Dynamics of a Multibody Mechanical System With Lubricated Long Journal Bearings

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Clearances exist in different kinds of joints in multibody mechanical systems, which could drastically affect the dynamic behavior of the system. If the joint is dry with no lubricant, impact occurs, resulting in wear and tear of the joint. In practical engineering design of machines, joints are usually designed to operate with some lubricant. Lubricated journal bearings are designed so that even when the maximum load is applied, the joint surfaces do not come into contact with each other. In this paper, a general methodology for modeling lubricated long journal bearings in multibody mechanical systems is presented. This modeling utilizes a method of solving for the forces produced by the lubricant in a dynamically loaded long journal bearing. A perfect revolute joint in a multibody mechanical system imposes kinematic constraints, while a lubricated journal bearing joint imposes force constraints. As an application, the dynamic response of a slider-crank mechanism including a lubricated journal bearing joint between the connecting rod and the slider is considered and analyzed. The dynamic response is obtained by numerically solving the constraint equations and the forces produced by the lubricant simultaneously with the differential equations of motion and a set of initial conditions numerically. The results are compared with the previous studies performed on the same mechanism as well a dry clearance joint. It is shown that in a multibody mechanical system, the journal bearing lubricant introduces damping and stiffness to the system. The earlier studies predict that the order of magnitude of the reaction moment is twice that of a perfect revolute joint. The proposed model predicts that the reaction moment is within the same order of magnitude of the perfect joint simulation case. [DOI: 10.1115/1.1864112]

Introduction

Modeling different kinds of joints in multibody mechanical systems has recently received considerable attention in many engineering applications. Hydrodynamic journal bearings have been widely used in various types of high speed rotating machinery. The concept of perfect kinematic joints is quite often used in modeling the multibody system. Clearance exists all the time in these joints, which makes the perfect joint concept unreal. The squeeze and viscous forces developed by the lubricant in lubricated journal bearings prevents the surface contact [1]. The lubricant in the mechanical systems provides protection against wear and tear. The force built up by the lubricant film supports the external load. In multibody mechanical systems, the external load varies with time in magnitude and direction, which results in a dynamically loaded lubricated journal bearing. Figure 1 shows a dynamically loaded journal bearing [2]. If the viscosity is assumed to be constant and the bearing is taken as long, then using the nomenclature Fig. 1, the lubricant forces in the x and y direction can be written as [3]

For $E>0$:

$$F_x=\frac{-\mu\omega LR_j^3}{c^2}\left[\frac{6\pi\xi(1-G)}{(2+\xi^2)(2-\xi^2)^{1/2}}\left(\frac{k+3}{k+\frac{3}{2}}\right)\sin\gamma + \frac{3E}{(2+\xi^2)(2-\xi^2)^{3/2}}\left(4k\xi^2+\pi(2+\xi^2)\frac{k+3}{k+\frac{3}{2}}\right)\cos\gamma\right] \quad (1)$$

$$F_y=\frac{\mu\omega LR_j^3}{c^2}\left[\frac{6\pi\xi(1-G)}{(2+\xi^2)(2-\xi^2)^{1/2}}\left(\frac{k+3}{k+\frac{3}{2}}\right)\cos\gamma - \frac{3E}{(2+\xi^2)(2-\xi^2)^{3/2}}\left(4k\xi^2+\pi(2+\xi^2)\frac{k+3}{k+\frac{3}{2}}\right)\sin\gamma\right] \quad (2)$$

For $E<0$:

$$F_x=\frac{-\mu\omega LR_j^3}{c^2}\left[\frac{6\pi\xi(1-G)}{(2+\xi^2)(2-\xi^2)^{1/2}}\left(\frac{k}{k+\frac{3}{2}}\right)\sin\gamma - \frac{3E}{(2+\xi^2)(2-\xi^2)^{3/2}}\left(4k\xi^2-\pi(2+\xi^2)\frac{k}{k+\frac{3}{2}}\right)\cos\gamma\right] \quad (3)$$

$$F_y=\frac{\mu\omega LR_j^3}{c^2}\left[\frac{6\pi\xi(1-G)}{(2+\xi^2)(2-\xi^2)^{1/2}}\left(\frac{k}{k+\frac{3}{2}}\right)\cos\gamma + \frac{3E}{(2+\xi^2)(2-\xi^2)^{3/2}}\left(4k\xi^2-\pi(2+\xi^2)\frac{k}{k+\frac{3}{2}}\right)\sin\gamma\right] \quad (4)$$

where

$$k=\sqrt{(1-\xi^2)\left[\left(\frac{1-G}{C}\right)^2+\xi^{-2}\right]} \quad (5)$$

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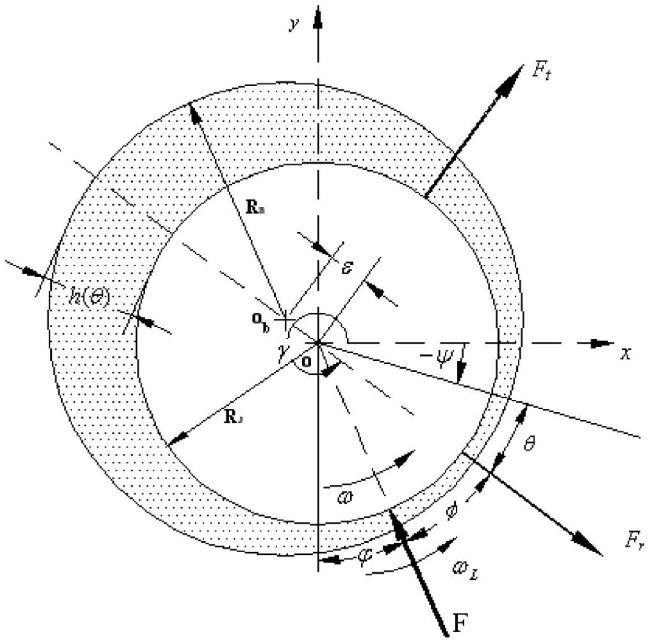

Fig. 1 Journal bearing under dynamic loading

$$\text{and } E=\frac{2}{\omega}\frac{d\xi}{dt}\quad G=\frac{2}{\omega}\frac{d\gamma}{dt}$$

Here ω is the relative angular speed between the journal and the bearing. R_B is the bearing radius and R_J is the journal radius. The radial clearance ($=R_B-R_J$) is denoted by c. The dimensionless eccentricity ratio at any instant of time is defined as the distance measure between the journal center and the bearing center divided by the radial clearance (c). γ is the angle between the line of centers and the x axis. This model showed a considerable deviation between the calculated forces and the ones obtained by numerically integrating the lubricant pressure distribution equation. It has been shown by Alshaer et al. [4], that the following relations for the lubricant forces yield accurate results.

For $d\xi/dt>0$:

$$F_t=12m\mu L\frac{R_j^3}{c^2}\left(\frac{-2(d\xi/dt)\xi\cos^3(\theta_1)}{\{1-[\xi\cos(\theta_1)]^2\}^2}+\frac{\xi\varpi(1-\xi^2)}{2(2+\xi^2)(1-\xi^2)^{3/2}}\{(2m\pi+\delta_2-\delta_1)+[\sin(2\delta_2)/2-\sin(2\delta_1)]/2+\xi(\sin(\delta_1)-\sin(\delta_2)\ \}\right) \tag{6}$$

$$F_r=12m\mu L\frac{R_j^3}{c^2}\left(\frac{-(d\xi/dt)(4m\pi+2\delta_2-2\delta_1)+\sin(2\delta_1)-\sin(2\delta_2)}{4(1-\xi^2)^{3/2}}+\frac{\xi^2\varpi\cos(\theta_1)[(2+\xi^2)\cos^2(\theta_1)-3]}{(2+\xi^2)\{1-[\xi\cos(\theta_1)]^2\}^2}\right) \tag{7}$$

For $d\xi/dt<0$:

$$F_t=12\mu L\frac{R_j^3}{c^2}\left(\frac{-2(d\xi/dt)\xi\cos^3(\theta_1)}{\{1-[\xi\cos(\theta_1)]^2\}^2}\frac{\xi\varpi(1-\xi^2)}{2(2+\xi^2)(1-\xi^2)^{3/2}}\{(\delta_2-\delta_1)+\sin(2\delta_2)/2-\sin(2\delta_1)/2\ \ +\xi[\sin(\delta_2)-\sin(\delta_1)]\right) \tag{8}$$

$$F_r=12m\mu L\frac{R_j^3}{c^2}\left(\frac{-2(d\xi/dt)(\delta_2-\delta_1)+\sin(2\delta_1)-\sin(2\delta_2)}{4(1-\xi^2)^{3/2}}+\frac{\xi^2\varpi\cos(\theta_1)[(2+\xi^2)\cos^2(\theta_1)-3]}{(2+\xi^2)\{1-[\xi\cos(\theta_1)]^2\}^2}\right) \tag{9}$$

where

$$\theta_1=-\tan^{-1}\left(-\frac{(2+\xi^2)(d\xi/dt)}{\xi\varpi}\right)$$

$$\delta\text{:Sommerfeld variable}[1+\xi\cos\vartheta=(1-\xi^2)/(1-\xi\cos\delta)]$$

$$\delta_1=\delta(\theta_1)$$

$$\delta_2=\delta(\theta_1+\pi)$$

$$\varpi=\omega-d\gamma/dt$$

$$m=1\ if\ \varpi>0$$

$$=-1\ if\ \varpi<0$$

The forces in the x and y direction can be now written as

$$F_x=F_r\cos(\gamma)-F_t\sin(\gamma)$$

$$F_y=F_r\sin(\gamma)+F_t\cos(\gamma)$$

For constrained multibody mechanical systems, the equations of motion can be written as [5]:

$$\boldsymbol{M}\ddot{\boldsymbol{q}}=\boldsymbol{g}+\boldsymbol{g}^{(c)} \tag{10}$$

where

$$\boldsymbol{M}=\text{diag}(\boldsymbol{M}_1,\boldsymbol{M}_2,\ldots\ \ldots\boldsymbol{M}_b)\quad \text{mass matrix of the system}$$

$$M_i=\begin{bmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I\end{bmatrix}\quad \text{mass matrix of body } i \text{ and } I\text{:}$$

$$\text{polar moment of inertia } I_{\zeta\zeta} \text{ of body } i$$

$$\boldsymbol{q}=[\boldsymbol{q}_1^T,\boldsymbol{q}_2^T,\ldots\ \ldots\boldsymbol{q}_b^T]^T\quad \text{Global Position coordinates for bodies in the system}$$

$$\boldsymbol{g}=[\boldsymbol{g}_1^T,\boldsymbol{g}_2^T,\ldots\ \ldots,\boldsymbol{g}_b^T]^T\quad \text{External force vector}$$

$$\boldsymbol{g}_i=[f_x,f_y,n]_i^T\quad \text{External forces and moments applied on body } i$$

$$\boldsymbol{g}^{(c)}=\boldsymbol{\Phi}_q^T\boldsymbol{\Gamma}$$

$$\boldsymbol{\Phi}=0\quad \text{constraint equations}$$

$$\boldsymbol{\Phi}_q=\text{Jacobian matrix for } \boldsymbol{\Phi}$$

$$\boldsymbol{\Gamma}=\text{vector of Langrange multipliers}$$

The equations of motion for a system of constraint bodies can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{q}}-\boldsymbol{\Phi}_q^T\boldsymbol{\Gamma}=\boldsymbol{g} \tag{11}$$

$$\boldsymbol{\Phi}=\boldsymbol{0}$$

In dynamic analysis, a unique solution is obtained when the constraint equations are considered simultaneously with the differential equations of motion with proper set of initial conditions. The main task now is to introduce the constraint equations for the lubricated dynamically loaded journal bearing into the equations of motion.

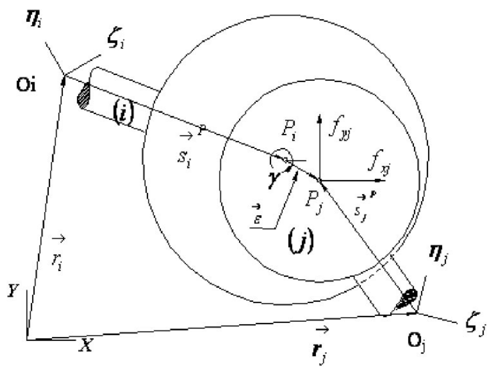

Fig. 2 Lubricated journal bearing joint

Modeling Lubricated Journal Bearings in Multibody Mechanical Systems

In traditional bearing design task, the load is known and the motion of the journal is calculated by solving the differential equations for the time dependent variables in the lubricated bearing journal bearing. In the present analysis, however, all the state variables $\omega, \gamma, d\gamma/dt, \xi, d\xi/dt$ are known and the instantaneous reaction forces on the journal from the fluid film need to be calculated. It implies that the journal bearing joint does not produce any kinematic constraints like the perfect revolute joint. Instead, it will act like a force element producing time dependent forces. These forces are a function of the time dependent parameters $\omega, \gamma, d\gamma/dt, \xi,$ *and* $d\xi/dt$, which can be evaluated at any given instant of time. Figure 2 shows two bodies i and j connected by a journal bearing joint, where part of body i is the bearing and part of body j is the journal. The center of mass of body i is O_i and the center of mass of body j is denoted by O_j. The local coordinates of the bodies are attached at their center of mass while the xy coordinates represent the global coordinate system. Point P_i indicates the center of the bearing, and the center of the journal is at point P_j. The vector $\vec{\varepsilon}$ is the vector of centers between the bearing center to the journal center and it makes an angle γ with the x axis as shown on the same figure. The forces produced by the journal are F_{xj} in the x direction and F_{yj} in the y direction. These forces are evaluated using Eqs. (6)–(9). In order to evaluate these forces, the different parameters on which these forces depend need to be evaluated.

Considering Fig. 2 again, the vector loop equation can be written as

$$\vec{r}_i + \vec{s}_i^P - \vec{s}_j^P - \vec{r}_j = \vec{\varepsilon} \tag{12}$$

This can be written in an expanded form as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \begin{bmatrix} x_i + \zeta_i^P \cos\phi_i - \eta_i^P \sin\phi_i - x_j - \zeta_j^P \cos\phi_j + \eta_j^P \sin\phi_j \\ y_i + \zeta_i^P \sin\phi_i + \eta_i^P \cos\phi_i - y_j - \zeta_j^P \sin\phi_j - \eta_j^P \cos\phi_j \end{bmatrix} \tag{13}$$

Equation (12) can be used to calculate the eccentricity as a vector. The magnitude of this vector is

$$\varepsilon = (\vec{\varepsilon}^T \vec{\varepsilon})^{1/2} \tag{14}$$

A unit vector $\vec{u}$ is defined as

$$\vec{u} = \frac{\vec{\varepsilon}}{\varepsilon} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \tag{15}$$

This unit vector has the same direction as the line of centers of the bearing and the journal, while γ is the angle between the line of centers and the x axis. Hence,

$$\begin{bmatrix} \cos\gamma \\ \sin\gamma \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \tag{16}$$

Equation (16) yields γ, which is the second parameter required for evaluating the bearing forces. The next step is to evaluate $d\varepsilon/dt$ which can be done by differentiating Eq. (13) with respect to time. This results in

$$\frac{d\vec{\varepsilon}}{dt} = \begin{bmatrix} \dfrac{d\varepsilon_x}{dt} \\ \dfrac{d\varepsilon_y}{dt} \end{bmatrix} = \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \end{bmatrix} + \dot{\varphi}_j \begin{bmatrix} -\sin\phi & -\cos\phi \\ \cos\phi & -\sin\phi \end{bmatrix}_j \begin{bmatrix} \zeta^P \\ \eta^P \end{bmatrix}_j - \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} - \dot{\varphi}_i \begin{bmatrix} -\sin\phi & -\cos\phi \\ \cos\phi & -\sin\phi \end{bmatrix}_i \begin{bmatrix} \zeta^P \\ \eta^P \end{bmatrix}_i \tag{17}$$

where $\dot{}$ represent d/dt. From Eq. (11), $d\varepsilon/dt$ can be written as

$$\frac{d\varepsilon}{dt} = \frac{(\vec{\varepsilon})^T \left(\dfrac{\vec{d\varepsilon}}{dt} \right)}{\varepsilon} \tag{18}$$

The last parameter required to calculate is $d\gamma/dt$. From Eq. (12):

$$\gamma = \tan^{-1}\left(\frac{u_y}{u_x} \right)$$

Thus

$$\frac{d\gamma}{dt} = \frac{\dot{u}_y u_x - \dot{u}_x u_y}{u_y^2 + u_x^2} = \frac{\dfrac{d\varepsilon_y}{dt}\varepsilon_x - \dfrac{d\varepsilon_x}{dt}\varepsilon_y}{\varepsilon_y^2 + \varepsilon_x^2} \tag{19}$$

From the eccentricity ratio $\xi = \varepsilon/c$ and Eqs. (12)–(19), the journal forces F_{xj} and F_{yj} can be evaluated from Eqs. (6)–(9). These forces needed to be transferred to the center of masses of both of the bearing and the journal (O_i and O_j, respectively). Again referring to Fig. 2, the forces and moments are given by

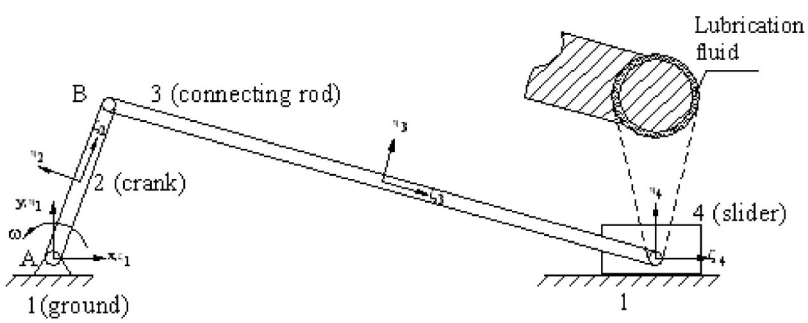

Fig. 3 Slider-crank mechanism with a lubricated journal bearing

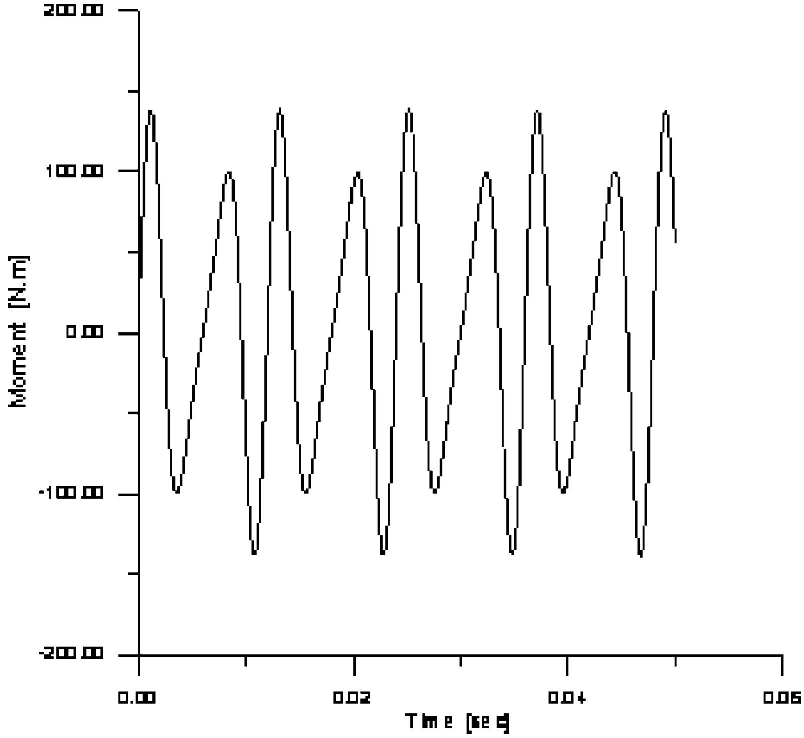

Fig. 4 Reaction moment on the crank (perfect joint)

For the journal at O_j

$$\begin{bmatrix} f_{xj} \\ f_{yj} \\ n_j \end{bmatrix}_j = \begin{bmatrix} f_{xj} \\ f_{yj} \\ f_{xj}(\zeta_j^P \cos\phi_j + \eta_j^P \sin\phi_j) - f_{yj}(\zeta_j^P \sin\phi_j + \eta_j^P \cos\phi_j) \end{bmatrix} \tag{20}$$

For the bearing at O_i

$$\begin{bmatrix} f_{xj} \\ f_{yj} \\ n_j \end{bmatrix}_i = \begin{bmatrix} f_{xj} \\ f_{yj} \\ -f_{xj}(\zeta_i^P \cos\phi_i + \eta_i^P \sin\phi_i) + f_{yj}(\zeta_i^P \sin\phi_i + \eta_i^P \cos\phi_i) + n_b \end{bmatrix} \tag{21}$$

where n_b is the moment produced by transferring the forces from the center of the journal to the center of the bearing, which can be evaluated from

$$n_b = f_{xj}\varepsilon_y - f_{yj}\varepsilon_x \tag{22}$$

Example

As an example, the dynamic response of a slider-crank mechanism shown in Fig. 3 is analyzed as an application using the various formulations. Three different simulations are considered. In the first simulation, the revolute joint between the connecting

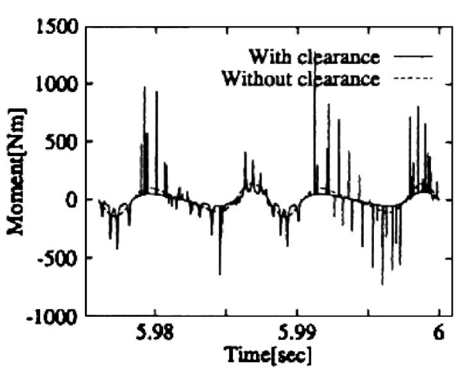

Fig. 5 Reaction moment on the crank (dry joint)

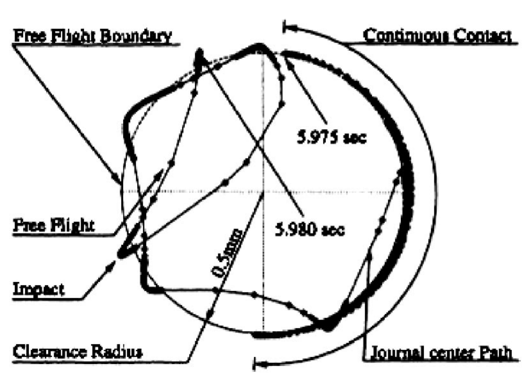

Fig. 6 Journal center path (dry clearance joint)

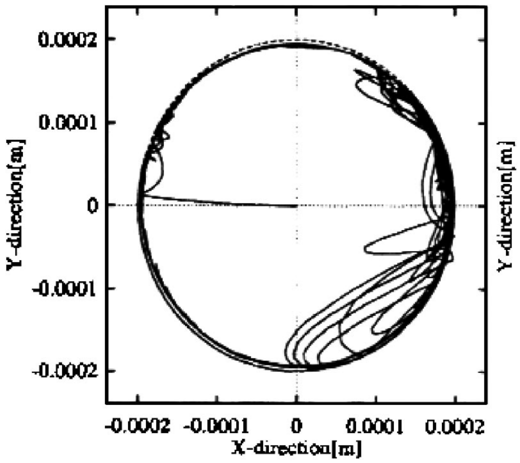

Fig. 7 Journal center path Ott's Solution

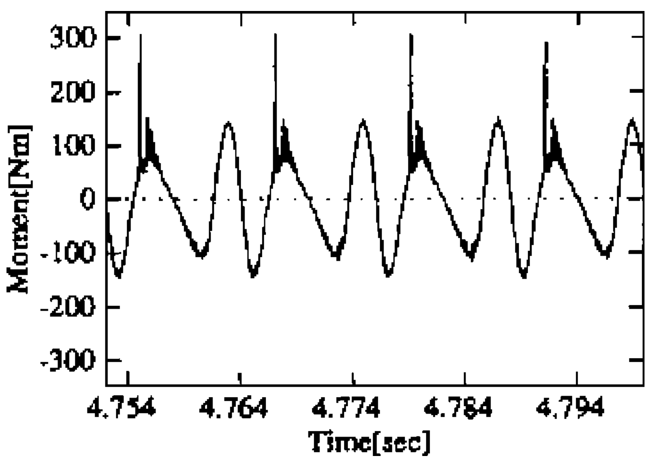

Fig. 8 Reaction moment on the crank Ott's Solution

rod and the slider assumed to be perfect. The crank reaction moment required to run the mechanism at a constant speed is shown in Fig. 4. With perfect joints there are no sharp peaks in the dynamic response of the mechanism and the reaction moment is of the order of 150 N M. When there is no lubricant (dry joint), internal impact occurs. The modified Hertzian equation is used to evaluate the contact force [6]. The moment on the crank required to maintain a constant angular velocity is shown in Fig. 5 and the motion of the pin center inside the circular clearance region is shown in Fig. 6 for a period of 0.05 s. As observed, due to dry lubrication, there are abrupt changes in the motion of the system. For such cases, the driving moment required to give the crank a constant speed of 5000 rpm would need sharp spikes of two-order of magnitudes larger when no clearance exists in the piston pin [6,7]. The slider-crank mechanism is analyzed next utilizing Eqs. (1)–(4) to evaluate the lubricant forces [6]. The result from the simulation is shown over four revolutions. Figure 7 shows the journal path inside the clearance circle and Fig. 8 shows the crank moment required to maintain a constant angular velocity. It can be observed that the crank moment shows considerably fewer and lower peaks compared to the case when no lubrication is utilized, indicating a more steady and desirable journal motion.

Finally, The analysis is performed adopting the proposed hydrodynamic lubrication formulation, i.e., Eqs. (6)–(9). Figure 9 shows the crank moment required to maintain a constant cranking angular velocity, while Fig. 10 shows the path of the journal center inside the clearance circle in terms of ξ components. It can be observed that the crank moment shows lower peaks compared to the two cases discussed before. The reaction moment in Fig. 9 is almost of the same order as the one for a perfect joint shown in Fig. 4. Also there is a considerable difference in the journal center path between the two solutions of the lubricated journal bearing. The solution presented in Fig. 10 is more likely to happen, due to the fact that the motion of mechanism is almost periodical.

Conclusions

A generalized model of a lubricated journal bearing in a multibody mechanical system is introduced. In multibody mechanical

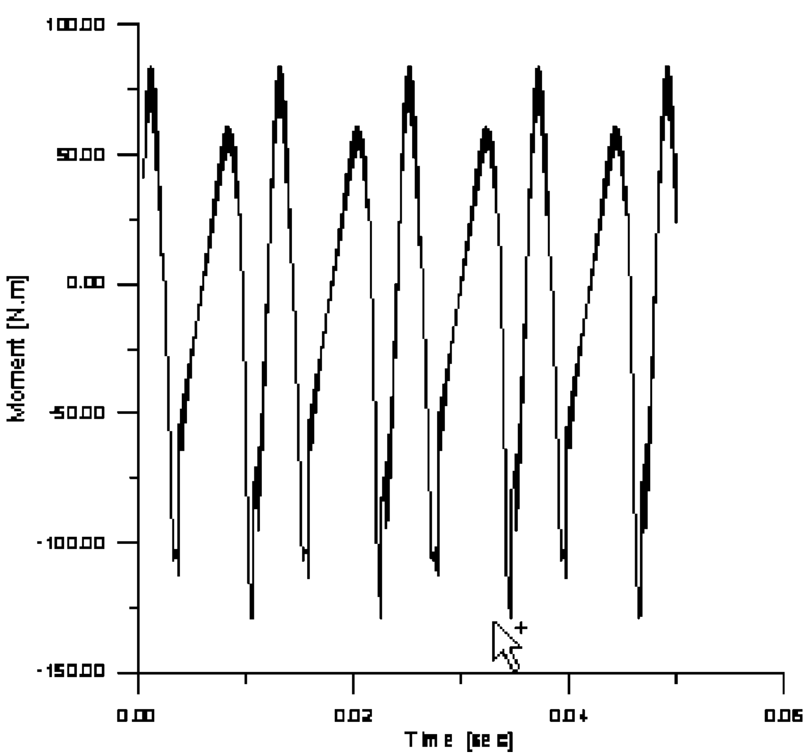

Fig. 9 Reaction moment on the crank (new Solution)

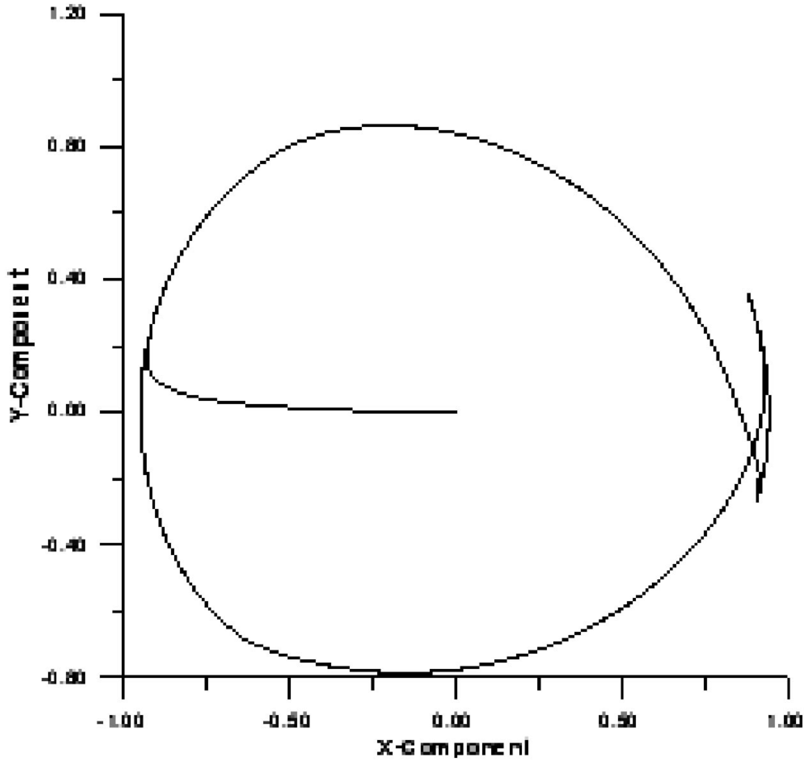

Fig. 10 Journal center path (New Solution)

systems, perfect joints provide kinematic constraints (unknown reaction forces) while lubricated journal bearings provide force constraints. The dry clearance revolute joints produce high impact forces in multibody mechanical systems. In a multibody mechanical system, the journal bearing lubricant introduces damping and stiffness into the system. Absorbing a part of the bodies' inertia by the lubricant results in reaction moments that have the same order of magnitude of the reaction moment using perfect joints. The new model presented is more accurate for evaluating the lubricant forces than the widely used Ott's solution.

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