydraulic conductivity). We tested our approach on the SP and hydraulic data sets from *Bogoslovsky and Ogilvy* [1973] to estimate the resolution of each hydraulic parameter.

2. Inversion Scheme of Surface SP Anomalies Induced by Water Pumping

2.1. Hydraulic Model

[5] Our inversion scheme for surface SP anomalies is based on a forward model of the electrokinetic effect of pumping in a homogeneous unconfined aquifer. This SP modeling is a three-step process that consists in solving the hydraulic problem, computing the streaming current sources and solving the electric problem.

[6] We assumed that the water flow occurs in a homogeneous unconfined aquifer in steady-state conditions limited at the bottom by impermeable bedrock. We assumed that the bedrock is horizontal and hence so is the flow (Dupuit's hypothesis). Moreover, we assumed that the flow is radial, that the water flux around the well is equal to the pumping flow rate Q and that the piezometric head in the vicinity of the well (at r_0) is h_0 (Figure 1). Under these assumptions, the solution of (1) is for $r > r_0$ [*De Marsily*, 1986]

$$h(r) = \sqrt{h_0^2 + \frac{Q}{\pi K}\ln(r/r_0)} \tag{1}$$

where h is the piezometric head (m) and K is the aquifer hydraulic conductivity (m.s^{-1}).

2.2. Electric Current Sources Calculation

[7] The second step is to convert the modeled piezometric head h into electrokinetic current sources I_s. For this purpose, we used the electric current conservation equation [cf. e.g., *Revil et al.*, 1999]

$$\vec{\nabla}.\left(\sigma\vec{\nabla}V\right) = -\left(\vec{\nabla}L.\vec{\nabla}P + L\nabla^2 P\right) \tag{2}$$

where P is the fluid pressure (Pa), V is the electric potential (V), L is the electrokinetic coupling parameter (V.S.Pa^{-1}.m^{-1}) and σ the ground electrical conductivity (S.m^{-1}). L depends mainly on the rock lithology and the fluid chemistry [*Revil et al.*, 1999]. Let us assume that the soil and the fluid are homogeneous and hence that L is constant, so the first term of the right hand side of equation (4) is negligible.

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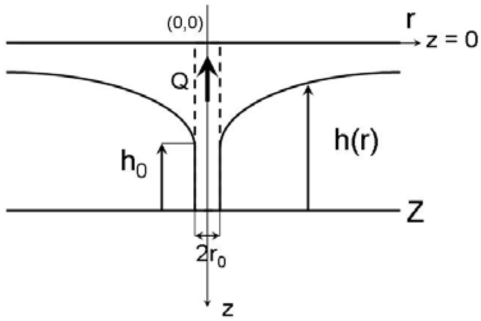

Figure 1. Sketch of the unconfined aquifer used to model the electrokinetic effect of a pumping test.

[8] Instead of using fluid pressure, we prefer to use the piezometric head (h = P/ρg) and (4) can be written as

$$\vec{\nabla}.\left(\sigma\vec{\nabla}V\right) = -L\nabla^2 P = -\rho g L\nabla^2 h \qquad (3)$$

where ρ is the fluid density (kg.m^{-3}) and g is the gravity acceleration (m.s^{-2}). In this equation, the source term on the right hand side can be interpreted as electrokinetic current sources I_s (A.m^{-3}). Combining (3) and (5), we get

$$I_s = -\rho g\nabla^2 h = \frac{\rho g L Q^2}{4\pi^2 K^2 r^2\left(h_0^2 + \frac{Q\ln(r/r_0)}{\pi K}\right)^{3/2}} \qquad (4)$$

2.3. Electrical Model

[9] The final step is to compute the electric potential distribution V by solving the electric current conservation equation (5) knowing the ground electrical conductivity distribution. In well pumping experiments, the high electrical conductivity of the metal casing disturbs the electric field near the wells. *Ishido et al.* [1983] proposed that the metal casing was channeling the electric currents generated at reservoir depth to the surface and hence to the surface electrodes. Therefore, our electrical conductivity model is a homogeneous half-space with a high-conductivity vertical cylindrical body representing the casing. We did not take into account the drop of electrical conductivity at the top of water table caused by water content decrease in the vadose zone because it is a second-order effect less than two orders of magnitude [e.g., *Revil et al.*, 1999] compared to the electrical conductivity contrast between the casing and the host-medium more than eight orders of magnitude [e.g., *Schenkel and Morrison*, 1990].

[10] We solve this problem using an integral formulation [*Schenkel and Morrison*, 1990] that gives a numerical estimate of the casing Green function G as a function of the ground and casing electrical conductivities and of the casing length, radius and thickness. G is obtained numerically by solving the electric current conservation equation (5) in both the half-space and the casing. Therefore, by integrating all electrokinetic sources induced by the water pumping (equation 6), we get the solution of (5) at the ground level,

$$V(r,z=0) = \int_0^{\infty}\int_0^{2\pi}\int_{Z-h(r')}^{Z} G(r,z=0,r',\theta',z')I_s(r',z')r'dz'd\theta'dr' \qquad (5)$$

The vertical integration is limited to the aquifer thickness because $I_s = 0$ outside the aquifer.

2.4. Inversion of SP Data

[11] Equation (7) describes the surface SP anomaly induced by a pumping test at rate Q in an unconfined homogeneous aquifer of hydraulic conductivity K, electrokinetic coupling parameter L and bedrock depth Z. Therefore, knowing the hydraulic head h_0 in the vicinity of the well (at r_0) and the surface SP anomaly during a pumping test of flow rate Q, we should be able to determine Z, K, and L. We used a genetic algorithm [*Dorsey and Mayer*, 1995] to find the best {Z, K, L} that minimizes a weighted quadratic error function *f* between the predicted and observed SP data

$$f = \sqrt{\frac{1}{\sum_{i=1}^{N} w_i}\sum_{i=1}^{N} w_i\left(V_{0i} - V_{Pi}\right)^2} \qquad (6)$$

where N is the number of measurements, w_i is the weight of measurement i ($0 < w_i < 1$) and V_{0i} and V_{Pi} are respectively the observed and predicted SP data i.

3. Application to the Inversion of *Bogoslovsky and Ogilvy* [1973] Data Set

[12] During a long-term pumping experiment, *Bogoslovsky and Ogilvy* [1973] have recorded simultaneously the surface SP anomaly and piezometric heads at nine observation wells (Figure 2). They observed a very good correlation between these two profiles that they attributed to the electrokinetic effect of the groundwater flow. Besides, two negative anomalies (at −65 and 110 m) were recorded above drainage ditches certainly caused by water infiltration.

[13] We performed an inversion of the SP data from only one side of well because our model requires a cylindrical

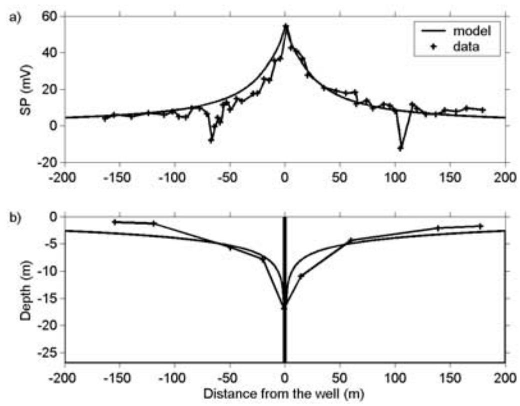

Figure 2. Result of the inversion of SP data from *Bogoslovsky and Ogilvy* [1973]. (a) Modeled (solid line) and recorded (cross line) surface SP anomaly. (b) Modeled (solid line) and recorded (cross line) piezometric heads. We found a bedrock depth of Z = 28 m, a ratio of the flow rate and the hydraulic conductivity Q/K = 290 m^2 and an electrokinetic coupling parameter L = 0.8 mV.S.MPa^{-1}.m^{-1} (C = −0.8 mV.m^{-1}).

symmetry; we used here the data that have positive abscissa. We assumed that the piezometric head at the well (h_0) was equal to the piezometric head recorded by the authors close to the well [i.e., −17 m (Figure 2)]. We chose weights of 1 for measurements located closer than 50 m from the well and of 0.5 further than 50 m to increase the influence of the significant measurements close to the well. We chose a weight of 0 for the small negative anomaly because it is not related to the pumping. For this inversion, we assumed that the casing length is 20 m, the inner radius 9 cm, the thickness 1.3 cm, the electrical conductivity 10^8 S.m^{-1} and the host medium electrical conductivity 100 Ω.m that are typical values for pumping casings [*Schenkel and Morrison*, 1990] in shallow formations like sandy gravels [*Aleshin et al.*, 1969].

[14] We found a bedrock depth of Z = 28 m, a ratio of the flow rate and the hydraulic conductivity Q/K = 290 m^2 and an electrokinetic coupling parameter L = 0.8 mV.S.MPa^{-1}.m^{-1}. We show the result of inversion on Figure 2 where for sake of comparison, we plotted the observed and predicted SP data (Figure 2a) and the observed and predicted hydraulic heads (Figure 2b).

[15] We obtained a satisfactory fit of the trend of hydraulic heads (Figure 2a). Nevertheless, this fit is not perfect because our inversion does not take into account local hydraulic or electrical conductivity heterogeneities. As we do not know the pumping flow rate, we can only estimate K: assuming a typical flow rate value of few cubic meters per hour (e.g., 10 m^3.h^{-1}) we found a hydraulic conductivity of around 10^{-5} m.s^{-1}; this is in agreement with what is expected for sandy gravels for which K ranges from 10^{-5} to 10^{-3} m.s^{-1} [*De Marsily*, 1986]. We found an electrokinetic coupling parameter L of 0.8 mV.S.MPa^{-1}.m^{-1} but to ease comparison with published values, we prefer to use the electrokinetic coupling coefficient C in mV.m^{-1}

$$C = -\frac{\rho g L}{\sigma} \tag{7}$$

where σ is the ground electrical conductivity. We found a C of −0.8 mV.m^{-1}, a reasonable value since C is usually in the range of −1 to −10 mV.m^{-1} for typical groundwater [*Revil et al.*, 2002].

4. Discussion

4.1. Robustness of the Inversion

[16] To investigate the robustness of the inversion, we have performed 125 successive inversions of the SP data. Each inversion stopped when the optimal solution did not change for 40 generations. The histograms of the different solutions are shown in Figure 3. The values of Z are clustered within 2% of its mean value, Q/K within 7% for and C within 13%. These small values indicate that the genetic algorithm gives similar solutions even though it explores a wide parameter space (shown by the abscissae of Figure 3).

4.2. Sensitivity of the Inversion

[17] To investigate the reliability of our inversion of SP data, we tested the sensitivity of the quadratic error function *f* to each parameter (Z, Q/K or C). As during pumping the flow rate Q is obviously known, we prefer to use K instead

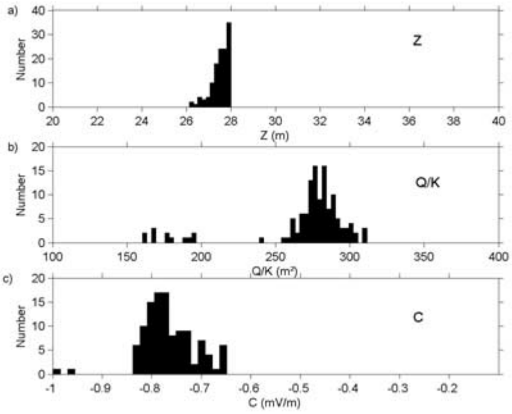

Figure 3. Distribution of the solutions found from 125 inversions of the SP data: (a) depth of the aquifer bedrock, (b) ratio flow rate Q above the hydraulic conductivity K and (c) electrokinetic coupling coefficient.

of Q/K; for this purpose, we chose as previously an arbitrary value of 10 m^3.h^{-1} for Q.

[18] We fixed two out of three parameters to their values obtained in the best-fitting model and set free the third parameter. The result is shown on Figure 3 where we plotted the quadratic error function *f* as function of Z (Figure 3a), the logarithm of K (Figure 3b) and C (Figure 3c). We chose to use the logarithm of K instead of K because it can easily vary over several orders of magnitude for shallow formations. The gradient of the error function *f* around the minimum gives an idea of resolution of each parameter: the strongest the gradient, the better the parameter. We found a resolution of 2 mV/m for Z, on average 50 mV/log(m/s) for K and 8 mV/(mV/m) for C. The best determined parameter is therefore the hydraulic conductivity K (or the ratio Q/K), followed by the coupling coefficient C and the bedrock depth Z. K is well determined because the electrokinetic current source term I_s depends on K squared (equation 6). C is also relatively well defined because it acts as a multiplicative factor of I_s. On the contrary, Z is poorly resolved because it only appears in the integration domain of equation (7) and does not affect the intensity of I_s (equation 6). Moreover, for great Z values, the additional sources have negligible effects at the surface, hence the low variation rate of *f* for large Z values (Figure 3a).

[19] Figure 3b shows that if we can estimate C (e.g., from laboratory tests on samples) and Z (e.g., from drilling or seismic method), we can reasonably expect to estimate the hydraulic conductivity from the inversion of surface SP data. However, as C depends on the chemistry of the water/rock interaction [*Revil et al.*, 1999], it can change dramatically within the same aquifer depending on the lithology and the water composition. Moreover, the depth of bedrock Z is also a parameter difficult to obtain because in real cases the bedrock is not horizontal and even sometimes discontinuous. Therefore, we can not expect to know very well C and Z for real cases and consequently, the accuracy on K will not be as good as than what is shown on Figure 3b.

[20] To improve the accuracy on Z, C and hence on K, we propose to over-determine the inversion problem by performing a joint inversion of several SP data sets acquired during pumping tests at different rates. Indeed, each test

should provide new information on the system as the SP response is a function of Q (equation 6). *Kawakami and Takasugi*, [1994] actually observed such a behavior during injection and production tests in a geothermal reservoir: they observed that the surface SP anomalies change their sign according to the flow direction and that they increase with the flow rate. Therefore, several hydraulic tests at different flow rates should give redundant information on Z, C, and K that should improve greatly the accuracy on Z, C, and K.

4.3. Surface SP: A Mirror Image of the Water Table?

[21] In our inversion, we assumed that the flow was radial around the well but it is obviously not the case as we can observe it on the hydraulic heads profiles on each side of the well (Figure 2b). This asymmetry seems related to the asymmetry of the surface SP data (Figure 2b) and explains why *Bogoslovsky and Ogilvy* [1973] concluded that "the natural electric field may be regarded as a mirror image of the cone surface". The more likely explanation for this SP asymmetry is that before pumping, the aquifer is not at equilibrium because away from the well, the piezometric head is higher on the left side than on the right side.

[22] This good correlation between both electric and hydraulic potentials can be explained by the fact that the fundamental equations governing both potentials are both diffusion equations (1) and (4) with sources roughly located at the same place (i.e., at the openhole). Indeed, as the electrokinetic sources vary roughly in radius squared (cf. equation 6), we can assume that they are concentrated around the well.

[23] The electrical conductivity model of our inversion takes into account a highly electrically conductive steel casing that is channeling the electric currents to the surface. Figure 4 shows the distribution of the normalized electric currents generated by the electrokinetic effect of the pumping. We can clearly observe that the surface SP anomaly is caused by the electric currents coming from the casing. There are therefore no direct relationship between the surface SP anomaly and the shape of the water table as suggested by *Bogoslovsky and Ogilvy* [1973]. We point out that all SP analysis methods based on Fournier's integral formula [*Fournier*, 1989; *Birch*, 1998; *Revil et al.*, 2002]

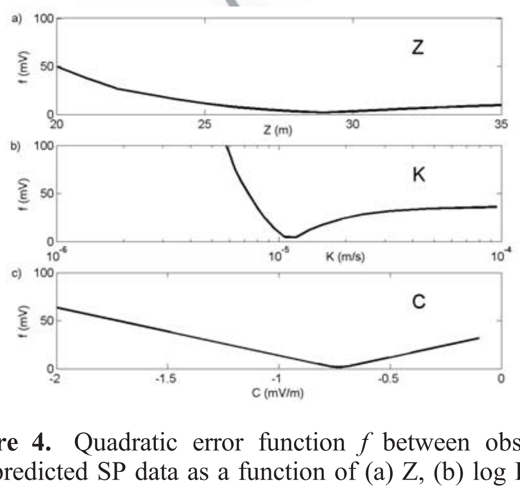

Figure 4. Quadratic error function f between observed and predicted SP data as a function of (a) Z, (b) log K and (c) C. The gradient of the error function f around the minimum gives an idea of resolution of each parameter: the strongest the gradient, the better the estimate.

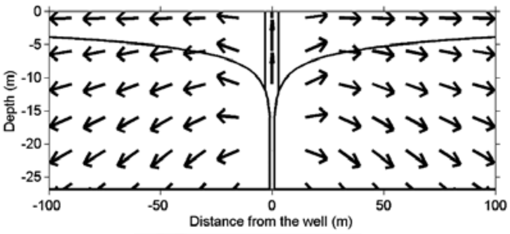

Figure 5. Distribution of electric currents normalized to their intensities generated by water flow during pumping. Note that the surface SP anomaly is caused by the electric currents coming from the reservoir through the steel casing situated at distance = 0.

assume that the ground electrical conductivity above the aquifer is constant; they can therefore not be applied for quantifying the shape of the water table when highly electrically conductive casings are buried in the ground.

[24] Tomographic algorithms of *Patella* [1997] and *Revil et al.* [2003] give a source depth of around 5 m that is much shallower than the true depth of the SP sources (between 17 and 25 m). This discrepancy is caused by the fact that these techniques are as sensitive to the ground electrical conductivity contrasts as to the SP source location; they can therefore not be applied for locating the SP source when the well is steel-cased.

5. Conclusions and Perspectives

[25] Our inversion scheme of Streaming Potential (SP) anomalies induced by water pumping allows us to give an estimate of the hydraulic conductivity and of the geometry of an aquifer. As it takes into account the presence of highly electrically conductive steel casings that channel the electric currents to the surface, it generalizes standard SP analysis methods.

[26] To improve the accuracy of the parameter estimates, we propose to perform a joint inversion of SP monitoring data acquired during pumping tests at different rates. However, this approach would require rewriting the proposed steady-state equations in their transient expressions. Nevertheless, this would allow to gain more insight into the transient regime of the pumping and therefore to get an estimate of the specific storage coefficient of the aquifer. Furthermore, time monitoring of SP data effectively removes any static effect caused by electrical conductivity contrasts, e.g., due to a casing as suggested by *Marquis et al.* [2002], and therefore to allow us the localization of the electrokinetic sources. Thus, surface SP monitoring can provide a picture of the groundwater flow distribution and perhaps allows to identify ground hydraulic conductivity heterogeneities without in-situ measurements.

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