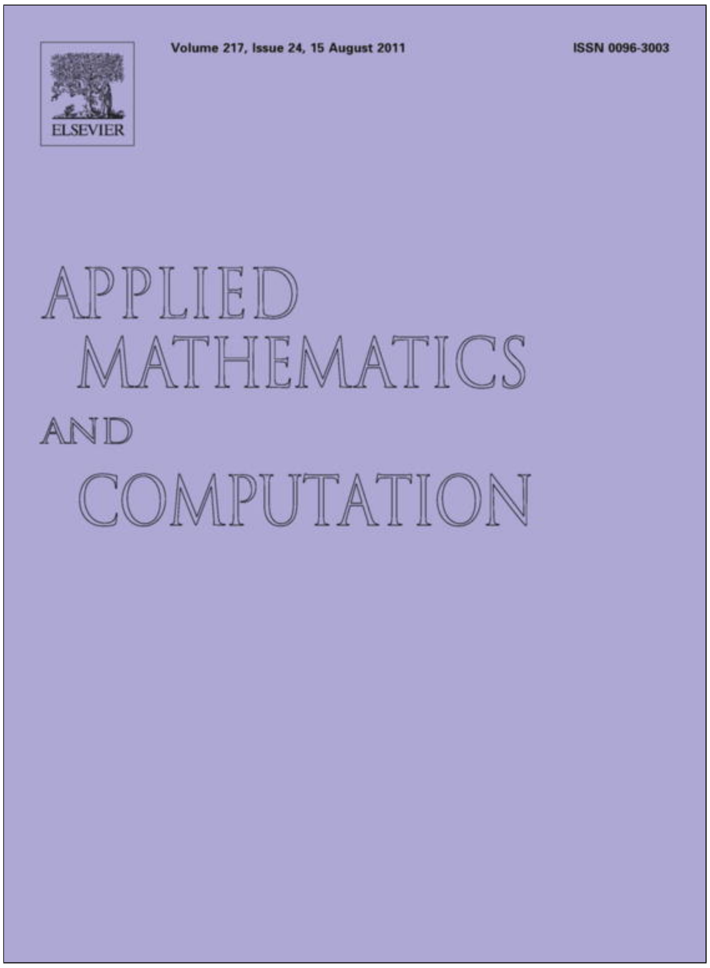
Volume 217, Issue 24, 15 August 2011
ISSN 0096-3003
ELSEVIER
APPLIED
MATHEMATICS
AND
COMPUTATION

Applied Mathematics and Computation 217 (2011) 10231–10238

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A new augmented Lyapunov–Krasovskii functional approach to exponential passivity for neural networks with time-varying delays

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ARTICLE INFO

Keywords:
Neural networks
Time-varying delays
Exponential passivity
LMI
Lyapunov method

ABSTRACT

In this paper, the problem of exponential passivity analysis for uncertain neural networks with time-varying delays is considered. By constructing new augmented Lyapunov–Krasovskii's functionals and some novel analysis techniques, improved delay-dependent criteria for checking the exponential passivity of the neural networks are established. The proposed criteria are represented in terms of linear matrix inequalities (LMIs) which can be easily solved by various convex optimization algorithms. A numerical example is included to show the superiority of our results.

1. Introduction

Recently, neural networks have been successfully applied to various fields such as signal processing, pattern recognition, fixed-point computations, control and other scientific areas. In the implementation of neural networks, time-delays frequently occur in many practical applications due to the finite switching speed of amplifies and the inherent communication of neurons [1–3]. It is well known that the existence of time-delay often causes poor performance or instability of the designed networks. Since the applications of delayed neural networks are heavily dependent on the dynamic behavior of the equilibrium points, there have been many results for asymptotic or exponential stability of neural networks during the last decade. For examples, see the papers [4–10] and references therein.

On the other hand, in numerous scientific and engineering problems, stability issues are often linked to the theory of dissipative systems with postulates that the energy dissipated inside a dynamic system is less than the energy supplied from external source [11]. In [12], the concept of dissipativeness was firstly introduced in the field of nonlinear control with the form of inequality including supply rate and the storage function. Passivity analysis is one of major tools for analyzing stability of nonlinear system. The majority of passivity theory is that the passive properties of a system can keep the system internal stability by only using input–output characteristics. In this regard, considerable efforts have been made to delay-dependent passivity analysis of delayed neural networks [11,13,14] because delay-dependent criteria are generally less conservative than delay-independent ones especially when the sizes of time delays are small. Very recently, inspired by the works of [15–17], exponential passivity analysis was studied for uncertain neural networks with time-varying delays [18]. It should be noted that if a system satisfies exponential passivity condition, then passivity can be guaranteed to the system, but the converse do not necessarily hold [15–17]. The exponential passivity condition proposed in [18] was independent of the parameter ρ which provides convergence information about an upper bound of storage function. This may lead to

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doi:10.1016/j.amc.2011.05.021

conservative results because the parameter ρ can be determined after obtaining LMI solution variables which satisfy exponential passivity criterion. Therefore, there is still a room for further improvement in this research.

In this paper, the problem of exponential passivity analysis for uncertain neural networks with time-varying delays is investigated. By use of augmented Lyapunov–Krasovskii's functionals, delay-dependent sufficient conditions such that the considered neural networks are exponentially passive are derived in terms of LMIs. Unlike the method of [18], the proposed conditions are dependent on the convergence information parameter ρ, which may provide larger feasible region of exponential passivity conditions. Another difference between the work [18] and ours is that our proposed methods do not include any free weighting matrices which increase computational burden. Instead, by taking more past history information about activation functions, new augmented Lyapunov–Krasovskii's functionals are proposed. Through one numerical example, it will be shown that the proposed criteria with fewer decision variables provides much larger feasible regions of exponential passivity conditions.

Notation. $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrix. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, the notation $\boldsymbol{X} > \boldsymbol{Y}$ (respectively, $\boldsymbol{X} \geqslant \boldsymbol{Y}$) means that the matrix $\boldsymbol{X} - \boldsymbol{Y}$ is positive definite, (respectively, nonnegative). $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. $\boldsymbol{X}_{[f(t)]} \in \mathbb{R}^{m\times n}$ means that the elements of matrix $\boldsymbol{X}_{[f(t)]}$ include the scalar value of $f(t)$.

2. Problem statements

Consider the following uncertain neural networks with time-varying delays:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= -(\boldsymbol{A} + \Delta\boldsymbol{A}(t))\boldsymbol{x}(t) + (\boldsymbol{W}_0 + \Delta\boldsymbol{W}_0(t))\boldsymbol{f}(\boldsymbol{x}(t)) + (\boldsymbol{W}_1 + \Delta\boldsymbol{W}_1(t))\boldsymbol{f}(\boldsymbol{x}(t-h(t))) + \boldsymbol{u}(t),\\ \boldsymbol{y}(t) &= \boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{f}(\boldsymbol{x}(t-h(t))) + \boldsymbol{u}(t),\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t),\ldots,x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, $\boldsymbol{y}(t) \in \mathbb{R}^n$ is the output vector of neuron networks, n denotes the number of neurons in a neural network, $\boldsymbol{f}(\boldsymbol{x}(t)) = [f_1(x_1(t)),\ldots,f_n(x_n(t))]^T \in \mathbb{R}^n$ means the neuron activation function, $\boldsymbol{f}(\boldsymbol{x}(t-h(t))) = [f_1(x_1(t-h(t))),\ldots,f_n(x_n(t-h(t)))]^T \in \mathbb{R}^n, \boldsymbol{u}(t) \in \mathbb{R}^n$ is an external input vector to neurons, $\boldsymbol{A} = diag\{a_i\} \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix, $\boldsymbol{W}_0 = (w_{ij}^0)_{n\times n} \in \mathbb{R}^{n\times n}$ and $\boldsymbol{W}_1 = \left(w_{ij}^1\right)_{n\times n} \in \mathbb{R}^{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, and $\Delta\boldsymbol{A}(t), \Delta\boldsymbol{W}_0(t)$, and $\Delta\boldsymbol{W}_1(t)$ are the uncertainties of system matrices of the form

$$[\,\Delta\boldsymbol{A}(t) \quad \Delta\boldsymbol{W}_0(t) \quad \Delta\boldsymbol{W}_1(t)\,] = \boldsymbol{D}\boldsymbol{F}(t)[\boldsymbol{E}_1 \quad \boldsymbol{E}_2 \quad \boldsymbol{E}_3\,], \tag{2}$$

where the time-varying nonlinear function $\boldsymbol{F}(t)$ satisfies

$$\boldsymbol{F}^T(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}, \quad \forall\ t \geqslant 0. \tag{3}$$

The delay, $h(t)$, is a time-varying continuous function that satisfies

$$0 \leqslant h(t) \leqslant h_U, \quad \dot{h}(t) \leqslant h_D, \tag{4}$$

where h_U is a positive scalar and h_D is any constant one.

The activation functions, $f_i(x_i(t)), i = 1,\ldots,n$, are assumed to be nondecreasing, bounded and globally Lipschitz; that is,

$$k_i^- \leqslant \frac{f_i(\xi_i) - f_j(\xi_j)}{\xi_i - \xi_j} \leqslant k_i^+, \quad \xi_i, \quad \xi_j \in \mathbb{R}, \quad \xi_i \neq \xi_j, \quad i,j = 1,\ldots,n, \tag{5}$$

where k_i^+ and k_i^- are constant values.

From Eq. (5), $f_j(\cdot)$ satisfies the following condition:

$$k_j^- \leqslant \frac{f_j(\xi_j)}{\xi_j} \leqslant k_j^+, \quad \forall\ \xi_j \neq 0, j = 1,\ldots,n, \tag{6}$$

which is equivalent to

$$\left[f_j(\xi_j) - k_j^-\xi_j\right]\left[f_j(\xi_j) - k_j^+\xi_j\right] \leqslant 0, \quad f_j(0) = 0, \quad j = 1,\ldots,n. \tag{7}$$

System (1) can be written as:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= -\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{W}_0\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{W}_1\boldsymbol{f}(\boldsymbol{x}(t-h(t))) + \boldsymbol{D}\boldsymbol{p}(t) + \boldsymbol{u}(t),\\ \boldsymbol{p}(t) &= \boldsymbol{F}(t)\boldsymbol{q}(t),\\ \boldsymbol{q}(t) &= -\boldsymbol{E}_1\boldsymbol{x}(t) + \boldsymbol{E}_2\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{E}_3\boldsymbol{f}(\boldsymbol{x}(t-h(t))),\end{aligned}$$

$$\boldsymbol{y}(t) = \boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{f}(\boldsymbol{x}(t-h(t))) + \boldsymbol{u}(t). \tag{8}$$

The objective of this paper is to investigate delay-dependent exponential passivity conditions for system (8) which will be conducted in 3.

Before deriving our main results, the following definition, and lemmas will be stated.

Definition 1 [18]. The neural networks are said to be exponentially passive from input $\boldsymbol{u}(t)$ to $\boldsymbol{y}(t)$, if there exists an exponential Lyapunov function (or, called the exponential storage function) V, and a constant $\rho > 0$ such that for all $\boldsymbol{u}(t)$, all initial conditions $\boldsymbol{x}(t_0)$, all $t \geqslant t_0$, the following inequality holds:

$$\dot{V}(\boldsymbol{x}(t)) + \rho V(\boldsymbol{x}(t)) \leqslant 2\boldsymbol{y}^T(t)\boldsymbol{u}(t), \quad t \geqslant t_0, \tag{9}$$

where $\dot{V}(\boldsymbol{x}(t))$ denotes the total derivative of $V(\boldsymbol{x}(t))$ along the state trajectories of $\boldsymbol{x}(t)$ of system (1).

Remark 1. As mentioned in [18], the reason called as *exponential Lyapunov function* in 1 is that if inequality (9), then the following inequality can be obtained

$$e^{\rho t}V(\boldsymbol{x}(t)) \leqslant e^{\rho t_0}V(\boldsymbol{x}(t_0)) + 2\int_{t_0}^{t} e^{\rho s}\boldsymbol{y}^T(s)\boldsymbol{u}(s)ds. \tag{10}$$

The parameter ρ provides an exponential convergence information about an upper bound of exponential Lyapunov function. If ρ increases, then tighter bound about Lyapunov function than the results in case of $\rho = 0$ can be provided.

Lemma 1 [19]. *For a positive matrix* $\boldsymbol{M}$*, the following inequality holds:*

$$-(\alpha-\beta)\int_{\beta}^{\alpha} \dot{\boldsymbol{x}}^T(s)\boldsymbol{M}\dot{\boldsymbol{x}}(s)ds \leqslant \begin{bmatrix} \boldsymbol{x}(\alpha) \\ \boldsymbol{x}(\beta) \end{bmatrix}^T \begin{bmatrix} -\boldsymbol{M} & \boldsymbol{M} \\ \star & -\boldsymbol{M} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(\alpha) \\ \boldsymbol{x}(\beta) \end{bmatrix}. \tag{11}$$

Lemma 2 [20]. *Let* $\zeta \in \mathbb{R}^n, \boldsymbol{\Phi} = \boldsymbol{\Phi}^T \in \mathbb{R}^{n\times n}$*, and* $\boldsymbol{B} \in \mathbb{R}^{m\times n}$ *such that* $\mathrm{rank}(\boldsymbol{B}) < n$*. Then, the following statements are equivalent:*

(1) $\zeta^T\boldsymbol{\Phi}\zeta < 0, \boldsymbol{B}\zeta = \boldsymbol{0}, \zeta \neq \boldsymbol{0}$,
(2) $(\boldsymbol{B}^\perp)^T\boldsymbol{\Phi}\boldsymbol{B}^\perp < \boldsymbol{0}$*, where* $\boldsymbol{B}^\perp$ *is a right orthogonal complement of* $\boldsymbol{B}$*.*

3. Main results

In this section, new exponential passivity criteria for neural networks with time-varying delays (8) will be proposed. For simplicity of matrix representation, $\boldsymbol{e}_i (i = 1, \ldots, 14) \in \mathbb{R}^{14n\times n}$ are defined as block entry matrices with an identity matrix in ith block and zero matrices in elsewhere. For example, $\boldsymbol{e}_3^T = [\boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{I}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}]$. The notations for some matrices are defined as:

$$\zeta^T(t) = \Big[\boldsymbol{x}^T(t)\ \boldsymbol{x}^T(t-h(t))\ \boldsymbol{x}^T(t-h_U)\ \dot{\boldsymbol{x}}^T(t)\ \dot{\boldsymbol{x}}^T(t-h_U) \int_{t-h(t)}^{t} \boldsymbol{x}(s)ds \int_{t-h_U}^{t-h(t)} \boldsymbol{x}(s)ds\ \boldsymbol{f}^{\,T}(\boldsymbol{x}(t))\ \boldsymbol{f}^{\,T}(\boldsymbol{x}(t-h(t)))\ \boldsymbol{f}^{\,T}(\boldsymbol{x}(t-h_U)) \int_{t-h(t)}^{t} \boldsymbol{f}^{\,T}(\boldsymbol{x}(s))ds \int_{t-h_U}^{t-h(t)} \boldsymbol{f}^{\,T}(\boldsymbol{x}(s))ds\ \boldsymbol{u}^T(t)\ \boldsymbol{p}^T(t)\Big],$$

$$\boldsymbol{\eta}^T(t) = [\boldsymbol{x}^T(t) \quad \dot{\boldsymbol{x}}^T(t) \quad \boldsymbol{f}^{\,T}(\boldsymbol{x}(t))],$$

$$\boldsymbol{\Gamma} = [-\boldsymbol{A} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad -\boldsymbol{I} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{W}_0 \quad \boldsymbol{W}_1 \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{I} \quad \boldsymbol{D}],$$

$$\boldsymbol{\Pi}_1 = [\boldsymbol{e}_1 \quad \boldsymbol{e}_3 \quad \boldsymbol{e}_6 + \boldsymbol{e}_7 \quad \boldsymbol{e}_{11} + \boldsymbol{e}_{12}], \quad \boldsymbol{\Pi}_2 = [\boldsymbol{e}_4 \quad \boldsymbol{e}_5 \quad \boldsymbol{e}_1 - \boldsymbol{e}_3 \quad \boldsymbol{e}_8 - \boldsymbol{e}_{10}],$$

$$\boldsymbol{\Pi}_3 = [\boldsymbol{e}_1 \quad \boldsymbol{e}_4 \quad \boldsymbol{e}_8], \quad \boldsymbol{\Pi}_4 = [\boldsymbol{e}_3 \quad \boldsymbol{e}_5 \quad \boldsymbol{e}_{10}],$$

$$\begin{aligned}\boldsymbol{\Omega}_{[h(t)]} = {} & \left(-2 + h_U^{-1}h(t)\right)[\boldsymbol{e}_6\boldsymbol{G}_{11}\boldsymbol{e}_6^T + 2\boldsymbol{e}_6\boldsymbol{G}_{12}\boldsymbol{e}_1^T - 2\boldsymbol{e}_6\boldsymbol{G}_{12}\boldsymbol{e}_2^T + \boldsymbol{e}_1\boldsymbol{G}_{22}\boldsymbol{e}_1^T - 2\boldsymbol{e}_1\boldsymbol{G}_{22}\boldsymbol{e}_2^T + \boldsymbol{e}_2\boldsymbol{G}_{22}\boldsymbol{e}_2^T + 2\boldsymbol{e}_6\boldsymbol{G}_{13}\boldsymbol{e}_{11}^T + 2\boldsymbol{e}_1\boldsymbol{G}_{23}\boldsymbol{e}_{11}^T \\ & -2\boldsymbol{e}_2\boldsymbol{G}_{23}\boldsymbol{e}_{11}^T + \boldsymbol{e}_{11}\boldsymbol{G}_{33}\boldsymbol{e}_{11}^T] + \left(-1 - h_U^{-1}h(t)\right)[\boldsymbol{e}_7\boldsymbol{G}_{11}\boldsymbol{e}_7^T + 2\boldsymbol{e}_7\boldsymbol{G}_{12}\boldsymbol{e}_2^T - 2\boldsymbol{e}_7\boldsymbol{G}_{12}\boldsymbol{e}_3^T + \boldsymbol{e}_2\boldsymbol{G}_{22}\boldsymbol{e}_2^T - 2\boldsymbol{e}_2\boldsymbol{G}_{22}\boldsymbol{e}_3^T + \boldsymbol{e}_3\boldsymbol{G}_{22}\boldsymbol{e}_3^T \\ & +2\boldsymbol{e}_7\boldsymbol{G}_{13}\boldsymbol{e}_{12}^T + 2\boldsymbol{e}_2\boldsymbol{G}_{23}\boldsymbol{e}_{12}^T - 2\boldsymbol{e}_3\boldsymbol{G}_{23}\boldsymbol{e}_{12}^T + \boldsymbol{e}_{12}\boldsymbol{G}_{33}\boldsymbol{e}_{12}^T].\end{aligned}$$

$$\boldsymbol{\Psi} = \boldsymbol{e}_1\boldsymbol{G}_{11}\boldsymbol{e}_1^T + 2\boldsymbol{e}_1\boldsymbol{G}_{12}\boldsymbol{e}_4^T + \boldsymbol{e}_4\boldsymbol{G}_{22}\boldsymbol{e}_4^T + 2\boldsymbol{e}_1\boldsymbol{G}_{13}\boldsymbol{e}_8^T + 2\boldsymbol{e}_4\boldsymbol{G}_{23}\boldsymbol{e}_8^T + \boldsymbol{e}_8\boldsymbol{G}_{33}\boldsymbol{e}_8,$$

$$\Phi = -2\boldsymbol{e}_1\boldsymbol{K}_m\boldsymbol{H}_1\boldsymbol{K}_p\boldsymbol{e}_1^T + \boldsymbol{e}_1(\boldsymbol{K}_m+\boldsymbol{K}_p)\boldsymbol{H}_1\boldsymbol{e}_8^T + \boldsymbol{e}_8\boldsymbol{H}_1(\boldsymbol{K}_m+\boldsymbol{K}_p)\boldsymbol{e}_1^T - 2\boldsymbol{e}_8\boldsymbol{H}_1\boldsymbol{e}_8^T - 2\boldsymbol{e}_2\boldsymbol{K}_m\boldsymbol{H}_2\boldsymbol{K}_p\boldsymbol{e}_2^T + \boldsymbol{e}_2(\boldsymbol{K}_m+\boldsymbol{K}_p)\boldsymbol{H}_2\boldsymbol{e}_9^T \\ + \boldsymbol{e}_9\boldsymbol{H}_2(\boldsymbol{K}_m+\boldsymbol{K}_p)\boldsymbol{e}_2^T - 2\boldsymbol{e}_9\boldsymbol{H}_2\boldsymbol{e}_9^T - 2\boldsymbol{e}_3\boldsymbol{K}_m\boldsymbol{H}_3\boldsymbol{K}_p\boldsymbol{e}_3^T + \boldsymbol{e}_3(\boldsymbol{K}_m+\boldsymbol{K}_p)\boldsymbol{H}_3\boldsymbol{e}_{10}^T + \boldsymbol{e}_{10}\boldsymbol{H}_3(\boldsymbol{K}_m+\boldsymbol{K}_p)\boldsymbol{e}_3^T - 2\boldsymbol{e}_{10}\boldsymbol{H}_3\boldsymbol{e}_{10}^T,$$

$$\Xi = -2\boldsymbol{e}_8\boldsymbol{e}_{13}^T - 2\boldsymbol{e}_9\boldsymbol{e}_{13}^T - \boldsymbol{e}_{13}\boldsymbol{e}_{13}^T + \varepsilon\boldsymbol{e}_1\boldsymbol{E}_1^T\boldsymbol{E}_1\boldsymbol{e}_1^T - 2\varepsilon\boldsymbol{e}_1\boldsymbol{E}_1^T\boldsymbol{E}_2\boldsymbol{e}_8^T, -2\varepsilon\boldsymbol{e}_1\boldsymbol{E}_1^T\boldsymbol{E}_3\boldsymbol{e}_9^T + \varepsilon\boldsymbol{e}_8\boldsymbol{E}_2^T\boldsymbol{E}_2\boldsymbol{e}_8^T + 2\varepsilon\boldsymbol{e}_8\boldsymbol{E}_2^T\boldsymbol{E}_3\boldsymbol{e}_9^T \\ + \varepsilon\boldsymbol{e}_9\boldsymbol{E}_3^T\boldsymbol{E}_3\boldsymbol{e}_9^T - \varepsilon\boldsymbol{e}_{14}\boldsymbol{e}_{14}^T,$$

$$\Sigma = \Pi_1\mathcal{R}\Pi_2^T + \Pi_2\mathcal{R}^T\Pi_1^T + \rho\Pi_1\mathcal{R}\Pi_1^T + \Pi_3\mathcal{N}\Pi_3^T - \Pi_4(e^{-\rho h_U}\mathcal{N})\Pi_4^T + h_U^2\Psi + \Phi + \Xi. \tag{12}$$

Firstly, when h_D is unknown, the following theorem is considered.

Theorem 1. *For given scalars $h_U > 0$ and $\rho > 0$, diagonal matrices $\boldsymbol{K}_p = diag\{k_1^+, \ldots, k_n^+\}$ and $\boldsymbol{K}_m = diag\{k_1^-, \ldots, k_n^-\}$, the system (1) is exponentially passive for $0 \leqslant h(t) \leqslant h_U$ if there exist positive scalar ε, positive diagonal matrices $\boldsymbol{H}_j = diag\{h_{j1}, \ldots, h_{jn}\} (j = 1, 2, 3)$, positive definite matrices $\mathcal{R} = [\boldsymbol{R}_{ij}]_{4\times4} \in \mathbb{R}^{4n\times4n}, \mathcal{N} = [\boldsymbol{N}_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}, \mathcal{G} = [\boldsymbol{G}_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}$, satisfying the following LMIs:*

$$(\Gamma^\perp)^T\{\Sigma + e^{-\rho h_U}\Omega_{[h(t)=0]}\}\Gamma^\perp < \boldsymbol{0}, \tag{13}$$
$$(\Gamma^\perp)^T\{\Sigma + e^{-\rho h_U}\Omega_{[h(t)=h_U]}\}\Gamma^\perp < \boldsymbol{0}, \tag{14}$$

where $\Omega_{[h(t)]}, \Sigma, \Gamma$ are defined in (12), and $\Gamma^\perp$ is the right orthogonal complement of Γ.

Proof. For positive definite matrices $\mathcal{R}, \mathcal{N}$, and $\mathcal{G}$, let us take the Lyapunov–Krasovskii's functional candidate:

$$V = \sum_{i=1}^{3} V_i, \tag{15}$$

where

$$V_1 = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-h_U) \\ \int_{t-h_U}^t \boldsymbol{x}(s)ds \\ \int_{t-h_U}^t \boldsymbol{f}(\boldsymbol{x}(s))ds \end{bmatrix}^T \mathcal{R} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-h_U) \\ \int_{t-h_U}^t \boldsymbol{x}(s)ds \\ \int_{t-h_U}^t \boldsymbol{f}(\boldsymbol{x}(s))ds \end{bmatrix},$$

$$V_2 = \int_{t-h_U}^t e^{\rho(s-t)}\boldsymbol{\eta}^T(s)\mathcal{N}\boldsymbol{\eta}(s)ds,$$

$$V_3 = h_U\int_{t-h_U}^t\int_s^t e^{\rho(u-t)}\boldsymbol{\eta}^T(u)\mathcal{G}\boldsymbol{\eta}(u)duds. \tag{16}$$

Calculation of the time-derivative of $\dot{V}_1$ yields

$$\dot{V}_1 = 2\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-h_U) \\ \int_{t-h(t)}^t \boldsymbol{x}(s)ds + \int_{t-h_U}^{t-h(t)} \boldsymbol{x}(s)ds \\ \int_{t-h(t)}^t \boldsymbol{f}(\boldsymbol{x}(s))ds + \int_{t-h_U}^{t-h(t)} \boldsymbol{f}(\boldsymbol{x}(s))ds \end{bmatrix}^T \mathcal{R} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}(t-h_U) \\ \boldsymbol{x}(t) - \boldsymbol{x}(t-h_U) \\ \boldsymbol{f}(\boldsymbol{x}(t)) - \boldsymbol{f}(\boldsymbol{x}(t-h_U)) \end{bmatrix} = 2\zeta^T(t)\Pi_1\mathcal{R}\Pi_2^T\zeta(t). \tag{17}$$

The time-derivative of V_2 can now be obtained as

$$\begin{aligned} \dot{V}_2 &= \frac{d}{dt}\left\{\int_{t-h_U}^t e^{\rho(s-t)}\boldsymbol{\eta}^T(s)\mathcal{N}\boldsymbol{\eta}(s)ds\right\} \\ &= -\rho e^{-\rho t}\int_{t-h_U}^t e^{\rho s}\boldsymbol{\eta}^T(s)\mathcal{N}\eta(\boldsymbol{s})ds + e^{-\rho t}\{\boldsymbol{\eta}^T(t)(e^{\rho t}\mathcal{N})\boldsymbol{\eta}(t) - \boldsymbol{\eta}^T(t-h_U)^T(e^{\rho(t-h_U)}\mathcal{N})\boldsymbol{\eta}(t-h_U)\} \\ &= -\rho V_2 + \boldsymbol{\eta}^T(t)\mathcal{N}\boldsymbol{\eta}(t) - \boldsymbol{\eta}^T(t-h_U)(e^{-\rho h_U}\mathcal{N})\boldsymbol{\eta}^T(t-h_U) = -\rho V_2 + \zeta^T(t)[\Pi_3\mathcal{N}\Pi_3^T - \Pi_4(e^{-\rho h_U}\mathcal{N})\Pi_4^T]\zeta(t). \end{aligned} \tag{18}$$

Calculation of $\dot{V}_3$ leads to

$$\begin{aligned}\dot{V}_3 &= \frac{d}{dt}\left\{h_U e^{-\rho t}\int_{t-h_U}^{t}\int_{s}^{t} e^{\rho u}\boldsymbol{\eta}^T(u)\mathcal{G}\boldsymbol{\eta}(u)duds\right\} = -\rho V_3 + e^{-\rho t}\frac{d}{dt}\left\{h_U\int_{t-h_U}^{t}\int_{s}^{t} e^{\rho u}\boldsymbol{\eta}^T(u)\mathcal{G}\boldsymbol{\eta}(u)duds\right\}\\ &= -\rho V_3 + e^{-\rho t}\left\{\left(h_U^2 e^{\rho t}\right)\boldsymbol{\eta}^T(t)\mathcal{G}\boldsymbol{\eta}(t) - h_U\int_{t-h_U}^{t} e^{\rho s}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\right\}\\ &\leqslant -\rho V_3 + h_U^2\boldsymbol{\eta}^T(t)\mathcal{G}\boldsymbol{\eta}(t) - e^{-\rho h_U}\left(h_U\int_{t-h_U}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\right), \end{aligned} \tag{19}$$

where $-e^{\rho(s-t)} \leqslant -e^{-\rho h_U}$ was used in Eq. (19).

Here, by the use of Lemma 1 ant the method of [21], the integral term $-h_U\int_{t-h_U}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds$ in (19) can be estimated as

$$\begin{aligned}-h_U\int_{t-h_U}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds &= -h_U\int_{t-h(t)}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds - h_U\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\\ &= -(h_U-h(t))\int_{t-h(t)}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds - h(t)\int_{t-h(t)}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds - (h_U-h(t))\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\\ &\quad - h(t)\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds \leqslant -h_U^{-1}(h_U-h(t))h(t)\int_{t-h(t)}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds - h(t)\int_{t-h(t)}^{t}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\\ &\quad - (h_U-h(t))\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds - h_U^{-1}h(t)(h_U-h(t))\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\\ &\leqslant (-2+h_U^{-1}h(t))\left(\int_{t-h(t)}^{t}\boldsymbol{\eta}(s)ds\right)^T\mathcal{G}\left(\int_{t-h(t)}^{t}\boldsymbol{\eta}(s)ds\right)\\ &\quad + \left(-1-h_U^{-1}h(t)\right)\left(\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}(s)ds\right)^T\mathcal{G}\left(\int_{t-h_U}^{t-h(t)}\boldsymbol{\eta}(s)ds\right)\\ &= \left(-2+h_U^{-1}h(t)\right)\begin{bmatrix}\int_{t-h(t)}^{t}\boldsymbol{x}(s)ds\\ \boldsymbol{x}(t)-\boldsymbol{x}(t-h(t))\\ \int_{t-h(t)}^{t}\boldsymbol{f}(\boldsymbol{x}(s))ds\end{bmatrix}^T\mathcal{G}\begin{bmatrix}\int_{t-h(t)}^{t}\boldsymbol{x}(s)ds\\ \boldsymbol{x}(t)-\boldsymbol{x}(t-h(t))\\ \int_{t-h(t)}^{t}\boldsymbol{f}(\boldsymbol{x}(s))ds\end{bmatrix} + \left(-1-h_U^{-1}h(t)\right)\\ &\quad \begin{bmatrix}\int_{t-h_U}^{t-h(t)}\boldsymbol{x}(s)ds\\ \boldsymbol{x}(t-h(t))-\boldsymbol{x}(t-h_U)\\ \int_{t-h_U}^{t-h(t)}\boldsymbol{f}(\boldsymbol{x}(s))ds\end{bmatrix}^T\mathcal{G}\begin{bmatrix}\int_{t-h_U}^{t-h(t)}\boldsymbol{x}(s)ds\\ \boldsymbol{x}(t-h(t))-\boldsymbol{x}(t-h_U)\\ \int_{t-h_U}^{t-h(t)}\boldsymbol{f}(\boldsymbol{x}(s))ds\end{bmatrix} = \zeta^T(t)\boldsymbol{\Omega}_{[h(t)]}\zeta(t), \end{aligned} \tag{20}$$

where $\boldsymbol{\Omega}_{[h(t)]}$ was defined in (12).

From Eq. (20), an upper bound of $\dot{V}_3$ can be

$$\dot{V}_3 \leqslant -\rho V_3 + \zeta^T(t)\left[h_U^2\boldsymbol{\Psi} + e^{-\rho h_U}\boldsymbol{\Omega}_{[h(t)]}\right]\zeta(t). \tag{21}$$

Here it should be noted that Eq. (6) means

$$\left[f_j(x_j(t)) - k_j^- x_j(t)\right]\left[f_j(x_j(t)) - k_j^+ x_j(t)\right] \leqslant 0 \quad (j = 1,\ldots,n), \tag{22}$$

$$\left[f_j(x_j(t-h(t))) - k_j^- x_j(t-h(t))\right]\left[f_j(x_j(t-h(t))) - k_j^+ x_j(t-h(t))\right] \leqslant 0 \quad (j = 1,\ldots,n), \tag{23}$$

$$\left[f_j(x_j(t-h_U)) - k_j^- x_j(t-h_U)\right]\left[f_j(x_j(t-h_U)) - k_j^+ x_j(t-h_U)\right] \leqslant 0 \quad (j = 1,\ldots,n). \tag{24}$$

From three inequalities Eqs. (22)–(24) for any positive diagonal matrices $\boldsymbol{H}_1 = diag\{h_{1i},\ldots,h_{1n}\}, \boldsymbol{H}_2 = diag\{h_{2i},\ldots,h_{2n}\}$, and $\boldsymbol{H}_3 = diag\{h_{3i},\ldots,h_{3n}\}$, the following inequality holds

$$\begin{aligned}0 &\leqslant -2\sum_{j=1}^{n} h_{1j}\left[f_j(x_j(t)) - k_j^- x_j(t)\right]\left[f_j(x_j(t)) - k_j^+ x_j(t)\right] - 2\sum_{j=1}^{n} h_{2j}\left[f_j(x_j(t-h(t))) - k_j^- x_j(t-h(t))\right]\\ &\quad \left[f_j(x_j(t-h(t))) - k_j^+ x_j(t-h(t))\right] - 2\sum_{j=1}^{n} h_{3j}\left[f_j(x_j(t-h_U)) - k_j^- x_j(t-h_U)\right]\\ &\quad \left[f_j(x_j(t-h_U)) - k_j^+ x_j(t-h_U)\right] = \zeta^T(t)\boldsymbol{\Phi}\zeta(t), \end{aligned} \tag{25}$$

where $\boldsymbol{\Phi}_{[h(t)]}$ was defined in Eq. (12).

From Eqs. (2) and (8), the inequality $\boldsymbol{p}^T(t)\boldsymbol{p}(t) \leqslant \boldsymbol{q}^T(t)\boldsymbol{q}(t)$ can be obtained. Then, there exists a positive scalar ε satisfying the following inequality

$$\varepsilon[\boldsymbol{q}^T(t)\boldsymbol{q}(t) - \boldsymbol{p}^T(t)\boldsymbol{p}(t)] \geqslant 0. \tag{26}$$

From Eqs. (16)–(26) and by application of S-procedure [22], an upper bound of $\dot{V} + \rho V - 2\boldsymbol{y}^T(t)\boldsymbol{u}(t)$ can be

$$\dot{V} + \rho V - 2\boldsymbol{y}^T(t)\boldsymbol{u}(t) \leqslant \zeta^T(t)\{\boldsymbol{\Sigma} + e^{-\rho h_U}\boldsymbol{\Omega}_{[h(t)]}\}\zeta(t), \tag{27}$$

where $\boldsymbol{\Sigma}$ and $\boldsymbol{\Omega}_{[h(t)]}$ are defined in (12).

It should be noted that the elements of $\boldsymbol{\Omega}_{[h(t)]}$ are affinely dependent on $h(t)$. By Lemma 2, $\zeta^T(t)\{\boldsymbol{\Sigma} + \boldsymbol{\Omega}_{[h(t)]}\}\zeta(t) < 0$ with $\boldsymbol{0} = \boldsymbol{\Gamma}\zeta(t)$ is equivalent to $(\boldsymbol{\Gamma}^\perp)^T\{\boldsymbol{\Sigma} + \boldsymbol{\Omega}_{[h(t)]}\}\boldsymbol{\Gamma}^\perp < \boldsymbol{0}$. Therefore, if LMIs Eqs. (13) and (14) hold, then $(\boldsymbol{\Gamma}^\perp)^T\{\boldsymbol{\Sigma} + \boldsymbol{\Omega}_{[h(t)]}\}\boldsymbol{\Gamma}^\perp < 0$ satisfies for $0 \leqslant h(t) \leqslant h_U$, which means

$$\dot{V} + \rho V < 2\boldsymbol{y}^T(t)\boldsymbol{u}(t). \tag{28}$$

This implies that the neural networks (1) is exponentially passive in the sense of Definition 1. This completes our proof. □

Theorem 1 does not have any constraints on time-derivative of $h(t)$ in Eq. (1). However, in case that the constraint $\dot{h}(t) \leqslant h_D$ is considered, then the following theorem is obtained.

Theorem 2. *For given scalars $h_U > 0, \rho > 0$ and h_D, diagonal matrices $\boldsymbol{K}_p = diag\{k_1^+, \ldots, k_n^+\}$ and $\boldsymbol{K}_m = diag\{k_1^-, \ldots, k_n^-\}$, the system (1) is exponentially passive for $0 \leqslant h(t) \leqslant h_U$ and $\dot{h}(t) \leqslant h_D$ if there exist positive scalar ε, positive diagonal matrices $\boldsymbol{H}_j = diag\{h_{j1}, \ldots, h_{jn}\} (j = 1, 2, 3)$, positive definite matrices $\mathcal{R} = [\boldsymbol{R}_{ij}]_{4\times4} \in \mathbb{R}^{4n\times4n}, \mathcal{N} = [\boldsymbol{N}_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}, \mathcal{G} = [\boldsymbol{G}_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}$, $\mathcal{Q} = [\boldsymbol{Q}_{ij}]_{2\times2} \in \mathbb{R}^{2n\times2n}$ satisfying the following LMIs:*

$$(\boldsymbol{\Gamma}^\perp)^T\{\boldsymbol{\Sigma} + e^{-\rho h_U}\boldsymbol{\Omega}_{[h(t)=0]} + \boldsymbol{\Upsilon}_{[h(t)=0]}\}\boldsymbol{\Gamma}^\perp < \boldsymbol{0}, \tag{29}$$

$$(\boldsymbol{\Gamma}^\perp)^T\{\boldsymbol{\Sigma} + e^{-\rho h_U}\boldsymbol{\Omega}_{[h(t)=h_U]} + \boldsymbol{\Upsilon}_{[h(t)=h_U]}\}\boldsymbol{\Gamma}^\perp < \boldsymbol{0}, \tag{30}$$

$$\begin{bmatrix} \boldsymbol{G}_{11} - \rho e^{\rho h_U} h_U^{-1}\boldsymbol{Q}_{11} & \boldsymbol{G}_{12} & \boldsymbol{G}_{13} - \rho e^{\rho h_U} h_U^{-1}\boldsymbol{Q}_{12} \\ \star & \boldsymbol{G}_{22} & \boldsymbol{G}_{23} \\ \star & \star & \boldsymbol{G}_{33} - \rho e^{\rho h_U} h_U^{-1}\boldsymbol{Q}_{22} \end{bmatrix} > \boldsymbol{0}, \tag{31}$$

where $\boldsymbol{\Omega}_{[h(t)]}, \boldsymbol{\Sigma}$, and $\boldsymbol{\Gamma}$ are defined in (12), $\boldsymbol{\Gamma}^\perp$ is the right orthogonal complement of $\boldsymbol{\Gamma}$, and

$$\begin{aligned}\boldsymbol{\Upsilon}_{[h(t)]} = {}& (-2 + h_U^{-1}h(t)\Big[\boldsymbol{e}_6\left(\rho h_U^{-1}\boldsymbol{Q}_{11}\right)\boldsymbol{e}_6^T + 2\boldsymbol{e}_6\left(\rho h_U^{-1}\boldsymbol{Q}_{12}\right)\boldsymbol{e}_{11}^T + \boldsymbol{e}_{11}\left(\rho h_U^{-1}\boldsymbol{Q}_{22}\right)\boldsymbol{e}_{11}^T\Big] + \boldsymbol{e}_1\boldsymbol{Q}_{11}\boldsymbol{e}_1 + 2\boldsymbol{e}_1\boldsymbol{Q}_{12}\boldsymbol{e}_8^T + \boldsymbol{e}_8\boldsymbol{Q}_{22}\boldsymbol{e}_8^T \\ & - (1 - h_D)\left[\boldsymbol{e}_2\boldsymbol{Q}_{11}\boldsymbol{e}_2 + 2\boldsymbol{e}_2\boldsymbol{Q}_{12}\boldsymbol{e}_9^T + \boldsymbol{e}_9\boldsymbol{Q}_{22}\boldsymbol{e}_9^T\right].\end{aligned}$$

Proof. For $\mathcal{R}, \mathcal{N}, \mathcal{G}$, and $\mathcal{Q}$, the following Lyapunov–Krasovskii's functional candidate is considered:

$$V = \sum_{i=1}^{4} V_i, \tag{32}$$

where

$$V_4 = \int_{t-h(t)}^{t} \begin{bmatrix} \boldsymbol{x}(s) \\ \boldsymbol{f}(\boldsymbol{x}(s)) \end{bmatrix}^T \mathcal{Q} \begin{bmatrix} \boldsymbol{x}(s) \\ \boldsymbol{f}(\boldsymbol{x}(s)) \end{bmatrix} ds \tag{33}$$

and $V_i (i = 1, 2, 3)$ are the same one as in Eq. (12).

Calculation of $\dot{V}_4$ leads to

$$\dot{V}_4 \leqslant \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{f}(\boldsymbol{x}(t)) \end{bmatrix}^T \mathcal{Q} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{f}(\boldsymbol{x}(t)) \end{bmatrix} - (1 - h_D) \begin{bmatrix} \boldsymbol{x}(t-h(t)) \\ \boldsymbol{f}(\boldsymbol{x}(t-h(t))) \end{bmatrix}^T \mathcal{Q} \begin{bmatrix} \boldsymbol{x}(t-h(t)) \\ \boldsymbol{f}(\boldsymbol{x}(t-h(t))) \end{bmatrix}. \tag{34}$$

From Eq. (19), the term ρV_4 can be incorporated in upper bound of $\dot{V}_3$ as follows

$$\begin{aligned}\dot{V}_3 + \rho V_4 &\leqslant -\rho V_3 + h_U^2\boldsymbol{\eta}^T(t)\mathcal{G}\boldsymbol{\eta}(t) - e^{-\rho h_U}\left(h_U \int_{t-h_U}^{t} \boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\right) + \rho V_4 \\ &= -\rho V_3 + h_U^2\boldsymbol{\eta}^T(t)\mathcal{G}\boldsymbol{\eta}(t) - e^{-\rho h_U}\left(h_U \int_{t-h(t)}^{t} \boldsymbol{\eta}^T(s)\widetilde{\mathcal{G}}\boldsymbol{\eta}(s)ds + h_U \int_{t-h_U}^{t-h(t)} \boldsymbol{\eta}^T(s)\mathcal{G}\boldsymbol{\eta}(s)ds\right),\end{aligned} \tag{35}$$

where

$$\widetilde{\mathcal{G}} = \begin{bmatrix} \boldsymbol{G}_{11} - \rho e^{\rho h_U} h_U^{-1} \boldsymbol{Q}_{11} & \boldsymbol{G}_{12} & \boldsymbol{G}_{13} - \rho e^{\rho h_U} h_U^{-1} \boldsymbol{Q}_{12} \\ \star & \boldsymbol{G}_{22} & \boldsymbol{G}_{23} \\ \star & \star & \boldsymbol{G}_{33} - \rho e^{\rho h_U} h_U^{-1} \boldsymbol{Q}_{22} \end{bmatrix}. \tag{36}$$

Therefore, if the inequality (31) holds, then by similar method in the proof of Theorem 1, it can be concluded that the condition $\dot{V} + \rho V - 2\boldsymbol{y}^T(t)\boldsymbol{u}(t) < 0$ holds under the inequality Eqs. (29) and (30). This completes our proof. □

Remark 2. Unlike the method of [18], the proposed method in Theorem 1 and 2 does not have any free variables. Instead of no using free variables, the augmented vector $\zeta(t)$ has integrating terms of activation function $\boldsymbol{f}(\boldsymbol{x}(t))$ which are $\int_{t-h(t)}^{t} \boldsymbol{f}(\boldsymbol{x}(s))ds$ and $\int_{t-h_U}^{t-h(t)} \boldsymbol{f}(\boldsymbol{x}(s))ds$. By these terms, more past history of $\boldsymbol{f}(\boldsymbol{x}(t))$ can be utilized in Theorem 1 and 2. Furthermore, the total decision variables of Theorem 1 and 2 are $17n^2 + 8n + 1$ and $19n^2 + 9n + 1$, respectively. However, the decision variables of Theorem 2 in [18] was $24n^2 + 7n + 3$. In next section, it will be shown that our proposed criteria with fewer decision variables provide larger feasible regions than that of [18] by comparison of the parameter ρ.

Remark 3. Another remarkable difference between the method developed in [18] and ours is that our proposed criteria in Theorem 1 and 2 are dependent on ρ, which may provide larger feasible regions of exponential passivity condition. The feasibility of parameter ρ in [18] is determined by utilizing the calculated LMI variables as shown in Eq. (17) of [18]. However, the results presented in this paper can check the exponential passivity condition of system (1) for a given ρ.

4. Numerical example

Consider the uncertain neural networks (1) with the parameters [18]:

$$\boldsymbol{A} = diag\{4,7\}, \quad \boldsymbol{W}_0 = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}, \quad \boldsymbol{W}_1 = \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix},$$

$$\boldsymbol{K}_p = diag\{1,1\}, \quad \boldsymbol{K}_m = 0, \quad h_U = 0.16, \quad h_D = 1.1. \tag{37}$$

The following uncertainties is considered as in [18]:

$$\Delta\boldsymbol{A}(t) = \begin{bmatrix} 0.02\sin(t) & 0.04\sin(t) \\ 0.03\sin(t) & 0.06\sin(t) \end{bmatrix},$$

$$\Delta\boldsymbol{W}_0(t) = \begin{bmatrix} 0.02\sin(t) & 0.04\sin(t) \\ 0.02\sin(t) & 0.04\sin(t) \end{bmatrix},$$

$$\Delta\boldsymbol{W}_1(t) = \begin{bmatrix} 0.03\sin(t) & 0.06\sin(t) \\ 0.02\sin(t) & 0.04\sin(t) \end{bmatrix}. \tag{38}$$

From (38), it can be chosen as

$$\boldsymbol{D} = \boldsymbol{I}, \quad \boldsymbol{E}_1 = \begin{bmatrix} 0.02 & 0.04 \\ 0.03 & 0.06 \end{bmatrix}, \quad \boldsymbol{E}_2 = \begin{bmatrix} 0.02 & 0.04 \\ 0.02 & 0.04 \end{bmatrix}, \quad \boldsymbol{E}_3 = \begin{bmatrix} 0.03 & 0.06 \\ 0.02 & 0.04 \end{bmatrix}. \tag{39}$$

It is shown in [18] that the system (1) with above parameters is exponential passive. $\rho = 0.003$ was presented by use of LMI solutions in [18]. However, by application of Theorem 2 to system (1), it can be concluded that the system with the parameters (37) is exponential passive for $\rho = 5.2864$. Also, it should be noted that the decision variables of Theorem 2 for this example is 95, whilst the number of variable in [18] was 113. Hence, Theorem 2 provides larger feasible region of exponential passive condition with fewer decision variables than the method of [18].

For various values of h_D when $h_U = 0.16$, the obtained maximum values of ρ are shown in Table 1.

5. Conclusion

In this paper, two exponential passivity criteria for uncertain neural networks with time-varying delays have been proposed in the form of delay-dependent LMIs. In Theorem 1, when the information of time-varying delays is unknown, the

Table 1
Maximum value of ρ with different h_D and $h_U = 0.16$ (Example 1).

h_D	0.1	0.5	0.9	unknown
Theorem 1	-	-	-	5.2864
Theorem 2	5.4753	5.3518	5.2864	-

exponential passivity criterion without any free-weighting matrices has been proposed by use of Lemma 2 and the proposed augmented Lyapunov–Krasovskii' functional. When the information of time-varying delays is available, the exponential passivity criterion was also proposed in Theorem 2 by introduction of augmented Lyapunov–Krasovskii functional V_4. The proposed two criteria are dependent on convergence parameter, ρ. Through one numerical example, the improvement of the proposed passivity criteria has been successfully verified.

Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2011–0001045).

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