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# Priority Setting in Health Care and Higher Order Degree Change in Risk

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## ABSTRACT

This paper examines how priority setting in health care expenditures is influenced by the presence of uncertainty surrounding first, the severity of the illness and second, the effectiveness of medical treatment. We provide necessary and sufficient conditions on social preferences for which a social planner will allocate more health care resources to the higher risk population. Change in risk is defined through the concept of stochastic dominance up to order  $n$ . The shape of the social utility function and an equity weighting function are considered to model the inequality aversion of the social planner. We show that for higher order risk changes, usual conditions on preferences such as prudence or relative risk aversion are not necessarily required to prioritise health care under more uncertainty.

JEL Classification: D61, D81, H51, I18

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## 1. Introduction

Uncertainty is an intrinsic characteristic of health care. As stressed by Arrow (1963) in his seminal work, uncertainty in health care relates mainly to two sources, the uncertainty surrounding the severity of illness and the uncertainty surrounding the effectiveness of medical treatment. Another characteristic of health care is that its consumption as a proportion of growth domestic product is steadily increasing in most countries worldwide, calling governments with a limited health care budget to prioritise health care expenditures among populations. Surprisingly no theoretical works, except those of Hoel (2003) and Bui et al. (2005) to the best of our knowledge, have addressed the issue of priority setting in health care in the face of more uncertainty. This paper tries to fill this gap.

As an illustration, consider a population confronted with the same disease for which a medical treatment is available. The population is composed of two types of individuals identical in all respects except for the fact that the severity of the disease is more uncertain for the first type of patients than for the second type of patients. This could be illustrated

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by the first type being confronted with co-morbidity or other health risks which increase the uncertainty of the health level. Consider the socially optimal allocation of a fixed health care budget that has to be made at the level of society. Should the social planner allocate more resources to the patient who is more at risk? The aim of this paper is to provide necessary and sufficient conditions on social preferences for which the social planner will allocate more health care resources to the higher risk population. Following Arrow's (1963) classification, this paper considers two sources of risk, either on the severity of the disease or on the effectiveness of the treatment. So as to define change in risk, we use the framework of  $n^{\text{th}}$ -order stochastic-dominance. Stochastic-dominance encompasses very general forms of risk changes and provides a useful tool to model these changes as shown in various works (see e.g. Gollier, 2001). In particular, stochastic-dominance includes the concepts of mean-preserving increase in risk introduced by Rothschild and Stiglitz (1970) as well as of increase in downside risk as defined by Menezes et al. (1980). This paper thus establishes conditions under which a social planner will allocate more resources to the patient who is in the more unfavourable situation, where the more unfavourable situation is defined in terms of stochastic dominance.

Our work relies on the utility approach to model the aggregation of health benefit by the social planner as introduced by Wagstaff (1991) and Dolan (1998). Under this approach, aversion to health inequalities is modelled through a concave social utility function over health outcomes, i.e. extra health is considered to be as more desirable when one is in poor health than when in close to perfect health. Yet, this approach has been criticised on the grounds that it does not allow one to dissociate attitudes towards outcome from attitudes towards inequality. Recently, an alternative approach has been proposed that weighs individuals with respect to their health level, reflecting the inequality aversion of the social planner through the shape of the equity weighting function. This approach is referred to as the Rank-Dependent QALY model (Bleichrodt et al., 2004). Hence we also consider the case where inequality aversion is modelled under an equity weighting function.

In the case of uncertainty on the severity of the disease, we show how the most commonly used social utility functions, i.e. those whose successive derivatives to any order  $n$  alternate in signs such as the logarithmic function and the power function (Bleichrodt et al., 2005), lead to prioritisation of the patients more at risk for any  $n^{\text{th}}$ -order increase in risk. In the case of a risk on the effectiveness of the medical treatment, conditions on preferences needed to prioritise the patients more at risk are more limiting and necessitate conditions on  $n^{\text{th}}$ -order relative risk aversion. While the signs of higher derivatives to order  $n$  as well as conditions on  $n^{\text{th}}$ -order relative aversion have been recently shown to drive various economic behaviours (see Eeckhoudt and Schlesinger, 2008; Chiu et al., 2011), it is the first time, to the best of our knowledge that these concepts are applied to health economics.

As said before, the works of Hoel (2003) and Bui et al. (2005) are the only ones we are aware of which address the issue of priority setting in health care in the face of more uncertainty. Our work differs from theirs in various respects. First Hoel (2003) and Bui et al. (2005) limit themselves to increase in risk in terms of second order increase in risk as they compare the certain case to the uncertain case. We consider higher order increase in risk up to order  $n$ , which make it possible to generalise results to higher orders and to compare two risky situations. Second, Hoel (2003) and Bui et al. (2005) do not differentiate between the forms of uncertainty as introduced by Arrow (1963) which

provide them with conditions only on the sign of the third derivative of the social planner's utility function to prioritise the patients at risk. We consider two types of uncertainty, either on the severity of the disease, or on the effectiveness of medical treatment. In the second case, we show that new conditions, i.e. on  $n^{\text{th}}$ -order relative risk aversion, drive the results. Finally, Hoel (2003) and Bui et al. (2005) consider only the utility approach, while we consider inequality aversion in terms of both the concavity of the social utility function and the equity weighting function.

This paper is organised as follows. In the next section, we introduce the general model of health care allocation under uncertainty. Section 3 presents the concepts of  $n^{\text{th}}$ -order stochastic dominance and of increase in  $n^{\text{th}}$ -degree risk (Ekern, 1980) as a special case of  $n^{\text{th}}$ -order stochastic dominance. Section 4 deals with uncertainty on the severity of the disease. Section 5 addresses the case of uncertainty on the effectiveness of health care. Section 6 considers the equity weighting function to define the inequality aversion of the social planner. Finally, a short conclusion is provided in the last section.

## 2. The model

The model is based both on the Dardanoni and Wagstaff (1990) model in the way uncertainty in health care is defined and on the Hoel (2003) and Bui et al. (2005) models in terms of the health care allocation problem. Consider a population composed of two types of individuals with  $\alpha_i$  representing the share of persons of type- $i$  (with  $i = 1, 2$ ) and such that  $\alpha_1 + \alpha_2 = 1$ . We assume that health can be quantified, for instance through quality-adjusted life-years (QALY). Health is a function  $H(c)$  of the form:<sup>2</sup>

$$H(c) = a + m(c), \quad (1)$$

where  $a$  can be interpreted as the basic level of health (health condition) reflecting the severity of the disease, and  $m(c)$  reflects the effectiveness or productivity of medical care  $c$ . We assume that higher investments in medical care improve the health of the patients, but that the marginal benefits from additional medical care decrease ( $m'(c) > 0 \forall c$ ,  $m''(c) \leq 0 \forall c$ ).

Uncertainty on the health level  $H(c)$  can take two forms, either  $\tilde{H}(c) = \tilde{a} + m(c)$ , or  $\tilde{H}(c) = a + \tilde{m}(c)$ . In the first situation, the effectiveness of health care is known with certainty but there is uncertainty on the severity of the disease, i.e.  $a$  is random. Uncertainty on the health level has an additive form. In the second situation, health condition is a deterministic variable, while there is uncertainty about the effectiveness of health care or the marginal product of medical care. This could also reflect uncertainty on the quality of health care as stressed by Arrow (1963). In that case, uncertainty can appear either in an additive or multiplicative form as further explained in section 5.

Consider the socially optimal allocation of a fixed health care budget  $r$ . The risk-averse social planner has a social utility function  $u$  such as  $u'(H) > 0$  and  $u''(H) < 0 \forall H$ . The social planner must choose the level of health care expenditures,  $c_1$  and  $c_2$ , respectively allocated to type-1 and type-2 patients, with the goal to maximise his expected welfare. The optimisation problem is then represented by the Lagrangian expression  $L$ :

$$L(c_1, c_2, \lambda) = \alpha_1 E[u(\tilde{H}_1(c_1))] + \alpha_2 E[u(\tilde{H}_2(c_2))] + \lambda(r - \alpha_1 c_1 - \alpha_2 c_2). \quad (2)$$

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<sup>2</sup>While Dardanoni and Wagstaff (1990) limited themselves to a linear health function, we consider a more general function to reflect decreasing marginal productivity of health care.

where the symbol  $E$  stands for the expectation. In the case of uncertainty on the severity of the disease, the patient  $i$  ( $i = 1, 2$ ) health is  $\tilde{H}_i(c_i) = \tilde{a}_i + m(c_i)$ , and patients differ in the uncertainty on the severity of the disease,  $\tilde{a}_i$ . In the case of uncertainty on the effectiveness of health care, the patient  $i$  ( $i = 1, 2$ ) health is  $\tilde{H}_i(c_i) = a + \tilde{m}_i(c_i)$ , and the two types of patients differ in the uncertainty on the effectiveness of health care.

### 3. Higher order degree change in risk

The changes of risk we consider in this paper are based on the concept of stochastic dominance. Stochastic dominance establishes a partial ordering of probability distributions. It is now documented that health and health care distributions are typically skewed, kurtotic (thick tailed) and heteroscedastic (see Blough et al., 1999; Hill and Miller, 2010) and the health econometrics literature is paying greater attention to higher order conditional moments (Manning et al., 2005; Cantoni and Ronchetti, 2006). The concept of stochastic dominance makes it possible to compare distributions that differ in their conditional moments of higher orders. A special case of stochastic dominance is the notion of increase in risk as developed by Ekern (1980). This includes the cases of mean preserving increase in risk of Rothshild and Stiglitz (1970) as well as of increase in downside risk defined by Menezes et al. (1980) as respectively a  $2^{nd}$ -degree and a  $3^{rd}$ -degree increase in risk.

So as to illustrate higher orders increase in risk, we rely on Eeckhoudt and Schlesinger's (2006) framework which provides a unified approach based on preferences over specific class of lotteries to explain the meaning of the signs of the successive derivatives.

Let us consider an individual with an initial health status  $a$  facing the binary lottery  $\tilde{H}_1 = [a - \delta_1, a; \frac{1}{2}, \frac{1}{2}]$  meaning that the individual has a fifty per cent chance of contracting a disease that decreases  $a$  by  $\delta_1$  units. Now let us assume that this individual is forced to undergo a second disease that decreases his health by  $\delta_2$  units. This second disease could occur either in the state of good health or in the state of bad health where the first disease already occurred, with equiprobable probability. Equivalently, this means that the individual is either confronted with the lottery  $\tilde{H}_1^0 = [a - \delta_1, a - \delta_2; \frac{1}{2}, \frac{1}{2}]$  or  $\tilde{H}_2^0 = [a, a - \delta_1 - \delta_2; \frac{1}{2}, \frac{1}{2}]$  where both lotteries have the same expected mean. The change from  $\tilde{H}_1^0$  to  $\tilde{H}_2^0$  illustrates a mean preserving increase in risk (Rothshild and Stiglitz, 1970). From Rothshild and Stiglitz (1970), we know that all risk-averse individuals prefer  $\tilde{H}_1^0$  to  $\tilde{H}_2^0$ , i.e.  $E[u(\tilde{H}_1^0)] \geq E[u(\tilde{H}_2^0)]$  for all concave functions  $u$ . Hence, a risk-averse individual prefers to be confronted with the second disease in the state of good health, i.e.  $\tilde{H}_1^0$ , rather than in the bad state of nature where the first disease already occurred, i.e.  $\tilde{H}_2^0$ .

Let us now present an illustration of a  $3^{rd}$ -degree increase in risk which relies on the skewness of the distribution. Skewness is a measure of the asymmetry of the probability distribution. Skewness is equivalent to what Menezes et al. (1980) call downside risk. Let us still consider an individual with an initial health status  $a$  facing the binary lottery  $\tilde{H}_1 = [a - \delta_1, a; \frac{1}{2}, \frac{1}{2}]$ . This individual is now forced to undergo an additional health risk  $\tilde{\epsilon}$  with  $E(\tilde{\epsilon}) = 0$  where  $\tilde{\epsilon}$  is also a number of life years (see Eeckhoudt (2002) for a similar formalisation). This risk could occur either in the state of good health or in the state of bad health where the first disease already occurred with equiprobable probability. Equivalently, this means that the individual is either confronted with the lottery  $\tilde{H}_1^* = [a + \tilde{\epsilon}, a - \delta_1; \frac{1}{2}, \frac{1}{2}]$  or  $\tilde{H}_2^* = [a, a - \delta_1 + \tilde{\epsilon}; \frac{1}{2}, \frac{1}{2}]$  where both lotteries have

the same expected mean and the same variance. The change from  $\tilde{H}_1^*$  to  $\tilde{H}_2^*$  represents an increase in downside risk (Menezes et al., 1980). From Menezes et al. (1980) and Eeckhoudt et al. (1996), we know that  $E[u(\tilde{H}_1^*)] \geq E[u(\tilde{H}_2^*)]$  if and only if  $u'''(x) \geq 0 \forall x$ , i.e. that all prudent individuals as defined by  $u'''(x) \geq 0$  (Kimball, 1990) prefer  $\tilde{H}_1^*$  to  $\tilde{H}_2^*$ . The intuitive explanation is that a prudent individual prefers to see attached the additional health risk  $\tilde{\epsilon}$  to the good state of health (health level  $a$ ) rather than to the bad one (health level  $a - \delta_1$ ).

To illustrate a 4<sup>th</sup>-degree increase in risk, let the health loss  $\delta_1$  be replaced in  $\tilde{H}_1^*$  and  $\tilde{H}_2^*$  by another health risk  $\tilde{\epsilon}_1$  such as  $E(\tilde{\epsilon}_1) = 0$ . The change from  $\tilde{H}_1^*$  to  $\tilde{H}_2^*$  now represents a 4<sup>th</sup>-degree increase in risk. A patient whose preferences verify temperance<sup>3</sup> (as defined by  $u''''(x) \leq 0$ ) prefers to see attached the additional health risk  $\tilde{\epsilon}$  to the no risky state of health (health level  $a$ ) rather than to the risky one (health level  $a + \tilde{\epsilon}_1$ ), i.e. he prefers  $\tilde{H}_1^*$  to  $\tilde{H}_2^*$ .

Such interpretation of the signs of higher derivatives relies on the idea of preference for harm disaggregation (Eeckhoudt and Schlesinger, 2006). This means that faced with two equally likely states of nature, a decision-maker always prefers to receive one of the two harms for certain (i.e. one in each state) as opposed to either facing the two harms jointly or facing none of them, the harm being either a sure loss or a zero-mean risk.

The examples provided so far are special cases of stochastic dominance of order two, order three and order four respectively, and illustrate their link with preferences. We can now present the general definition of stochastic dominance of order  $n$ . Let  $F$  and  $G$  denote two cumulative distribution functions, defined over a probability support<sup>4</sup> contained with the open interval  $(a, b)$ . Define  $F_1 = F$  and  $G_1 = G$ . Now define  $F_{k+1}(z) = \int_a^z F_k(t)dt$  for  $k \geq 1$  and similarly define  $G_{k+1}$ . The distribution  $F$  dominates the distribution  $G$  via stochastic dominance of order  $n$  (denoted  $nSD$ ) if and only if  $F_n(z) \leq G_n(z)$  for all  $z$ , and if  $F_k(b) \leq G_k(b)$  for  $k = 1, \dots, n - 1$ .

If the random health variables  $\tilde{H}_1$  and  $\tilde{H}_2$  have distribution  $F$  and  $G$  respectively,  $\tilde{H}_2$  is said to be riskier than  $\tilde{H}_1$  in terms of  $n^{th}$ -order stochastic dominance or equivalently that  $\tilde{H}_1 nSD \tilde{H}_2$ . As said earlier, the concept of an increase in risk as developed by Ekern (1980) restricts to pairs of random variables that have the  $(n - 1)$  first moments identical is a special case of stochastic dominance. Following Ekern (1980),  $\tilde{H}_2$  has more  $n^{th}$ -degree risk than  $\tilde{H}_1$  if and only if  $\tilde{H}_1 nSD \tilde{H}_2$ , and  $E(\tilde{H}_1^k) = E(\tilde{H}_2^k) \forall k = 1, 2, \dots, n - 1$ . Ekern's (1980) definition includes the concepts of mean-preserving increase in risk introduced by Rothschild and Stiglitz (1970) as well as of increase in downside risk as defined by Menezes et al. (1980).

From Ingersoll (1987), we know that  $\tilde{H}_1 nSD \tilde{H}_2$  if and only if  $E[u(\tilde{H}_1)] \geq E[u(\tilde{H}_2)]$ , for all functions  $u$  such that  $sgn(u^{(k)}) = (-1)^{k+1}$  for  $k = 1, \dots, n$  where  $u^{(k)}$  denotes the  $k^{th}$  derivative of the function  $u$ . In the special case of Ekern (1980)'s increase in risk, this extends to  $\tilde{H}_2$  has more  $n^{th}$  degree risk than  $\tilde{H}_1$  if and only if  $E[u(\tilde{H}_1)] \geq E[u(\tilde{H}_2)]$ , for all functions  $u$  such that  $sgn(u^{(n)}) = (-1)^{n+1}$ .

## 4. Uncertainty on the severity of the disease

<sup>3</sup>A negative fourth derivative of the utility function was labelled temperance by Kimball (1992) who showed its relevance for risk management in the presence of a background risk.

<sup>4</sup>The probability support of a cumulative distribution is the definition set of the realisations of the random variable defined by this distribution.

Let us first consider the case where the two types of individuals in the population differ in the uncertainty on the severity of the disease, with  $\alpha_i$  representing the share of persons with a risk  $\tilde{a}_i$  on their basic level of health. The optimisation problem is then represented by the Lagrangian expression  $L$ :

$$\underset{c_1, c_2, \lambda}{Max} L = \alpha_1 E[u(\tilde{a}_1 + m(c_1))] + \alpha_2 E[u(\tilde{a}_2 + m(c_2))] + \lambda(r - \alpha_1 c_1 - \alpha_2 c_2) \quad (3)$$

With an interior solution, the first order conditions imply that the optimal allocations, denoted  $c_1^*$  and  $c_2^*$ , satisfy:

$$m'(c_1^*)E[u'(\tilde{a}_1 + m(c_1^*))] = m'(c_2^*)E[u'(\tilde{a}_2 + m(c_2^*))]. \quad (4)$$

Using Ingersoll (1987), we obtain the following proposition (see the proof in appendix A).

**Proposition 1.** *Given two types of patients confronted with the same illness for which the risk on the severity of illness is respectively  $\tilde{a}_1$  and  $\tilde{a}_2$  such that  $\tilde{a}_1 \text{ nSD } \tilde{a}_2$ , the social planner should allocate more resources to type-2 patients than to type-1 patients if and only if  $(-1)^{k+1}u^{(k+1)}(x) \leq 0 \quad \forall k = 1, 2, \dots, n$ .*

Using the concept of  $n^{\text{th}}$ -degree increase in risk defined by Ekern (1980), we can easily extend proposition 1 to the following corollary.

**Corollary 1.** *Given two types of patients confronted with the same illness for which the risk on the severity of illness is respectively  $\tilde{a}_1$  and  $\tilde{a}_2$  such that  $\tilde{a}_2$  has more  $n^{\text{th}}$ -degree risk than  $\tilde{a}_1$ , the social planner should allocate more resources to type-2 patients than to type-1 patients if and only if  $(-1)^{n+1}u^{(n+1)}(x) \leq 0$ .*

Note that while in section 3 we linked  $n^{\text{th}}$ -order increase in risk to the signs of the derivatives of order  $n$ , we now link  $n^{\text{th}}$ -order increase in risk to the sign of the derivatives of order  $n + 1$ . This is the case as we are now dealing with a resource allocation problem and are focusing on marginal utility rather than utility as shown in equation (4).

Let us illustrate these results for the case of  $n^{\text{th}}$ -degree increase in risk. First consider the case where  $\tilde{a}_1 = a$  and  $\tilde{a}_2 = a - \delta$  with  $\delta \geq 0$ . Type-2 patients face the disease with higher severity than type-1 patients, i.e. type-2 patients have a lower level of health than type-1 patients. The shift from  $\tilde{a}_1$  to  $\tilde{a}_2$  is a first-degree change in risk. In that case, proposition 1 says that a social planner should allocate more resources to the patient for whom the severity of the disease is higher if and only if  $u'' \leq 0$ . Obviously if  $\delta = 0$ , the severity of the disease is the same for the two patients and thus  $c_1^* = c_2^*$ . Note that an increase in the probability of being in the worst state of health is also a first-degree change in risk, meaning that a risk-averse social planner should prioritise the patients with the higher probability of being in the worst state. The probability of being in the worst state of the world is often found in the literature as a indicator of health risk (see e.g. Edwards, 2008).

Next, let us consider the case where  $\tilde{a}_1 = a$  and  $\tilde{a}_2 = a + \tilde{\epsilon}$  with  $E(\tilde{\epsilon}) = 0$ . For example  $\tilde{\epsilon} = [\delta, -\delta; \frac{1}{2}, \frac{1}{2}]$ , meaning that type-2 patients have a fifty-fifty chance to have their health

either improved or deteriorated at the same level. The shift from  $\tilde{a}_1$  to  $\tilde{a}_2$  is a second-order change in risk. Type-2 patients face a risk on their health but type-1 patients do not. In that situation, the social planner should prioritise the patients at risk if and only if  $u''' \geq 0$ , so that the social planner's preferences are characterised by prudence (Kimball, 1990). The explanation is that by allocating a higher amount of health care to the patient at risk, the social planner allows him to support this risk in a higher state of health. Prudence is required as we know from the preceding section that a prudent decision-maker prefers to see the health risk attached to the good state of health rather than to the bad one. This has to be linked to the results of Hoel (2003) and Bui et al. (2005) who highlighted the dominant role of prudence in prioritising health care. As indicated before, a mean-preserving increase in risk as defined by Rothschild and Stiglitz (1980) is also a second-order increase in risk. A mean-preserving spread represents any situation in which two illnesses have the same expected loss of health but different severities. As an illustration, Jappelli et al. (2007) take as health risk the variance associated with falling into the worst possible state of health. Also Palumbo (1999) measures health risk as the variance of out-of-pocket health expenditure, a second-order phenomenon.

Going one step further, third-order change in risk can be illustrated by the shift from  $\tilde{H}_1^* = [a + \tilde{\epsilon}, a - \delta_1; \frac{1}{2}, \frac{1}{2}]$  to  $\tilde{H}_2^* = [a, a - \delta_1 + \tilde{\epsilon}; \frac{1}{2}, \frac{1}{2}]$  defined in the previous section where  $\tilde{a}_1 = \tilde{H}_1^*$  and  $\tilde{a}_2 = \tilde{H}_2^*$ . Under  $\tilde{a}_1$ , the individual is confronted with either a sure loss or a zero-mean risk on his health with the same probability. While under  $\tilde{a}_2$ , the individual is confronted with either both a sure loss and a zero-mean risk on his health or with nothing with the same probability. Still applying corollary 1, a social planner should allocate more resources to the patients whose health status is more risky in terms of downside risk or skewness if and only if  $u^{(4)}(x) \leq 0$ , so that the social planner's preferences are characterised by temperance (Kimball, 1992). The explanation is that by allocating a higher amount of health care to the riskier patient, the social planner allows him to support this risk in a less risky health state. Temperance is required as a temperant decision-maker exhibits preferences for harm disaggregation as explained in the previous section. Hence, in the case of third-order change in risk, prudence is not required to prioritise health care under uncertainty.

In the same vein and still relying on Eeckhoudt and Schlesinger's (2006) framework (2006), consider the two following lotteries,  $\tilde{a}_1 = [a + \tilde{\epsilon}_1, a + \tilde{\epsilon}_2; \frac{1}{2}, \frac{1}{2}]$ , and  $\tilde{a}_2 = [a, a + \tilde{\epsilon}_1 + \tilde{\epsilon}_2; \frac{1}{2}, \frac{1}{2}]$  (with  $\tilde{\epsilon}_1$  and  $\tilde{\epsilon}_2$  being two independent and actuarially neutral health risks such as  $E(\tilde{\epsilon}_1) = E(\tilde{\epsilon}_2) = 0$ ). Under  $\tilde{a}_1$ , the individual is confronted with one of the two zero-mean health risks for certain, while under  $\tilde{a}_2$ , the individual is confronted with either facing the two health risks jointly or facing none of them with the same probability. Applying corollary 1, we have that  $c_2^* \geq c_1^*$  if and only if  $u^{(5)}(x) \geq 0$ , so that the social planner's preferences are characterised by edginess<sup>5</sup> (Lajeri-Chaherli, 2004).

It is worth stressing that the logarithmic function ( $u(x) = \ln(x)$ ) and the power function ( $u(x) = x^b$  with  $0 < b < 1$ ), which are widely used social utility functions in medical decision-making (see Bleichrodt et al., 2005), have the property of having their successive higher derivatives alternate in signs, i.e. such that  $(-1)^{n+1}u^{(n+1)}(x) \leq 0$  for all  $n \geq 0$ . This illustrates that the conditions on social preferences exhibited in proposition 1 are rather common in the health literature.

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<sup>5</sup>A positive fifth derivative of the utility function was labelled edginess by Lajeri-Chaherli (2004) to explain precautionary saving behaviour in the presence of a background risk.



## 5. Uncertainty on the effectiveness of treatment

We now assume that the two types of population differ in the uncertainty about the effectiveness of health care captured by  $\tilde{\epsilon}_i$  such as  $\tilde{m}_i(c) = m(c, \tilde{\epsilon}_i)$ . The uncertainty can appear in two different forms, either  $m(c, \tilde{\epsilon}_i) = m(c) + \tilde{\epsilon}_i$ , or  $m(c, \tilde{\epsilon}_i) = \tilde{\epsilon}_i m(c)$ .

In the first case, the uncertainty appears in an additive form and thus does not modify the marginal productivity of health care. Technically, this case is identical to the case analysed in the previous section. Indeed, the health level  $\tilde{H}_i(c)$  writes as  $\tilde{H}_i(c) = \tilde{a}_i + m(c)$  with  $\tilde{a}_i = a + \tilde{\epsilon}_i$ . Results are given by proposition 1 and corollary 1.

In the second case, the uncertainty appears in a multiplicative form. The difference is important because the presence of uncertainty makes the marginal productivity of health care random<sup>6</sup>. Following Dardanoni and Wagstaff (1990) and for the sake of simplicity, we consider  $a = 0$ . The optimisation problem is now represented by the Lagrangian expression  $L$ :

$$\underset{c_1, c_2, \lambda}{Max} L = \alpha_1 E[u(\tilde{\epsilon}_1 m(c_1))] + \alpha_2 E[u(\tilde{\epsilon}_2 m(c_2))] + \lambda(r - \alpha_1 c_1 - \alpha_2 c_2) \quad (5)$$

If interior solutions prevail, the first order conditions imply:

$$E[\tilde{\epsilon}_1 m'(c_1^*) u'(\tilde{\epsilon}_1 m(c_1^*))] = E[\tilde{\epsilon}_2 m'(c_2^*) u'(\tilde{\epsilon}_2 m(c_2^*))]. \quad (6)$$

Using Ingersoll (1987), we obtain the following proposition (see the proof in appendix B).

**Proposition 2.** *Given two types of patients confronted with the same illness for which the risk on the effectiveness of health care is respectively  $\tilde{\epsilon}_1$  and  $\tilde{\epsilon}_2$  such that  $\tilde{\epsilon}_1$  nSD  $\tilde{\epsilon}_2$ , the social planner should allocate more resources to type-2 patients than to type-1 patients if and only  $[-xu^{(k+1)}(x)]/u^{(k)}(x) \geq k \quad \forall k = 1, 2, \dots, n$ .*

Similarly to Corollary 1, we can induce from Proposition 2 the following corollary.

**Corollary 2.** *Given two types of patients confronted with the same illness for which the risk on the effectiveness of health care is respectively  $\tilde{\epsilon}_1$  and  $\tilde{\epsilon}_2$  such that the risk  $\tilde{\epsilon}_2$  has more  $n^{th}$ -degree risk than  $\tilde{\epsilon}_1$ , the social planner should allocate more resources to type-2 patients than to type-1 patients if and only  $[-xu^{(n+1)}(x)]/u^{(n)}(x) \geq n$ .*

Corollary 2 says that priority setting results are governed by the value of the  $n^{th}$ -order relative risk aversion (RRA- $n$ ) coefficient<sup>7</sup>,  $\frac{-xu^n(x)}{u^{n-1}(x)}$ . The importance of the  $2^{nd}$ -order relative risk aversion coefficient or also labelled relative risk aversion, i.e.  $\frac{-xu''(x)}{u'(x)}$ , has been long known. Indeed, portfolio decisions, insurance decisions or saving decisions depend, among other things, on a comparison between unity and the value of the RRA-2

<sup>6</sup>It is assumed that realisations of the random variable  $\tilde{\epsilon}$  are all strictly positive to ensure that health care is always effective. In the case where the realisation of  $\tilde{\epsilon}$  is  $\epsilon$  such that  $0 < \epsilon < 1$  ( $\epsilon = 1, \epsilon > 1$ ), the effectiveness of health care is lower (equal to, bigger) than the one under certainty.

<sup>7</sup>Note that if we consider  $a \neq 0$ , then the results would be governed by the  $n^{th}$ -order partial risk aversion coefficient  $\frac{-xu^n(a+x)}{u^{n-1}(a+x)}$  (for further details on this coefficient, see Chiu et al. (2011)).

coefficient (e.g. Rothshild and Stiglitz, 1971). Since the concept of prudence is more recent, the 3<sup>rd</sup>-order relative risk aversion coefficient also known as relative prudence, defined by  $\frac{-xu'''(x)}{u''(x)}$ , is much less discussed. Yet, in some recent papers, the comparison between the RRA-3 coefficient and 2 is shown to drive various economic decisions (e.g. Choi et al., 2001). This is confirmed by Eeckhoudt et al. (2009) and Chiu et al. (2011) who show that relative prudence in excess of two seems rather natural.

As before, let us illustrate these results. If the effectiveness of health care is higher for type-1 patients than for type-2 patients, i.e.  $\tilde{\epsilon}_1 = \epsilon_1$  and  $\tilde{\epsilon}_2 = \epsilon_2$  with  $\epsilon_1 > \epsilon_2$ , then the social planner should allocate more resources to the patient for whom effectiveness of health care is lower if and only if relative risk aversion exceeds one. While such a result might be surprising at first glance, since one could think that more resources should be allocated to health care whose effectiveness is higher, the implicit decision rule is to allocate more resources to the patient who is in the more unfavourable situation, where the more unfavourable situation is defined in terms of stochastic dominance. In the current illustration, the more unfavourable situation is to receive health care whose effectiveness is lower.

Next, let us consider the case where the effectiveness of health care for type-1 patients is certain and is equal to  $\epsilon$ , while it is risky for type-2 patients and such that  $\tilde{\epsilon}_2 = \epsilon + \tilde{\theta}$  with  $E(\tilde{\theta}) = 0$ . The social planner should then allocate more resources to the patient for whom effectiveness of health care is risky if and only if relative prudence exceeds two.

In the same way, if there is an increase in downside risk of the effectiveness of health care, the social planner should prioritise the patients more at risk if and only if the RRA-3 coefficient is superior to 3, meaning that conditions on relative risk aversion and relative prudence can become obsolete to prioritise health care resources in the face of more uncertainty. In the same vein, we could illustrate higher risk changes and link them to higher orders of relative risk aversion.

## 6. Attitude towards inequality

Under the utility model used so far, the health benefit of individuals is aggregated by unweighted summation, i.e. the weight each individual gets is equal to his proportion in society. Recently, an alternative model has been proposed that weighs individuals with respect to their health level. This model is known as Rank-Dependent QALY model (Bleichrodt et al., 2004)<sup>8</sup>.

Under the Rank-Dependent QALY model, the proportion of patients involved is transformed to reflect the inequality aversion of the social planner. The ranking of individuals' preferences is crucial as higher weights are assigned to individuals who are worse-off under inequality aversion. This section investigates whether previous results are modified by the introduction of the equity weighting function.

Consider the case of uncertainty on the severity of the disease where  $\tilde{a}_1 \sim nSD \tilde{a}_2$ . Let  $w$  be the equity weighing function such that  $w(\alpha_2) > \alpha_2$  to reflect inequality aversion, since for any given medical care  $c$ , type-2 patients are in a more unfavourable situation.

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<sup>8</sup>In a recent paper, Bleichrodt et al. (2008) compared some priority setting's results under both models and showed that policy implications could be highly sensitive to the choice of model.

The optimisation problem is then represented by the Lagrangian expression:

$$L(c_1, c_2, \lambda) = (1 - w(\alpha_2))E[u(\tilde{a}_1 + m(c_1))] + (w(\alpha_2))E[u(\tilde{a}_2 + m(c_2))] + \lambda(r - (1 - \alpha_2)c_1 - \alpha_2c_2) \quad (7)$$

If interior solutions prevail, denoted  $c_1^{**}$  and  $c_2^{**}$ , the first order conditions imply:

$$\frac{1 - w(\alpha_2)}{(1 - \alpha_2)}m'(c_1^{**})E[u'(\tilde{a}_1 + m(c_1^{**}))] = \frac{w(\alpha_2)}{\alpha_2}m'(c_2^{**})E[u'(\tilde{a}_2 + m(c_2^{**}))] \quad (8)$$

Comparing this equation to the respective FOC of the preceding model (see equation (4)) shows that the optimal allocation of health care depends on the weight assigned to the two types, and thus on the degree of inequality aversion. It is easy to show that  $w(\alpha_2) > \alpha_2$  is equivalent to  $w(\alpha_2)/\alpha_2 > 1 > (1 - w(\alpha_2))/(1 - \alpha_2)$ . Hence equation (8) becomes

$$m'(c_1^{**})E[u'(\tilde{a}_1 + m(c_1^{**}))] \geq m'(c_2^{**})E[u'(\tilde{a}_2 + m(c_2^{**}))] \quad (9)$$

We can then show (see appendix C) that  $c_2^{**} \geq c_1^{**}$  is equivalent to  $(-1)^{k+1}u^{(k+1)} \leq 0 \forall k = 1, 2, \dots, n$ . Therefore, the introduction of the equity weighting function does not modify the results in the sense that the social planner continues to allocate more resources to patients in the worst situation for the same conditions on the utility function. However, it amplifies the difference in the allocation of health care. Indeed, it is easy to show that  $c_2^{**} - c_2^* > 0 > c_1^{**} - c_1^*$ . Under inequality aversion as defined through the equity weighting function, the social planner allocates even more health care to the patient that is more at risk than under inequality neutrality. These results also apply in the case of uncertainty on the effectiveness of health care.

Bleichrodt et al. (2005) performed an empirical elicitation of the Rank-Dependent QALY model using a power utility function. They found that the power coefficient was positive and just below one, meaning that such a utility function shares the property of having its successive derivatives alternate in signs which is in line with the assumptions made on the social utility functions in this paper.

## 7. Conclusion

Confronted with a limited budget for health care, countries need to prioritise health care expenditures among their population. This paper addressed this issue when populations differ in the uncertainty on their health. In particular, it establishes conditions under which a social planner will devote more resources to patients facing the worst risk on their health, when risk is defined in terms of stochastic dominance of order  $n$ . Following Arrow (1963), we consider two sources of uncertainty, one on the severity of the illness and the other on the effectiveness of the treatment. When the severity of illness is risky, the social planner should invest more resources in the patient whose risk of the severity of the illness is  $n^{th}$ -degree more at risk than the other if and only if  $(-1)^{n+1}u^{(n+1)} \leq 0$ . When there is an increase in  $n^{th}$ -degree risk in the effectiveness of health care, the social planner should allocate more resource to the patient who is more at risk if and only if  $[-xu^{(n+1)}(x)]/u^{(n)} \geq n$ .

These results are valid when inequality aversion is modelled either under the utility approach or under an equity weighting function. Yet, inequality aversion as defined through

the equity weighting function amplifies the difference in the allocation of health care. Indeed, under the above conditions on health aggregation, the social planner allocates even more health care to the patient who is more at risk under inequality aversion than under inequality neutrality.

While these conditions look rather complex, they encompass many common assumptions used in health literature. Indeed, when change in risk is measured by an increase in the probability of the worst state of health, then only risk aversion is required. Yet, when changes in risk concern higher orders such as variance, skewness or kurtosis, conditions such as prudence, temperance or edginess are required in the allocation of health care. This shows that depending on the change of risk considered, further conditions on preferences of the social planner need to be considered so as to prioritise health care amongst risky populations. As health and health care distribution differ in their conditional moments of higher orders, this paper shows the relevance of better knowing the preferences of the social planner when allocating health care resources amongst risky populations.

While this paper considers the case of uncertainty either on the severity of the disease or on the effectiveness of health care, it may happen that individuals differ both in the severity of illness and the effectiveness of care. A natural extension of this current work would be to consider priority setting in health care where individuals differ in multiple sources of uncertainty.

## Appendix A.

So as to compare  $c_1^*$  and  $c_2^*$ , we follow Ingersoll (1987) showing that if the risk  $\tilde{a}_1$  dominates  $\tilde{a}_2$  via  $n^{th}$ -order stochastic dominance, this is equivalent to  $E[v(\tilde{a}_1)] \geq E[v(\tilde{a}_2)]$  for all functions  $v$  such that  $(-1)^{k+1}v^{(k)} \geq 0 \forall k = 1, 2, \dots, n$ . Using this result, and substituting  $v$  by  $u'$ , we can write that  $E[u'(\tilde{a}_1)] \leq E[u'(\tilde{a}_2)]$  or equivalently  $E[u'(\tilde{a}_1 + x)] \leq E[u'(\tilde{a}_2 + x)] \forall x$ . This inequality equivalently rewrites as  $m'(c_1^*)E[u'(\tilde{a}_1 + m(c_1^*))] \leq m'(c_1^*)E[u'(\tilde{a}_2 + m(c_1^*))]$  for all utility function  $u$  such that  $(-1)^{k+1}u^{(k+1)} \leq 0 \forall k = 1, 2, \dots, n$ . Hence, using equation (4), we obtain  $m'(c_2^*)E[u'(\tilde{a}_2 + m(c_2^*))] \leq m'(c_1^*)E[u'(\tilde{a}_2 + m(c_1^*))]$ . Let's denote  $G(c) = m'(c)E[u'(\tilde{a}_2 + m(c))]$ . The previous inequality rewrites as  $G(c_2^*) \leq G(c_1^*)$  that is equivalent to  $c_2^* \geq c_1^*$ , because the function  $G$  is decreasing in  $c$  under our assumptions ( $m''(c) \leq 0 \forall c$  and  $u''(x) < 0 \forall x$ ).

## Appendix B.

So as to compare  $c_1^*$  and  $c_2^*$ , let's first define the function  $f$  as follows:  $f(\epsilon) = \epsilon m'(c)u'(\epsilon m(c))$  for all  $\epsilon$ .

Calculations show that  $f'(\epsilon) = m'(c)u'(\epsilon m(c)) + m(c)\epsilon m'(c)u''(\epsilon m(c))$ . Thus  $f'(\epsilon) \leq 0$  holds for all  $\epsilon$  if and only if  $\frac{-xu''(x)}{u'(x)} \geq 1 \forall x > 0$ . It follows from standard induction arguments, for any  $k > 1$ , that  $f^{(k)}(\epsilon) \leq (\geq 0)$  holds for all  $k$  if and only if  $\frac{-xu^{(k+1)}(x)}{u^{(k)}(x)} \geq (\leq)k \forall x > 0$ , as long as  $u^{(k)}(x) \neq 0$ .

Assuming that  $\tilde{\epsilon}_1$   $n$ SD  $\tilde{\epsilon}_2$ , following Ingersoll (1987) we have  $E[f(\tilde{\epsilon}_1)] \leq E[f(\tilde{\epsilon}_2)]$  for all function  $f$  such that  $(-1)^{(k+1)}f^{(k)} \leq 0 \forall k = 1, 2, \dots, n$ , that rewrites as  $E[\tilde{\epsilon}_1 m'(c_1)u'(\tilde{\epsilon}_1 m(c_1^*))] \leq E[\tilde{\epsilon}_2 m'(c_1)u'(\tilde{\epsilon}_2 m(c_1^*))]$ . Using equation (5), we obtain  $E[\tilde{\epsilon}_2 m'(c_2)u'(\tilde{\epsilon}_2 m(c_2^*))] \leq E[\tilde{\epsilon}_2 m'(c_1)u'(\tilde{\epsilon}_2 m(c_1^*))]$ . Let's define  $Z(c) = E[\tilde{\epsilon}_2 m'(c)u'(\tilde{\epsilon}_2 m(c))]$ . The previous inequality rewrites as  $Z(c_2^*) \leq$

$Z(c_1^*)$ . We obtain then  $c_2^* \geq c_1^*$  since  $Z'(c) < 0$  for all  $c$  since  $m''(c) \leq 0$  for all  $c$  and  $u'' < 0$  for all  $x$ .

## Appendix C.

Reasoning as before, we know that if the risk  $\tilde{a}_1$  dominates  $\tilde{a}_2$  via  $n^{\text{th}}$ -order stochastic dominance, then  $m'(c_1^{**})E[u'(\tilde{a}_1 + m(c_2^{**}))] \leq m'(c_1^{**})E[u'(\tilde{a}_2 + m(c_2^{**}))]$  for all utility function  $u$  such that  $(-1)^{k+1}u^{(k+1)} \leq 0 \forall k = 1, 2, \dots, n$ . Hence, using equation (9), we have  $m'(c_2^{**})E[u'(\tilde{a}_2 + m(c_2^{**}))] \leq m'(c_1^{**})E[u'(\tilde{a}_1 + m(c_1^{**}))]$  which is equivalent to  $c_2^{**} \geq c_1^{**}$  (see Appendix A).

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