Recent Developments in Contact Conductance Heat Transfer

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The characteristics of thermal contact conductance are increasingly important in a wide range of technologies. As a consequence, the number of experimental and theoretical investigations of contact conductance has increased. This paper reviews and categorizes recent developments in contact conductance heat transfer. Among the topics included are the theoretical/analytical/numerical studies of contact conductance for conforming surfaces and other surface geometries; the thermal conductance in such technological areas as advanced or modern materials, microelectronics, and biomedicine; and selected topics including thermal rectification, gas conductance, cylindrical contacts, periodic and sliding contacts, and conductance measurements. The paper concludes with recommendations for emerging and continuing areas of investigation.

Introduction

The thermal contact conductance depends upon the thermal and physical characteristics of the material junction and the junction environment. The relative importance of the thermal conductance is directly related to the desirability of isolating or enhancing the energy transfer at a junction. As a consequence, the understanding of the thermal contact conductance phenomenon has taken on new dimensions as more and more researchers develop solutions to immediate problems.

When two surfaces are brought into contact, whether in the presence of an interstitial medium (epoxy, solder, grease, or other material) or a bare junction, an imperfect junction exists. While a uniform temperature gradient may exist in a homogeneous material, the junction between two surfaces creates a temperature difference between the surfaces. This difference is dependent upon the mechanical and thermophysical properties of the contacting materials, the characteristics of the contacting surfaces, the presence of gaseous and nongaseous interstitial media, and the overall environment of the junction.

Because two surfaces in contact are not perfectly flat, most of the heat or energy passes through a limited number of actual contact spots, as shown in Fig. 1. The contact resistance, then, is the resistance to heat flow because the actual area of contact is only a small fraction of the nominal or apparent area. This contact resistance is defined as the ratio of the temperature drop at the interface to the average heat flux across the junction, as shown in Fig. 2. The thermal contact resistance is related to the thermal contact conductance; however, the results of most analytical and experimental studies are reported in terms of thermal contact conductance.

The type of experimental facility generally used to obtain thermal contact conductance data consists of a vertical cylindrical column under axial load, with the contacting surfaces located at approximately the midpoint of the column. One end of the column serves as a heat source (heated by cylindrical heaters or other means), and the other end serves as the heat sink (cooled by water, liquid nitrogen, or other cooling media). The test materials, heat source, guard heaters, radiation shields, and heat sink usually are instrumented with thermocouples to provide information on the operation of the system and to measure axial temperatures in the column. Column loading is generally provided by dead weight, hydraulic, or pneumatic means. The test facility is generally located in a vacuum chamber to minimize convective heat transfer around the test materials and through gaseous interstitial media.

Since studies of thermal contact conductance or thermal contact resistance cover such a broad and diverse range and have been conducted over a substantial period of time, this paper is restricted to those investigations and studies that have been reported since 1980. This paper is further divided into recent reviews and summaries, theoretical and analytical studies, thermal conductance in such technological areas as advanced or modern materials, microelectronics, and biomedicine, and selected topics including thermal rectification, gas conductance, cylindrical contacts, periodic and sliding contacts, and conductance measurements.

Recent Reviews

A comprehensive review of thermal contact conductance literature for the period 1950 to 1980 was provided by Madhusudana and Fletcher (1981a, 1986). This review considered the advances in such areas as the resistance of single constrictions, including disk, conical, and others, as well as the effects of large-scale surface irregularities. Correlations were reviewed for thermal contact conductance in a vacuum through multiple contact spots for nominally flat, rough surfaces. The effects of both gaseous and nongaseous interstitial media and materials were summarized, including the effects of surface films and heat flow direction. Also considered were special topics of the time such as stacks of laminations, bolted joints, cylindrical joints, and nuclear reactor fuel elements.

Snaith et al. (1986) reviewed the literature in the area of the thermal resistance of pressed contacts in an effort to assist the designer in understanding the factors involved. The paper discussed mechanisms for heat transfer across solid metal/solid metal interfaces, influential parameters affecting thermal contact resistance, analytical predictions of the thermal resistance of pressed contacts with limitations, and empirical correlations. The comparison of the correlations suggested that a suitable general correlation for a bare metal junction has not yet been achieved, primarily because there is no standard method for measuring and reporting the topography of the contacting surfaces.

There have been other more recent reviews focused on selected areas. Antonetti and Yovanovich (1984) provided a selective review of the state of the art of thermal contact resistance as applied to microelectronic equipment. The review summarized analytical methods for determining the spreading resistance in microelectronic packages and presented techniques to predict, measure, and minimize the contact resistance across mechanical joints. The prediction and measurement of the contact resistance of bonded joints in integrated circuit packages was discussed, noting that there is considerable information on contact resistance available to the designer of microelectronic packages, but that the data

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published are not comparable and are often difficult to interpret.

Because of the importance of heat dissipation in microelectronic components, the National Science Foundation sponsored a workshop on research needs in electronic cooling. As part of the workshop, Yovanovich et al. (1986b) summarized the issues related to internal resistance of microelectronic components, noting the dependence on internal design features, material selection, the integrity of assembly procedures, and mounting and cooling techniques. The conclusions and recommendations were divided into materials behavior and problems associated with thermal contact resistance. Standardized procedures for identification and assessment of materials used in multilayer composites and printed wiring boards were called for. It was suggested that the existing parameters and models pertaining to thermal contact conductance should be identified and disseminated to the microelectronics industry (noting the inaccuracy and limitations) and that a standardized test method for thermal resistance should be established.

The increasing focus on development of cooling technologies for electronic equipment resulted in an international symposium on the subject, at which Yovanovich (1987) presented a summary of the theory and applications of constriction and spreading resistance concepts for the thermal management of microelectronic components. The paper provided a timely review of the state of the art of thermal contact resistance problems and models, discussed the effect of thin metallic layers bonded to one surface of the contact, and presented recently developed dimensionless spreading resistance expressions suitable for solving many microelectronic thermal problems. A prediction expression for the thermal resistance of soldered joints was presented. The effects of thin metallic foils for minimization of joint resistances were also reported.

Yovanovich (1986) summarized recent developments in contact conductance models for point and line contacts and conforming rough surfaces. Theory was compared with experimental results, noting good to excellent agreement for point contact and conforming rough-surface models. The limited agreement observed at light loads was attributed to errors in form, or crowning, for line contact models. One of the major factors that caused differences in the conforming model at light loads was the high local strain due to very large local temperature gradients.

The influence of interstitial media on thermal contact conductance is of importance to the aerospace industry, as well as the electronics industry. Fletcher (1984) presented a review of the thermal contact conductance with interstitial materials useful in spacecraft thermal control. The paper was restricted to those investigations of appropriate spacecraft environments, and dealt with both gaseous and nongaseous interstitial media. The paper further divided nongaseous materials into four classifications, including synthetic or processed natural sheets, ceramic sheets and powders, metallic foils and screens, and greases and oils. An efficiency factor for thermal control materials was defined, which related the thermal conductance of the journal with an interstitial material to a bare metallic junction under the same conditions. Based on the range of materials reported, heat transfer was improved by as much as 40 percent or reduced by as much as 100 percent for the same junction conditions.

Fletcher and Peterson (1986) expanded on the review of interstitial materials for spacecraft systems to include a wide range of thermal systems with composite materials and similar and dissimilar metals. Both gaseous and nongaseous materials were considered, including polyethylene and elastomeric materials, sintered metallic fibers and powders, foils and grease-filled joints. It was noted that judicious use of interstitial materials could result in systems whose thermal behavior was not only optimized but predictable. A conductance efficiency was used to evaluate the relative merits of the various interstitial materials.

While there are numerous studies of sliding contacts with heat transfer, the review by Kennedy (1984) provides perspective on the thermal and thermomechanical effects in dry sliding. Among the topics reviewed are thermal deformation around sliding contacts, the changes in contact geometry caused by thermal deformation and thermoelastic instability, the thermomechanical stress distribution around frictionally heated and thermally deformed contact spots, the measurement and analysis of surface and near-surface temperatures resulting from frictional heating, and the mechanisms of frictional heating. The influence of thermal and thermomechanical contact phenomena on various modes of failure was also discussed.

General Theoretical/Analytical Studies

While there is thermal resistance associated with essentially all physical geometries in which heat flows across an interface, recent literature has focused on selected areas. Of specific interest has been the theoretical and analytical characterization of the thermal resistance problem.

Point Contacts. The reviews by Antonetti and Yovanovich (1984), Yovanovich et al. (1986), and Yovanovich (1987) provided an excellent overview of the state of the art in theoretical and analytical thermal contact resistance problems and models, particularly in the area of point contacts and conforming rough surfaces. In the last few years there have been some advances in the analyses of thermal contact resistance problems, and several of these are described here.

A numerical solution for the heat conduction through a joint composed of surfaces characterized as frustrums of cones was developed by Madhusudana (1980a) as a model for the asperities on contacting surfaces. The conical asperity was surrounded by a conducting medium or a vacuum. The model provided temperature profiles and demonstrated that the constriction resistance was reduced significantly because of the presence of a conducting fluid and that the resistance decreased with increasing cone semi-angle.

Yovanovich et al. (1982b) presented a series of thermal contact conductance models incorporating surface microhardness distributions. These models were based on the bulk hardness or isotropic hardness throughout the surface, an iterative approach based on a surface microhardness distribution, an in-

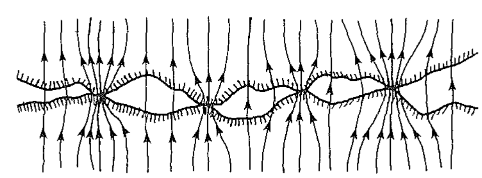

Fig. 1 Heat flow through a joint

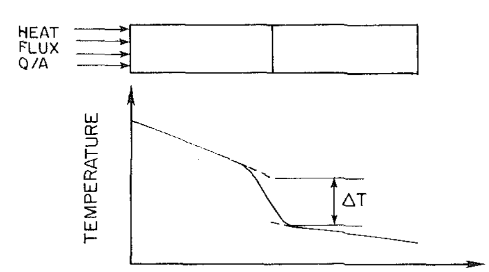

Fig. 2 Temperature variation along two bars joined together

tegral approach that assumed that each asperity has a hardness distribution corresponding to the hardness layer, and a direct approximation method, which involves the determination of an effective hardness for a particular surface at the mean contact pressure. The predictions of the contact conductance as determined with these models were compared with experimental data obtained by Yovanovich et al. (1983). In these comparisons, the use of the bulk hardness model significantly overpredicted the experimental data, whereas the iterated model resulted in reasonable agreement. The integral approach and direct approximation models resulted in excellent agreement with the data. Based on the comparisons, it was suggested that the direct approximation method would be the most suitable model to use for predicting the contact conductances for rough conforming surfaces.

Negus and Yovanovich (1984a) used an approximate technique for the solution of the thermal constriction resistance parameter for a wide range of relative contact sizes. The technique, which was used for an isothermal, circular disk supplying heat to a semi-infinite coaxial cylinder, consisted of superposing Neumann-specified solutions to Laplace's equation. A least-squares criterion was used so that the mixed boundary conditions could be approximated. Results of the analysis were compared by means of a correlation equation. Negus and Yovanovich (1984b) also obtained solutions to other combinations of other geometries for the case of uniform flux and various isothermal boundary conditions.

A thermal constriction resistance expression for an isotropic rough surface was developed by Negus et al. (1985) utilizing the method of infinite images. An elemental heat flux tube consisting of an elliptical contact on a semi-infinite adiabatic rod of rectangular cross section was used in the analysis. For small contact sizes, the constriction resistance was shown to be a perturbation of the half-space resistance. Through use of the surface element method and the method of infinite images, approximate expressions for the two low-order perturbations were developed. This approximation of the dimensionless thermal constriction resistance parameter provides a useful technique for analyzing the heat flow paths between real contacting surfaces.

The effect of a coating material (with different mechanical and thermal properties) on the short-time and steady-state constriction resistance was analyzed by Dryden et al. (1985) for an arbitrary axisymmetric contact spot flux. In the limit of very short times, the constriction resistance was equivalent to one-dimensional transient heat flow through a two-layer wall. For steady state, the effect of the coating was primarily dependent on the relative thermal properties of the coating and substrate.

Multiple Contacts. An approximate technique for determining the thermal contact resistance of a set of multiple circular contacts on half-spaces or circular flux tubes was developed by Saabas and Yovanovich (1985), using both surface element methods and superposition techniques. The results of this novel analytical-numerical method compared favorably with other models, and in the limiting case of a single elliptic contact on a half-space, compared favorably with published analytical solutions. Tabular results were presented for both half-spaces and flux tubes.

Ronzon et al. (1986) utilized the finite element method to analyze the thermal resistance across contiguous, rough interfaces. The interface shapes included saw-tooth, sine-wave, and square-wave profiles. Parametric studies were conducted to ascertain the influence of the interface profile on the thermal resistance. Results indicated that the resistance was decreased below the one-dimensional, smooth interface value. Solutions presented were estimated to be within 1 percent of the actual value of the incremental resistance. Observations indicated that the reduction in resistance increased as the interface material increased.

The contact pressure is extremely important in predicting the thermal contact conductance, and Song and Yovanovich (1987a) developed an explicit expression for the contact hardness and the relative contact pressure for nominally flat but microscopically rough surfaces. This expression was used for parametric studies and demonstrated the relative dependence of the contact pressure and hardness on the surface roughness, asperity slope, and Vickers microhardness correlations. Predictions utilizing the explicit expressions were in excellent agreement with experimental data obtained in their investigations and those of Eid and Antonetti (1986) and Peterson and Fletcher (1988b).

The theory of conforming rough surfaces was used by Negus et al. (1987) to predict the thermal contact resistance for dissimilar metal junctions, where mass diffusion processes produced a continuous alloy distribution. The diffusion process was found to raise or lower the constriction factor depending on whether it was a conductive or resistive alloy layer. The general expression for the thermal constriction factor for a circular flux tube with a layer of linearly varying thermal conductivity was determined for a circular contact with flux-specified boundary conditions. Results were presented in dimensionless thermal and geometric constriction parameters, using a copper–nickel diffusion interface as an example.

A new physically realistic thermomechanical model was developed by DeVaal et al. (1987) for predicting the contact conductance of both nominally flat, rough surfaces having variations in the surface slope with direction, and surfaces prepared by normal machining processes. The model was based on the assumption that the heat transfer across the interface was controlled by the constriction of heat passing through the individual contact regions, and that if the geometry of these individual contact regions were known, thermal models could be developed. The model represented a direct theoretical extension of previous correlations for isotropic, or slope invariant, surfaces, and was expected to be useful for finely ground, conforming surfaces. Based on a comparative study with the model, anisotropic surface pairs with aligned slope direction were found to be more resistive than similar surface pairs with perpendicular slope directions, and preliminary experimental data verified the suitability of the model for such surface geometries.

The thermal accommodation coefficient is used to represent the extent to which an interchange of energy takes place when a stream of gas molecules impacts a solid surface. A thermal accommodation coefficient correlation for engineering surfaces was developed by Song and Yovanovich (1987b) using experimental data for monatomic gases on several types of surfaces. It should be noted that the accommodation coefficient was significantly greater for engineering surfaces than that for clean surfaces. The correlation was used to predict the thermal accommodation coefficient for diatomic/polyatomic gases within 25 percent.

In recent years there have been significant advances in theoretical/analytical models used to predict the thermal contact resistance at an interface. Models and correlations suitable for selected conditions and geometries have been developed and refined with remarkable success and accuracy, and each new model has improved the overall understanding and knowledge of the phenomenon. Nevertheless, the diverse nature of the thermal contact conductance phenomenon is such that a generalized prediction technique has not been developed.

Thermal Conductance in Advanced Materials

There have been a number of investigations of heat conduction in engineering materials. Conduction occurs in such diverse media as insulation materials, reinforced laminates, filament wound structures, geological strata, printed circuit boards, and aerospace components. As a consequence, there has been increasing interest in understanding the thermal contact conductance phenomenon and its relationship to porous, layered, and composite materials.

Porous Materials. Transient conduction and radiation heat transfer through planar porous thermal insulation was analyzed by Tong et al. (1984). The analysis was directed toward the use of transient methods for measuring the thermal conductivity of porous insulation, since steady-state methods require long times to establish thermal equilibriums. The analysis incorporated the thermal conductance of the material and the two flux equations for radiation heat transfer. The material was treated as an isotropic homogeneous porous medium that absorbed, scattered, and emitted thermal radiation. Results of the analysis suggested that when the porous material was a lightweight fiberglass insulation, neglecting radiation would result in significant errors in predicting the hot-wall temperature rise.

Zwart and Yovanovich (1985) modeled a simple packed system of identical spheres as an equivalent homogeneous substance having an effective thermal diffusivity, and they analyzed the transient, one-dimensional conduction within the system. The analysis incorporated the influence of the constriction resistance within the spheres and the gap resistance between the contacting spheres. The temperature profile in the material and the effective thermal conductivity of the heterogeneous material were evaluated. Results of the analysis showed that the effective thermal diffusivity is dependent upon the lumped parameter effective diffusivity, time, position within the system of spheres, and the total number of spheres. The sphere-packing arrangement and boundary conditions were also found to affect the effective diffusivity.

A theoretical model for the thermal conductivity of a consolidated mixture of two metal powders was developed by Hasley (1986), using the technique of volume averaging to produce working equations applicable to thermal conduction through mixtures. Modeling of mixtures is complex; therefore, the analysis was restricted to components that were well known and characterized. An experimental investigation was conducted for different mixture volume fractions of 70/30 Brass and 316 Stainless Steel to above 90 percent theoretical density, using both air and water as saturants. Experimental results were presented in both tabular and graphic form. Both the experimental data and the published data compared favorably with the values predicted by the model.

An inversion method for determining effective thermal conductivities of porous materials was developed by Kamiuto and Iwamoto (1987a). The method utilized numerical simulations and observed mean effective thermal conductivities. The effective thermal conductivity of glass beads was used successfully to predict temperature profiles within the glass bead layer. In an extension of the work, Kamiuto and Iwamoto (1987b) analyzed the combined conductive and radiative heat transfer through a layer of glass particles, using different layer thicknesses. The theory for predicting the effective conductivity of a glass particle layer by conduction was found to be accurate for ratios of the disperse phase to continuum phase of about 0.5 to 100. The present analytical method reported accurately predicts heat transfer and temperature profiles for a layer of glass particles having comparatively larger diameters.

As a result of increasing interest in moist porous media, Woodbury and Thomas (1985) conducted an experimental investigation to determine the variation of the effective thermal conductivity of a highly porous glass fiber insulation with moisture content ranging from zero to 100 percent on a dry mass basis. The experimental study utilized a spherical thermistor operating in a self-heating mode and providing local effective conductivity values. Results indicated that there was a strong nonlinear dependence of thermal conductivity on mositure content and that currently available models do not adequately reflect the observed thermal behavior of the moist fiberglass insulation.

Vafai and Alkire (1984) investigated the effects of an external boundary and variable porosity on heat transfer for flow through a variable-porosity medium. Packed beds for 5 and 8-mm-dia beads were used for different packings. Variable porosity close to an impermeable boundary led to flow maldistribution and channeling, or the occurrence of maximum velocity in a region close to an external boundary. It was noted that different packings do not significantly alter the heat transfer process for a given bead size. The average Nusselt number, which incorporates the effect of thermal work on the activity of the bed, was correlated with the Reynolds number based on bead diameter using a linear correlation. The experimental results were found to compare favorably with the resulting correlation for different packings.

Peterson and Fletcher (1987a) conducted an experimental investigation to determine the thermal contact conductance of packed beds of spherical particles in contact with flat surfaces. Beds comprised of Aluminum 2017-T4, Yellow Brass, Stainless Steel 304, and Chromium Alloy AISI 52100 were placed in contact with flat Stainless Steel 304 surfaces. Tests were conducted at a mean interface temperature of 66°C over an apparent contact pressure range of 0.2 to 8.0 MPa. An expression for the correlation of the data was developed utilizing the earlier work for single, large sphere-flat contacts and for large numbers of microscopic asperities. Data for both large single-point contacts and small, multiple contacts compared favorably with the correlation expression because the interface between beds of spherical particles and flat surfaces could be predicted with a high degree of accuracy.

Since heat pipe wicking is frequently comprised of sintered metal materials, Peterson and Fletcher (1986) conducted an experimental investigation of the effective thermal conductivity of sintered heat pipe wicks. The investigation included both sintered Nickel 200 and sintered copper powders over a temperature range of 25°C to 100°C, with several porosities in both dry and water-saturated conditions. The experimental results were found to compare favorably with Dul'nev (1965) and with other published experimental data. The effective thermal conductivity of the sintered materials was found to be a function of the thermal conductivity of the metal from which the sintered powder was made, the thermal conductivity of the fluid filling the pores, the porosity, and the mean sample temperature.

Layered Materials. The thermal contact resistance of layered materials has taken on increased importance in printed circuit boards, transformer systems, insulation, motor/generator armatures and fields, and surface oxides or material coatings. The determination of the thermal resistance of layered slabs is particularly important when energy dissipation is required.

In order to consider the influence of oxide films on the thermal resistance of contacts, Al-Astrabadi et al. (1980) developed an analysis for the thermal resistance of oxidized flat, randomly rough metallic surfaces using stochastic representations of surface microtopography with assumed uniform film thicknesses. Experimental data for oxidized contacts between EN3B mild steel surfaces were obtained and surface topographies and oxide film thicknesses were measured. The resulting data compared favorably with the theory developed for oxide films; however, the experimental data were obtained only for first loading of new surfaces.

Sheffield et al. (1980) investigated the steady-state thermal contact conductance of multilayered, electrically insulated sheets. Experimental tests were conducted for twelve combinations of sheet materials and surface coatings in vacuum and low-pressure helium environments. The analysis considered a stack of laminations composed of a number of similar contact elements and evaluated the model utilizing the steady-state temperature distribution in a plate. The analysis compared favorably with Veziroglu et al. (1979) and comparison with experimental results indicates excellent agreement. Sheffield indicated that the predictive model could be improved, however, through the inclusion of an effective microhardness.

The heat transfer through a two-material, periodically layered, semi-infinite solid was investigated by Hagen (1987). The analysis utilized a Fourier series, and the specific heat and density of the layers were assumed equal. A perturbation expansion was used to provide a solution to the unsteady conduction problem. The analysis provided a reasonable prediction of the temperatures within the periodically layered, semi-infinite medium.

Luu et al. (1986) investigated the heat transfer in evacuated multilayer and mass insulations, considering relatively large amounts of residual gases. Models were used to study the contributions of gas conduction, solid conduction, and radiation. These models were based on the temperature jump theory and existing fibrous insulation formulations. Experimental data were obtained for two multilayer insulation specimens with residual gases of air and hydrogen. Results of the investigation indicated that the characteristic dimensions of an evacuated insulation significantly affected the performance in the presence of residual gases. The thermal accommodation coefficients between air and the materials were higher than those for hydrogen and the multilayer insulation materials.

Composite Materials. Although most composite material research has been directed toward the determination of mechanical properties, the use of composite materials in high-temperature applications and the thermal characteristics of such materials are equally important. The thermal contact conductance and thermal conductivity of composite materials are particularly important as new composite materials are developed. Han and Cosner (1981) summarized an analytical study of the steady-state effect of thermal conductivities of fiber-matrix type composites. Of particular importance was the proximity effect of embedded fibers dispersed uniformly in a matrix of region. The dispersion patterns included unidirectional fibers in a matrix and a configuration with fibers perpendicular to each other. The analysis was based on a two-region steady-state heat conduction solution. The results provided a technique for estimating the thermal conductivity of the fibrous composite.

Continuing the study of short-fiber reinforced composites, Takao and Taya (1986) developed a formulation for the effect of the thermal expansion coefficient and the thermal stresses induced around the fibers. Particular emphasis was placed on a carbon fiber/aluminum material. A parametric study was conducted, and the results were useful for the prediction of the thermal expansion coefficient for short fiber composites, as well as for the analysis of the stress in and around the fiber. Han et al. (1985) conducted an investigation of the effective thermal conductivity of graphite-epoxy composites with various fiber orientations and fiber content of 60 percent by volume. Analysis of the results suggested that there was a correspondence between the directional effect of conductivity of composites with unidirectional fibers. Further, the correspondence principle permitted an estimate of the conductivity of other balanced composite materials.

An investigation of the thermal resistance between cryogenic components was conducted by Yoo and Anderson (1983) in order to enhance the thermal resistance between mechanical contacts at temperatures below 4 K. A number of material junctions were considered, including copper, lithium fluoride, oxidized aluminum, sapphire, fiberglass, and thermal bonding cement. Results indicated that a thermally bonded metal/dielectric joint had the lowest resistance and that the use of sapphire plates separated by alumina powder increased the thermal impedance by a factor of more than 106.

Experimental thermal contact resistance values for selected metals and composites were obtained by Marchetti et al. (1987) for aerospace applications. Observed average values of thermal resistance indicate the relevance of the whole thermal resistance, particularly for small composite material thickness and metal/composite interfaces. Results were presented in graphic form for graphite-epoxy laminates and glass-epoxy laminates, as well as for traditional metals.

Madhusudana (1980b) conducted an experimental investigation of Zircaloy-2/uranium dioxide interfaces, and Madhusudana and Fletcher (1983) developed a power-law correlation to predict the dependence of nondimensional thermal contact conductance on nondimensional contact pressure for Zircaloy-2/uranium dioxide surfaces in contact. The correlation incorporated a wide range of data for joints formed by flat surfaces; however, cylindrical surfaces were not included since the contact pressure may only be inferred indirectly for these surfaces. The analysis took into account the effect of the mean interface temperature on the contact conductance, and utilized simple power laws to deduce the relationship between nondimensional conductance and nondimensional pressure. The correlation compared favorably with all of the data and results within a standard deviation of only 6.78 percent, as noted in Fig. 3.

The effect of the thermal conductivity of uranium dioxide fuel materials was investigated by Peterson et al. (1987a, 1987b) for the case in which fuel elements are comprised of packed beds of spherical or near-spherical particles. Analytical and experimental investigations were reviewed in order to develop a technique for determining the thermal conductivity of beds of packed spherical particles used in reactor fuel elements. Analytical techniques were presented and compared with existing experimental data. The results of the investigation indicated that additional analytical and experimental investigations are necessary in order to provide accurate predictions of the effective thermal conductivity of sphere-pac reactor fuels.

Thermal Conductance in Microelectronics

The optimization of microelectronic systems poses significant thermal design and analysis problems because of the in-

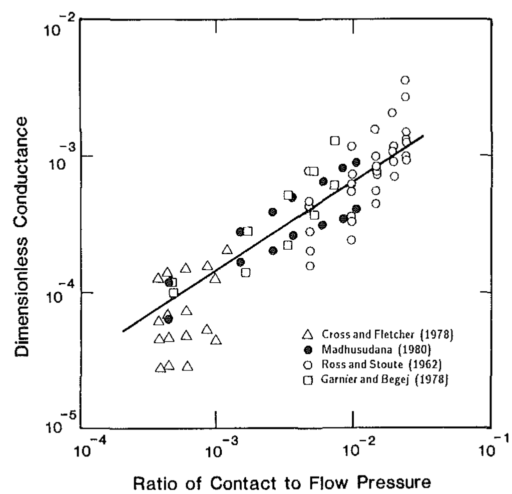

Fig. 3 Comparison of thermal contact conductance data for Zircaloy 2/uranium dioxide with a proposed correlation by Madhusudana and Fletcher (1983)

creased thermal loads that occur when components are miniaturized. The heat generation within microelectronic circuits, printed circuit boards, and multichip modules has become the primary factor that limits the physical size of both the individual components and multichip modules (Kraus and Bar-Cohen, 1983). The thermal environment and associated high temperatures could lead to overheating, significantly reducing component performance and increasing the possibility of total failure. As a consequence, there is an increasing interest in improving the thermal conductance at interfaces within microelectronic systems.

As a result of the increased interest in the cooling problems associated with microelectronic systems, an International Symposium on Cooling Technology for Electronic Equipment was held in 1987. This Symposium addressed such issues as air cooling techniques, liquid cooling techniques, thermal systems modeling, conduction cooling, and internal resistances. The conduction cooling and internal resistance area includes various aspects of thermal contact resistance. The recent literature concerned with thermal contact resistance in microelectronic systems may be divided into two general categories, those investigations dealing primarily with theoretical/analytical or numerical studies of heat transfer in microelectronic systems and components, and those investigations which are more experimental in nature.

Analytical Studies. In the area of theoretical analyses, Yovanovich et al. (1986a) developed analytical one and two-dimensional solutions for annular fins of constant thickness in a configuration that might be used as a heat sink for a microelectronic chip. The solutions were reduced to polynomial form for use in calculating the heat loss ratio as a function of dimensionless geometric and fin parameters. Numerical computations were performed for selected conditions, and graphic results were provided for the heat loss rate ratio as a function of the Biot number for selected fin-diameter ratios.

The contribution of contacting surfaces was statistically estimated by Haji-Sheikh and Lakshminarayana (1987) in an analytical/numerical Monte Carlo solution for temperature distribution within a silicon chip. The contact resistance values for the silicon chip analysis were developed by Chu et al. (1982). The resulting solution provided the temperature at critical points within the chip and surrounding bodies.

A three-dimensional finite difference numerical model with thermal resistance measurements was used by Chyu and Aghazadeh (1987) to study the thermal performance of an internally molded heat spreader for a 48-lead, plastic, dual, in-line package (PDIP) and a 68-lead, plastic, leaded chip carrier (PLCC). A parametric study of the heat spreader characteristics was conducted, and the results indicated that the package thermal resistance was strongly influenced by the area and location of the heat spreader but relatively insensitive to the heat spreader thermal conductivity and thickness. Favorable agreement was noted between the model and experimental measurements, and the use of such models was recommended for predictions of IC package thermal performances.

Negus and Yovanovich (1987) developed a thermal analysis procedure for semiconductor dies using fundamental solutions with surface element methods and arrays of images. The thermal analysis procedure was demonstrated with a complex contact distribution example, and the results indicated that for high power applications, silicon performed significantly better than gallium arsenide. As a consequence, the layout of gallium arsenide circuits appeared to be of importance due to the internal resistance. The thermal analysis procedure was developed as an efficient computational tool for designers involved in the circuit layout of semiconductor dies.

In view of the increased demands on microelectronic system design, Vanoverbeke et al. (1987) developed an approximate expression for the thermal resistance of bolted microelectronic chip carriers. The system investigated consisted of a semiconductor die mounted on a cylindrical carrier, which was bolted to a large heat sink. The thermal resistance of the carrier represented a significant portion of the overall thermal resistance. Parametric studies were conducted for a selection of geometric and loading criteria to provide the lowest possible thermal resistance. For carriers with a thickness less than optimum, there was a reduction in the thermal resistance. Conversely, for carriers with thicknesses above the optimum value, there will be an increase in the resistance.

Theoretical/analytical/numerical models of the heat transfer or thermal resistance of microelectronic systems and modules provide excellent opportunities for parametric studies. These studies permit the development of system designs that have reduced thermal resistance, greater heat dissipation, optimum spacing and sizing, and better materials selection. Such studies are extremely useful in analyzing new components and designs. While some of these theoretical/analytical/numerical investigations have included experimental data for comparison, other investigations have not. In most instances, real semiconductor systems and geometries are sufficiently complex that modeling techniques are difficult to use and experimental investigations have been conducted to provide data for analysis.

Experimental Studies. Because a majority of the thermal contact resistance data are not directly applicable to microelectronics applications, Eid and Antonetti (1986) conducted an experimental investigation of aluminum in contact with a silicon chip in air, argon, and vacuum environments. The test surfaces were appropriate for microelectronics applications, i.e., ground aluminum and polished silicon surfaces. The contact pressures were lower than those of previous studies, and the test samples were similar in size to semiconductor chips and modules, on the order of 4 mm^2. The experimental data compared favorably with correlations for the solid component of the thermal resistance in a vaccum, and the gap component compared well with the correlations of Yovanovich et al. (1983). Accommodation coefficients estimated from the data also compare well with published values.

The thermal contact resistance occurring at the bonded joints between silicon chips and substrate materials was investigated by Peterson and Fletcher (1987b), utilizing seven conductive epoxies (with a wide range of thermal conductivities) in contact with ground aluminum surfaces. Although the thermal contact resistances were relatively constant with respect to mean junction temperature, the contact resistance component of the overall joint resistance increased markedly with respect to the thermal conductivity of the diebond materials. Based on these data, an expression was developed for predicting the overall thermal resistance as a function of the void fraction, the thermal conductivity and thickness of the diebond material, and the void fraction.

Peterson and Fletcher (1988b) also evaluated the thermal resistance between substrate and mold compound materials. The investigation included four mold compounds and three heat spreader materials over a range of 20°C to 70°C at interface pressures of 0.5 to 5.0 MPa. Results of the investigation indicated that the mold compound and substrate/spreader interface in electronic equipment was relatively constant with respect to the mean interface temperature but varied significantly with changes in the interfacial pressure. The correlation proposed by Yovanovich et al. (1983) for metal-to-metal interfaces was shown to predict the thermal contact resistance at these interfaces accurately.

Techniques for enhancing the thermal conductance of a joint have been discussed by Fletcher (1984), Fletcher and Peterson (1986), and Yovanovich (1987). Thermal enhancement generally involves the use of interstitial fluids, greases, or metal foils at the interface. In view of the fact that greases migrate with time, metallic foils or metallic coating on surfaces may provide a more suitable technique for thermal enhancement of electronic chips and modules.

Peterson (1987) presented a summary of two methods by which the thermal contact resistance occurring at the metal-to-metal interfaces might be reduced. These methods included the use of thin metallic foils and metallic surface coatings. Comparative published data are presented for metallic foils and coatings for selected material combinations.

Peterson and Fletcher (1988a) presented a brief review of recent experimental and analytical investigations utilizing thin metallic foils with applications to microelectronic systems, along with the results of an experimental investigation of selected soft metallic foils. This investigation evaluated the effects of surface roughness along with foil hardness and thermal conductivity for optimum foil thicknesses. Results indicated that for optimum conditions of foil thickness and surface roughness, the thermal resistance could be reduced by a factor of as much as seven over the bare junction resistance. The study concluded that for metallic foils to be effective for thermal enhancement uses, the foils must be very thin, and that the parameter proposed by Yovanovich (1987), k/H, could be used to rank the thermal enhancement of various foils. Comparisons of metallic foil and coating characteristics are given in Figs. 4 and 5.

Chou and Jou (1987) conducted experiments to ascertain the effects of thermal greases with metal powder at the interface between two metallic surfaces in contact. The thermal grease was observed to serve as an interface filling medium, eliminating the air or gas trapped at the interface. Thermal grease with 10 to 20 weight percentage of copper powder decreases the thermal resistance; however, larger quantities of powder increase the thermal resistance and decrease the fluidity of the grease mixture.

Pinto and Mikic (1987) proposed a novel design concept for reduction of the equivalent thermal contact resistance for electronic equipment applications. The concept incorporated an increase in the apparent interface area without changing the projected area between the two components, utilizing two or three-dimensional matching features such as ribs and grooves, or pins and holes. The analysis indicated that the equivalent thermal contact resistance could be reduced substantially by modifying the contact area. The minimum contact resistance ratio possible was essentially the ratio between the flat interface and the extended surface areas.

Thermal Conductance in Biomedicine

Heat transfer mechanisms in and between biomaterials have received increasing attention with the development of synthetic organs, tissues, and other implantable devices. Knowledge of the thermal properties of biomaterials, as well as in situ tissue, is essential to modeling and to the development of new materials and organs. There are numerous circumstances in which the temperature gradient and the temperature difference between body tissues or fluids and implantable materials affect the overall performance or comfort of implantable devices. These circumstances include coronary angioplasty (removal of atherosclerotic plaque on arterial walls), the selection of appropriate bone cements for implantable replacement joints, measurement systems requiring

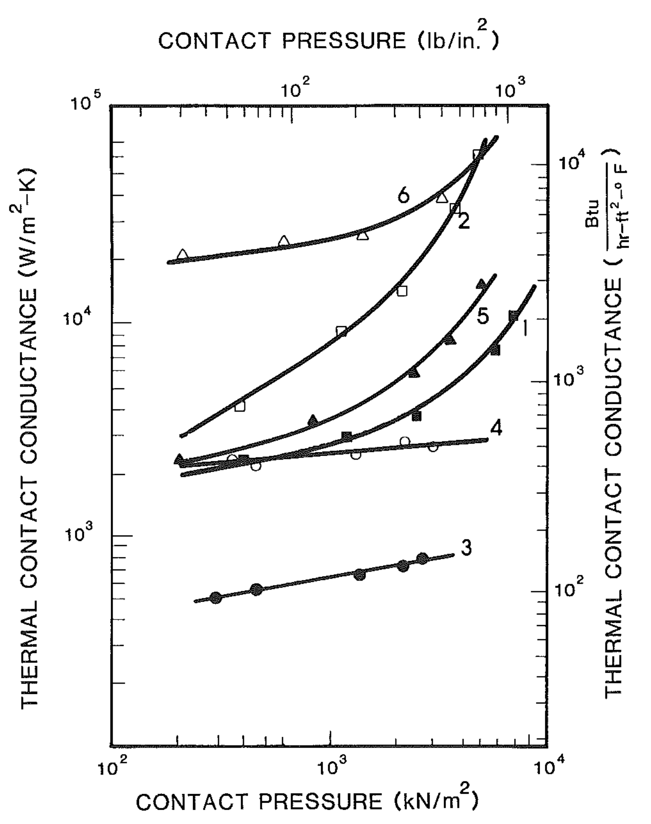

Fig. 4 Effect of metal foils on thermal contact conductance (Peterson, 1987): (1) SSKh18N9T bare interfaces; (2) SSKh18N9T interfaces with copper foil; (3) SSKhN789T bare interfaces; (4) SSKhN789T interfaces with copper foil; (5) molybdenum VM-1 bare interfaces; (6) molybdenum VM-1 interfaces with copper foils

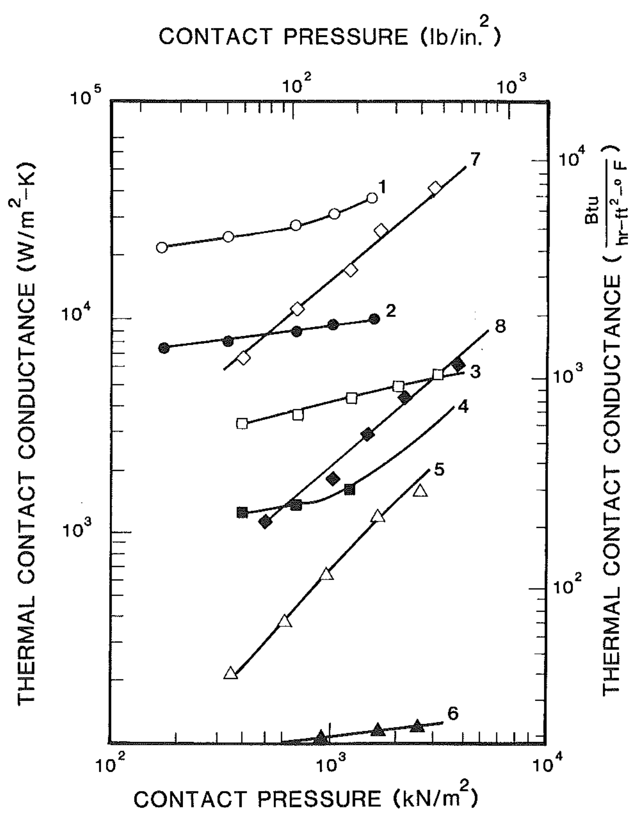

Fig. 5 Effect of metallic coatings on thermal contact conductance (Peterson, 1987): (1) bronze surface coated with tin/nickel alloy; (2) uncoated bronze alloy; (3) SS surface coated with silver; (4) uncoated SS surfaces; (5) SS surface coated with aluminum alloy; (6) uncoated SS surface; (7) nickel surface coated with nickel alloy; (8) uncoated nickel surfaces

intrusive catheters and probes, dental cements for installation of crowns or caps, and other biomedical applications.

A number of methods have been proposed for the dissolution or removal of atherosclerotic plaque utilizing surgical, chemical, and mechanical means. One recent technique deals with the use of a laser catheter for removal of plaque in both normal and diseased vessels by means of vaporization (Lee et al., 1981, 1983). Because excessive temperature at the artery wall may damage the artery as well as the surrounding tissue, the removal of plaque using laser techniques requires accurate temperature control as well as knowledge of the temperature gradients, tissue thermal conductivity, and thermal contact conductance of the plaque–arterial wall interface. Valvano et al. (1984) provided experimental measurements of the thermal conductivity and thermal diffusivity of atherosclerotic plaque and arterial walls from the human circulation system, using self-heated microthermistor probes over the temperature range of 25°C to 95°C. The atherosclerotic plaque investigated was categorized as fibrous, calcified, or fatty.

There are a number of factors that contribute to the failure of total joint prostheses, and one of the major factors is a loosening of the cement–bone interface, as noted by Liu et al. (1981). Failure at this interface is attributed to fatigue cracking of the bone–cement interface and resorption of bone. In addition, according to DiPisa et al. (1976), the high temperature attained by the cement during polymerization may cause tissue necrosis at the cement–bone interface. Further, the widely different material properties of bone and bone cement lead to a thermal discontinuity or a thermal resistance at the interface. This thermal contact resistance contributes to the loosening of the cement–bone interface. As in all thermal contact resistance problems, the surface characteristics of the bone and the replacement component are important, and Geiger and Greenwood (1981) proposed several surface preparation techniques. The characteristics of the thermal resistance at the cement–bone interface are not clearly understood and warrant further investigation.

Octerbeck et al. (1988) investigated the thermal contact conductance at the interface between bone, polymethylmethacrylate (PMMA)-based bone cements, and stainless steel. Measurements were made for both the thermal contact conductance of the bone–cement interfaces and the thermal conductivity of the bone for both saturated and dry conditions. The temperatures used, 286 K to 300 K, were similar to those presented previously for noncancellous bone. Both high-viscosity and low-viscosity bone cements were used for the investigation, with the high-viscosity cement resulting in the lower thermal contact resistance values.

Due to the surgical inaccessibility of some tumors, lesions, and cancerous regions of the human body, focused heat is used as a means for destroying diseased cells. Localized heat may be used as a therapeutic tool for precisely localized necrosis, particularly with reference to the central nervous system. Techniques for providing localized heating include thermal probes, radiofrequency probes, or ultrasound. Each of these techniques is appropriate for certain therapeutic uses. Thermal probes heated by temperature-controlled fluids or by electricity may be used for soft tissue as well as bones. Radiofrequency probes, such as those used in neurosurgery, may also have potential applications in bone surgery. Ultrasound has obvious advantages due to its nonintrusive nature, and offers selective heating in soft tissue. The use of ultrasound is not appropriate in bone due to the high absorption coefficient compared to soft tissue. Thermal probes or radiofrequency probes may be used for the treatment of certain cancers. The temperature of the treated area, however, must be carefully controlled and is generally measured with catheters composed of thermocouples and hypodermic needles. The primary problem with such measurements is perceived to be the thermal contact resistance between the thermocouple and hypodermic needle.

Selected Topics

Although the primary areas of recent contact heat transfer work have been reviewed in earlier sections of this paper, there are other equally interesting aspects of contact heat transfer that warrant consideration in this review. Special topics include the thermal conductance of dissimilar metals and the associated thermal rectification phenomenon, the contribution of interstitial gases to thermal contact resistance, the problems occurring in cylindrical contact surfaces, as well as pe.:odic and sliding contacts, and the problems associated with the measurement of thermal contact conductance.

Thermal Rectification. There are many situations in which the thermal resistance of dissimilar metal interfaces may be important, including fin-tube heat exchangers, nuclear fuel rods, printed circuit boards, and cryogenic systems. The phenomenon of thermal rectification occurs when the thermal contact conductance of a junction composed of dissimilar metals varies with the direction of heat flow. Occasionally, thermal rectification is also observed for similar metal surfaces with different surface characteristics. Selected data for the thermal conductance of dissimilar metal interfaces are shown in Fig. 6. Padgett and Fletcher (1982) experimentally investigated the thermal contact conductance of aluminum/stainless steel interfaces for apparent contact pressure from 68.9 to 1.33×103 kPa, with mean junction temperatures up to 195°C, for a selected range of surface conditions. Data indicated thermal rectification effects at higher apparent contact pressures.

Williams and Fletcher (1983) reviewed the various, often conflicting, investigations of thermal rectification and found that directional effects existed but that experimental data were inconsistent. Contact behavior was found to be very sensitive to deviations in surface characteristics. Elastic deformations permitted directional effects to exist indefinitely as the heat flow direction was cycled, but plastic deformations resulted in diminishing effects after a few cycles. Published data conflict because of the wide variety of test materials and surface characteristics involved in thermal rectification tests.

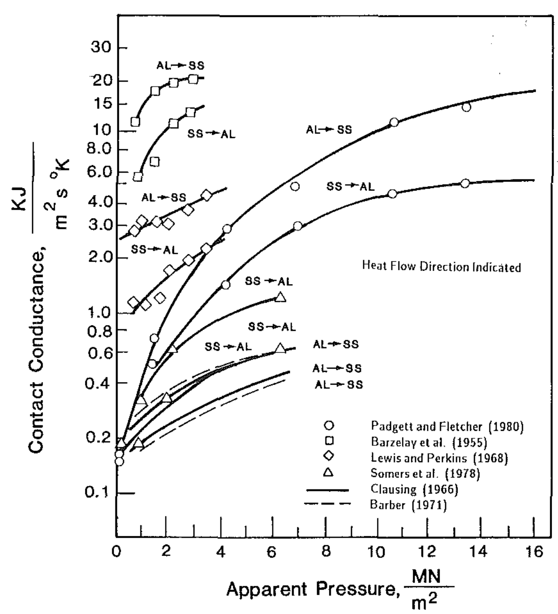

Fig. 6 Variation of contact conduct with apparent interface pressure for selected dissimilar metal surfaces (Padgett and Fletcher, 1982)

In an effort to understand the physical basis of thermal rectification, Somers (1983) and Somers et al. (1987) conducted a semi-quantitative study of an idealized contact between two smooth, frictionless spheres utilizing Hertzian methods. An expression defining the relationship between the contact length and the heat flux through the contact was derived. Thermal rectification was shown to exist between two elastic spheres through which heat was flowing, and this rectification was found to be a function of the thermal distortions on the contact interface which, in turn, were a function of the temperature fields within the contacting bodies. Observations of experimental trends indicated a thermal rectification increase due to increases in apparent contact load, a thermal rectification reversal due to contact surface changes, a thermal rectification reversal due to heat flux magnitude changes, and a reduction in thermal rectification due to the action of microscopic asperity contacts. The contact model developed may be used to explain the existence of these directional bias trends on a quantitative basis.

Gas Conductance. The gases filling the voids between a pair of contacting surfaces may significantly affect the overall heat transfer at a junction between contacting materials. Mentes et al. (1981) developed a model using mixed boundary conditions to determine the thermal contact conductance of coated, multilayered sheets. The fluid thermal conductivity and accommodation coefficient were introduced into the formulation as the equivalent fluid conductivity. Experimental contact conductance data were obtained with environments of vacuum, air, and helium, and the effective hardness values for coated surfaces were ascertained. Based on these values, the accommodation coefficients for the interfacial gas and surface coating combinations were estimated. Tabulated results for accommodation coefficients were included.

Madhusudana and Fletcher (1981c) compared the relationships between the peak-to-peak surface roughness, the effective gap width, and the temperature jump distances proposed by several authors. These comparisons required knowledge of the accommodation coefficient and the relationship between profilometer surface roughness readings and peak-to-peak roughness heights. The relationship between the effective gap width, peak-to-peak height, and temperature jump distance was compared for several different prediction techniques, as shown in Fig. 7. The gas conduction contributed to the overall heat transfer, started to decrease at a gas pressure called the threshold pressure, and became negligible below pressures of 0.13 kPa.

Cylindrical Contacts. The thermal contact conductance of concentric cylinders made of similar or dissimilar materials is somewhat different from planar thermal contact conductance. The apparent contact pressure in coaxial cylinders depends on the initial interference or clearance between cylinders, the differential expansion or contraction due to the temperature gradient, and the differential expansion or contraction due to the

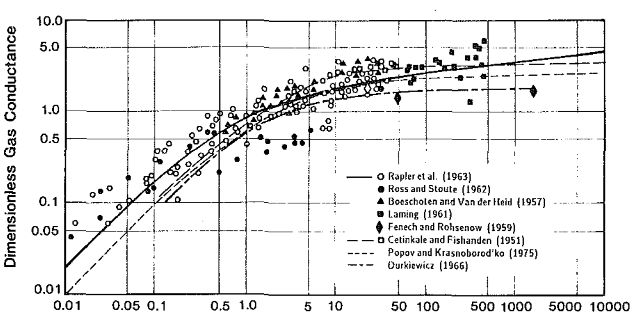

Fig. 7 Comparison of dimensionless gas conductance with dimensionless effective gap width (Madhusudana and Fletcher, 1981c)

temperature difference at the interface between cylinders. For these reasons, it is generally more appropriate to express the contact conductance as a function of heat flux rather than interface contact pressure.

Madhusudana and Fletcher (1981b) summarized an experimental investigation of heat transfer occurring across the interface of composite cylinders formed by stainless steel–stainless steel and armco iron–armco iron cylinders. Experimental tests were conducted in vacuum and in an air environment at atmospheric pressure. The data indicated that the conductance increased with increased contact pressure, which resulted from the differential expansion of the cylinders, caused by the radially outward heat flux.

The thermal constriction resistance for two cylindrical surfaces, one with longitudinal grooves, was evaluated by Wang nd Nowak (1981). In this geometric configuration, the lands were in contact, but the grooved area had no contact. This configuration was representative of a double tube heat exchanger for process fluids and lethal gases. The analysis of the cylindrical contacts was based upon steady heat flow, constant thermal conductivities, a two-dimensional temperature distribution, and a perfect contact at the contact land. In view of the mixed boundary conditions, no analytical solution was available. An equivalent heat flux distribution function was employed, however, and results were developed numerically. The results defined the limit of the thermal constriction resistance for cylindrical surfaces.

Chu et al. (1986) utilized the hyperbolic heat conduction equation to investigate the transient response of a finite, composite, hollow cylinder composed of two different materials. The cylinder was heated internally by a moving line source and cooled convectively on the exterior surface. Results of the investigation, which were calculated numerically, showed that the relaxation time and thermal conductivity ratio played an important role in the transient heat transfer. This work, however, assumed perfect contact at the interface.

A theory for prediction of the thermal contact conductance of concentric cylinders was developed by Madhusudana (1986). The contact pressure was calculated in terms of the temperature gradient in one of the cylinders, the initial allowance between cylinders, the surface characteristics, the cylinder radius ratios, the cylinder material properties, and the interstitial medium. The resulting formulation could be used to establish a relationship between the heat flux and thermal contact conductance. These results showed that conductance increased with increased heat flux, because of the resulting increased contact pressure. A surprising conclusion was that for cylindrical joints, a material combination with lower effective thermal conductivity may, in fact, exhibit higher thermal conductance than a combination with higher effective conductivity.

The design and construction of an apparatus to measure the heat transfer performance of cylindrical joints was reported by Madhusudana and Litvak (1987). The apparatus incorporated test specimens that were heated internally and cooled externally. Experimental data were presented for stainless steel interfaces and stainless steel–aluminum interfaces, and the results compared reasonably well with theoretical predictions.

Periodic and Sliding Contacts. Vick and Özişik, (1981) utilized the finite integral transform technique to obtain quasi-steady-state solutions for the problem of two periodically contacting finite regions with imperfect thermal contact at the interface. This investigation focused on the effects of contact duration, thermal contact conductance, thermal conductivity, and thermal diffusivity on the temperature distribution across two contacting surfaces. Moses and Johnson (1987) experimentally examined the problem of periodically contacting similar metal surfaces and obtained results that are comparable in both form and magnitude to those of Vick and Özişik (1981).

Moses and Johnson (1988) also experimentally examined the approach to the quasi-steady-state condition for similar metallic surfaces in periodic contact, focusing on the behavior of the thermal contact conductance during the contact portion of a cycle and on the number of cycles required to reach a quasi-steady-state condition. Results of these experiments indicated that, for cycles with short contact times, the thermal contact conductance varied throughout the contact period, while for longer contact times, a uniform value of thermal contact conductance was ultimately attained. The experimentally obtained temperature distributions from Moses and Johnson (1988) also were examined by Beck (1988) as an example of a parameter and function estimation technique for the inverse heat conduction problem. Utilizing this technique for computing values of the thermal contact conductance, Beck obtained values similar in both form and magnitude to those obtained by the experimental analysis. Utilizing the function estimation analysis, in which the transient conduction equation was solved numerically and the thermal contact conductance was computed at each measured time step, Beck obtained values for the thermal contact conductance that are considerably more uniform than either the experimental analysis or the parameter estimation analysis.

Dodd and Moses (1988) extended this set of experiments to consider periodic contacts between aluminum and stainless steel. Results for this single case of periodic contact between dissimilar metals indicated that the same transient behavior of the thermal contact conductance was observed for all metal pairs.

The flash temperatures of circular and elliptical contact spots for two materials in sliding contact were evaluated by Kuhlmann-Wilsdorf (1987). The parameters used in the analysis included experimental data, the coefficients of friction, known material properties, and the ellipticity and number of contact spots. The prediction equations were simplified for the limiting cases of very high and very low speeds. The effect of sliding rate on the flash temperature through the strain rate dependence of local hardness was considered, and demonstrated to be of considerable importance.

Yuen (1988) developed an analysis of the conduction between sliding solids with heat energy generated along the region of contact. An asymptotic solution for the heat flux position to each solid was used to develop closed-form expressions for the temperature fields in the solids. Based on the analysis, an appropriate parameter was recommended for use with correlations to estimate the thermal penetration into the solids.

Conductance Measurements. Intrinsic problems associated with accurately measuring the thermal contact conductance were discussed by Snaith et al. (1983). The specimen geometry and physical details, the apparatus design and temperature measurement procedures, the test schedules, and the experimental error all affect measurement accuracy. This suggests the need for a standard test procedure for measurement of thermal contact conductance. The authors recommended a number of special criteria for thermal contact conductance testing and analysis including a specimen form ratio, the reduction of transverse heat losses in the measurement system, specific arrangements for facility design and heat metering, temperature measurement and uncertainty procedures, and data presentation. While such a concept has merit, there has been little serious consideration or implementation of these recommendations.

The characteristic of thermal contact resistance was used in the development of a hardness tester by Goldsmid and Johnston (1981). When a probe of relatively low thermal conductivity was pressed against softer materials of higher conductivity, the thermal contact resistance depends upon the radius of contact. A ruby-tipped, heated, copper-constantan thermocouple was applied to several materials of high conductivity, and the resultant thermoelectric EMF generated was found to be directly related to the diagonal length of the indentation produced. Goldsmid (1982) extended the work utilizing a probe with a diamond tip. The load on the diamond-tipped probe was varied until the thermal contact resistance was the same as that of a heated copper-tipped probe. Experimental results for a range of materials indicated that the load required for balance was proportional to the Vickers hardness number.

Problems related to beaded and intrinsic thermocouples were studied both theoretically and experimentally by Cassagne et al. (1986). Analysis of the thermocouples indicated a temperature disturbance and the influence of thermal contact resistance. The perturbations produced in the temperature field were assumed to be the result of several phenomena including macroconstriction of the flux lines, contact resistance at the interface between the thermocouple bead and the material, and the wire environment heat transfer. Results of the analysis were experimentally verified and the errors introduced in the two methods of measurement were analyzed.

Antonetti and Eid (1987) proposed a new procedure for measuring steady-state thermal contact resistance that relied on monitoring the contact resistance rather than the specimen temperatures. The concept was based on the premise that the actual contact resistance should be invariant with time because the capacitance of the contact is extremely small compared to the thermal capacitance of the measurement systems involved. Based on experimental verification using transient techniques, apparent contact resistance reached actual contact resistance values considerably faster than with the traditional steady-state approach.

Conclusions and Recommendations

This review of recent developments in contact conductance heat transfer, although by no means exhaustive, has delineated areas in which recent experimental and theoretical studies have been conducted. The categorization of studies by area permits a more detailed comparison among similar analyses and experimental investigations. These comparisons have suggested several features to be considered in further analysis of contact conductance heat transfer.

Based on recent theoretical analyses, it is clear that the surface parameters are a major factor in thermal contact conductance. Conforming surfaces have been evaluated, and reliable models have been developed for predicting the apparent contact area. Further, effects of geometry have been studied, as have various surface preparation techniques. Nevertheless, further studies are needed to explore areas such as the combination of macroscopic elastic deformations and microscopic plastic distortions.

Inasmuch as each thermal contact conductance geometry is unique, knowledge of the surface characteristics and the material properties is essential to an analysis of the heat transfer. In addition, it can be expected that other characteristics such as load conditions, temperature regime, interstitial media, and geometry should be considered in any analysis of heat transfer at a junction.

There are a number of techniques available for the prediction of thermal contact conductance, although each prediction technique is appropriate for only selected conditions. Very few of these, however, appear to be successful, even over a limited range of conditions. At present, it does not appear possible to develop a generalized prediction technique for all test environments and conditions. While most prediction techniques are suitable for moderate-to-large loads, the studies of low apparent contact pressures have not led to comparable techniques for the prediction of heat transfer. Hence, it would appear that further investigations would be desirable at low apparent contact pressures.

In view of the increasing miniaturization of microelectronic components and the low apparent contact pressures involved, the dissipation of heat is directly related to the thermal contact resistance of chips, heat spreaders, and associated materials. Continuing investigations of new heat spreaders or chip carriers in conjunction with chip attachment techniques would be desirable.

The increasing focus on biomedical instrumentation and the limited knowledge of the thermal properties and characteristics of biological tissues suggest that further study of the thermal resistance of biological materials is warranted. Of particular interest would be bone/dental cements, catheters, body tissues, and prosthesis components.

With the advent of superconductivity and the continuing quest for higher temperature superconducting materials, it is essential that the thermal contact resistance of such materials be investigated. Initial studies should be conducted at cryogenic temperatures to assess the potential for superconducting material installation.

Plans for the development of a space station, and the continuing desire for more efficient space thermal power systems, suggest the need for a better understanding of thermal contact resistance as it relates to space applications. The desirability of providing optimum spacecraft systems necessitates the improvement of thermal control systems. Further study of contact conductance utilizing selected materials in space environments is important.

Modern composite materials, including metallic ceramics, are being considered for many high-temperature applications, especially in the gas turbine industry. The thermal characteristics of these materials, and the associated thermal contact resistance, will be important design parameters. Continuing experimental and theoretical studies of new materials would be instructive.

The characteristics of rolling and sliding contacts have been investigated from the perspective of frictional heating and the associated tribological behavior. There has been limited study of the thermal and thermomechanical effects in such contacts, and further studies are warranted. Significant thermal contact resistance leads to thermocracking and other forms of failure in rolling and sliding mechanical parts.

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