Analysis of an Electromagnetic Boundary Layer Probe for Low Magnetic Reynolds Number Flows

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Electromagnetic (EM) flow meters are used to measure volume flow rates of electrically conductive fluids (e.g., low magnetic Reynolds number flows of seawater, milk, etc.) in pipe flows. The possibility of using a modified form of EM flow meter to nonobtrusively measure boundary-layer flow characteristics is analytically investigated in this paper. The device, named an electromagnetic boundary layer (EBL) probe, would have a velocity integral-dependent voltage induced between parallel wall-mounted electrodes, as a conductive fluid flows over a dielectric wall and through the probe's magnetic field. The Shercliff-Bevir integral equation, taken from EM flow meter theory and design, is used as the basis of the analytical model for predicting EBL probe voltage outputs, given a specified probe geometry and boundary layer flow conditions. Predictions are made of the effective range of the nonobtrusive EBL probe in terms of electrode dimensions, the magnetic field size and strength, and boundary layer velocity profile and thickness. The analysis gives expected voltage calibration curves and shows that an array of paired electrodes would be a beneficial feature for probe design. A key result is that the EBL probe becomes a displacement thickness meter, if operated under certain conditions. That is, the output voltage was found to be directly proportional to the boundary layer displacement thickness, δ_1, for a given free stream velocity.

Introduction

As an electrically conductive fluid flows over a dielectric wall and through an imposed magnetic field, voltages are induced in the fluid and on the dielectric surface. This phenomenon is a consequence of Faraday's law of electromagnetic induction. The voltage field is directly dependent (among other things) on the velocity distribution in the fluid.

This velocity distribution dependency has led to the very successful development of commercial electromagnetic pipe flow rate meters. For conventional pipe flows, such a meter usually consists of two wall-mounted diametrically opposed electrodes which are used to measure the induced voltage produced as a conductive fluid with an axisymmetric velocity profile flows through an imposed magnetic field in an insulated section of pipe. The induced voltage is directly proportional to volume flow rate. A linear calibration curve can be made which is dependent on the magnetic field and the electric conductivity of the fluid. Electromagnetic (EM) flow meters are discussed in detail by Shercliff (1962) and in literature available from manufacturers (e.g., Foxboro Co., Foxboro, MA, USA).

The possibility of using a modified form of the efficacious EM flow meter to measure characteristics of a boundary layer flow, is examined in this paper. A sketch of the modified EM flow meter is shown in Fig. 1. It is not clear at this point what boundary layer flow characteristic (e.g., velocity) such a device would measure, so that it will simply be called an electromagnetic boundary layer (EBL) probe, here. As shown in Fig. 1, the EBL probe consists of a single pair of line electrodes aligned with the nominal flow direction and mounted on a dielectric solid plane surface. An electrically conductive moving fluid (e.g., sea water) forms a boundary layer as the fluid moves parallel to the electrode pair, 1 and 2, through a magnetic field of flux density **B**. The magnetic field is produced by a magnet (or a current carrying coil) embedded in the dielectric surface. For the most part, the flow field considered in this paper will be the two-dimensional boundary layer over the flat plate shown in Fig. 1, and it is assumed that there is no variation of the boundary layer thickness in the region surrounding the electrodes of the EBL probe. Only steady state or time-averaged velocities will be considered.

An electrical current is induced in the fluid as it moves through the magnetic field, and the resulting current density **j** is given by Ohm's law in the form,

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \tag{1}$$

where σ is the fluid conductivity, **E** is the electric field vector, **V** is the fluid velocity and **B** is the magnetic flux density produced by the magnet (or an electromagnetic coil). The induced voltage between electrodes 1 and 2, $\Delta\Phi = \Phi_1 - \Phi_2$, is related to **E** by

$$\mathbf{E} = -\nabla\phi \tag{2}$$

The crucial step here is to obtain the vector velocity **V** in (1)

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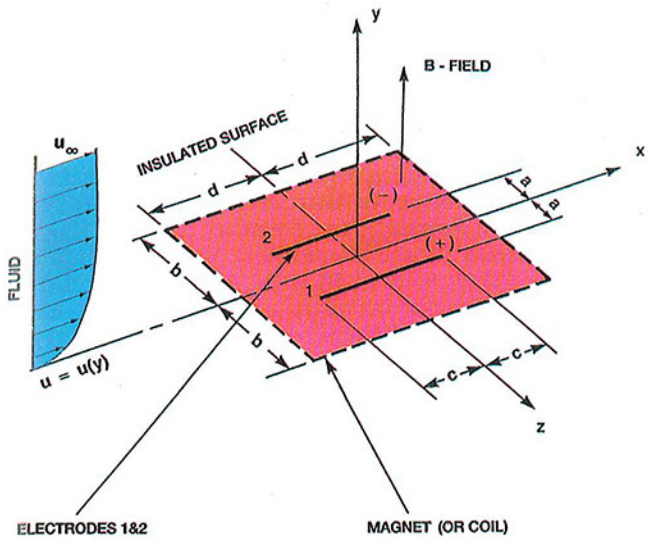

Fig. 1 Electromagnetic boundary-layer probe. As shown, a boundary-layer flow of an electrically conducting fluid in a magnetic field is in the x direction, which generates an induced current in the flow (normal to the electrodes at $x=0$), and a voltage difference $\Delta\Phi$ between the two electrodes.

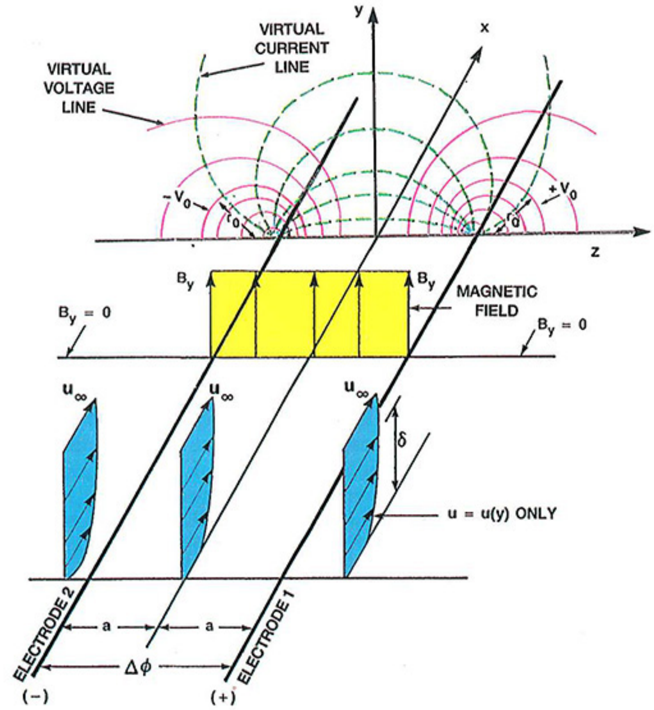

Fig. 2 Two-dimensional model of the EBL probe. Note that virtual current and voltage lines shown are for a no-flow condition.

from the scalar electrode voltage $\Delta\Phi$. An analytical model for doing this is presented in the section that follows this one.

Each particle of moving fluid in the magnetic field is acted upon by a Lorentz force given by

$$\mathbf{F}_L = \mathbf{j} \times \mathbf{B} \tag{3}$$

where $\mathbf{F}_L$ is the Lorentz force per unit volume of fluid. As discussed by Shercliff (1965) (and also Hemp (1988)) the magnetic Reynolds number is given by

$$\mathrm{Re}_m = \sigma \mu_m u_\infty x = \sigma \mu_m \nu \,\mathrm{Re}_x \tag{4}$$

where μ_m is the magnetic permeability of the fluid, ν is the kinematic viscosity, u_∞ is the free-stream velocity (Fig. 1), and Re_x is the conventional Reynolds number based (in this case) on the streamwise coordinate x. If the magnetic Reynolds number is much less than unity, the Lorentz force will be small and the fluid flow field will not be effected by the magnetic field (and conversely the $\mathbf{B}$ field will not be effected by the induced current density $\mathbf{j}$). At a flat plate Reynolds number of $\mathrm{Re}_x = 5\times10^5$, Re_m for sea water ($\sigma = 4$ (ohm m)$^{-1}$) is of the order 10^{-6}. (Re_m for liquid sodium (300°C) would be very much higher, on the order 10.) Thus the fluid flows considered in this paper (tap water, sea water and conducting fluids at conventional temperatures) will all be at a very low magnetic Reynolds number, where the Lorentz forces (Eq. (3)) can be neglected.

Devices similar to the EBL probe shown in Fig. 1 have been discussed briefly by Shercliff (1962) (wall velometers) and have been proposed by Smith and Slepian (1917), and Bruno and

Nomenclature

- a = electrode half-spacing (Fig. 1)
- $\mathbf{B}$, B = magnetic flux density (vector, scalar)
- b = magnetic field (or magnet) half-width (Fig. 1)
- c = electrode half-length (Fig. 1)
- d = magnetic field (or magnet) half-length (Fig. 1)
- $\mathbf{E}$ = electric field
- EBL = electromagnetic boundary layer
- EM = electromagnetic
- $\mathbf{F}_L$ = Lorentz force
- G = function defined by Eq. (20)
- I_v = virtual current
- i_v = virtual current per unit length of electrode
- $\mathbf{j}$ = current density
- $\mathbf{J}_v$, J_v = virtual current density per unit of virtual current
- K = constant value of weight function
- n = exponent for turbulent boundary layer velocity profile
- Re_m = magnetic Reynolds number (Eq. (4))
- Re_x = conventional Reynolds number based on x, xu_∞/ν
- SB = Shercliff-Bevir
- $u = u(y)$ = velocity in x-direction as a function of y
- u_∞ = free-stream velocity
- V = volume of fluid
- $\mathbf{V}$ = fluid velocity
- $\mathbf{W}$ = weight vector (Eq. (6))
- x = streamwise coordinate (Fig. 1)
- y = coordinate normal to surface (Fig. 1)
- z = coordinate in z (spanwise) direction (Fig. 1)
- α = constant in Eq. (21)
- β = constant in Eq. (21)
- Δ = difference
- δ = boundary layer thickness
- δ_1 = boundary layer displacement thickness defined in Eq. (9)
- ϵ = location and size of velocity deficit or excess (Fig. 4)
- μ_m = fluid magnetic permeability
- ν = kinematic viscosity
- π = 3.14159····
- σ = fluid electric conductivity
- Φ = electrode voltage

Subscripts

- 1, 2, 3, 4, 5 = electrode pairs
- I, II = boundary layers I and II
- y = y-component
- z = z-component

Kasper (1989). The Smith-Slepian device (patented well before the 1950's development of the EM flow meter) was to be mounted below the water line on a ship's hull to act as a ship's log. However, the effects of induced currents and three dimensionality were erroneously neglected, so that the device would not have given a correct measure of the ship's velocity. The more recent Bruno-Kasper device is designed to produce a voltage signal that is indicative of the velocity fluctuations in the turbulent boundary layer on the hull of a submarine or surface ship. It is not apparent how the signal could be used to obtain steady state or time-averaged boundary layer properties.

The scalar voltage output of the EBL probe shown in Fig. 1 is a function of many independent variables. They are the free stream velocity, u_∞, the boundary layer thickness, shape factor and state (laminar or turbulent), the electrode spacing ($2a$) and length ($2c$), the magnetic field size ($2b \times 2d$) and flux density (**B**) and the electric conductivity of the fluid, σ. (Use of finite width electrodes rather than line electrodes, would add another independent variable.)

One need only look at the large number of these independent variables to see that a simple analytical model of the EBL probe is called for. The purpose of this paper is to develop such a model to predict the voltage characteristics for given boundary-layer conditions. The goal will be to see what boundary-layer flow property is measured by the scalar voltage output. Results of the analysis will also provide guidance for the actual design of an EBL probe and for an experimental program to evaluate and calibrate it.

Analysis and Results

The goal in this section is to derive an expression for the open-circuit voltage $\Delta\Phi = \Phi_1 - \Phi_2$ between the electrodes (Fig. 1) in terms of the fluid velocity and geometric and magnetic terms that characterize the EBL probe. First the Shercliff-Bevir equation which is basic to the analysis will be discussed. Then a two-dimensional model (infinitely long electrodes and magnet) will be derived to show important features. Finally a three-dimensional model (finite electrodes and magnet) will be formulated and used to show that an array of paired electrodes would be a key feature in making boundary layer measurements.

(a) The Shercliff-Bevir Equation. In the study and design of electromagnetic volume flow meters, Shercliff (1962) introduced the concept of a weight function which was later made more general by Bevir (1970) as the weight vector. Their work resulted in a much-used, well-documented equation (see Bevir (1970) for the derivation, and also an alternate derivation given by Hemp (1988)) that is widely used for the design of EM volume flow rate meters. It will be referred to here as the Shercliff-Bevir (SB) equation. The equation is valid for small magnetic Reynolds numbers only. It is given by

$$\Delta\phi = \phi_1 - \phi_2 = \int_V \mathbf{V} \cdot \mathbf{W}\, dV \qquad (5)$$

where $\Delta\Phi$ is the voltage measured between the parallel electrodes (see Fig. 1) (volts), V is the volume in the conducting fluid over which the EBL probe acts (m^3) (all of the fluid above the insulated surface in Fig. 1), **V** is the velocity vector at a point in the volume V of the fluid (m/s), and **W** is defined as:

$$\mathbf{W} = \mathbf{B} \times \mathbf{J}_v \qquad (6)$$

where **B** is the magnetic flux density of the EBL probe (Fig. 1) (Webers/m^2, or volt s/m^2) and $\mathbf{J}_v$ is the virtual current density per unit of virtual current, (amps/amp m^2). The virtual current is a "calibration" (EBL probe geometry and magnetic field dependent) quantity. It is an electrical current that is caused to flow between the two electrodes, when there is no ($\mathbf{V} = 0$) fluid motion. It is not the actual induced current (which causes the measured voltage, $\Delta\Phi$). The induced current is produced when the conducting fluid is in motion through the magnetic field (see Fig. 1).

The weight vector, **W**, is a very useful concept that is used by volume flow rate meter designers to "correct" (usually by means of the magnetic field) for nonsymmetrical velocity profiles in pipe flows (e.g., when a flow meter is mounted close to an upstream elbow). To more clearly see what the probe voltage (Eq. (5)) represents for a boundary layer flow (Fig. 1), let it be assumed for the moment that **W** can be controlled (say by a suitable choice of **B**) in the following way. Let **W** be a constant K in the boundary layer ($y < \delta$ where δ is the boundary layer thickness) and between the electrodes (1, 2) $|x| < a$, $|z| < c$. Everywhere outside of this region, **W** is assumed to be zero (or very small). Also, it is assumed that $|\mathbf{V}| = u_\infty$ (a constant) for $y \geq \delta$ and $|\mathbf{V}| = u(y)$ and $y < \delta$. From (5) the EBL probe voltage will be

$$\Delta\phi = 4ac\,K \int_0^\delta u(y)\,dy \qquad (7)$$

Now consider a uniform flow ($u = u_\infty$ for $y > 0$ in Fig. 1) over the same EBL probe. Equation (5) yields

$$\Delta\phi_{\text{uniform}} = 4ac\,K\,u_\infty \int_0^\delta dy \qquad (8)$$

Subtracting (7) from (8) and using the definition of δ_1, the boundary-layer displacement thickness, one gets

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy = \frac{\Delta\phi_{\text{uniform}} - \Delta\phi}{4ac\,u_\infty K} \qquad (9)$$

Thus in this particular case where **W** has been specified to be a constant K, it is seen that with knowledge of $\Delta\Phi_{\text{uniform}}$, the EBL probe voltage provides a direct measure of a boundary layer integral quantity, the displacement thickness, δ_1. (Recall that δ_1 is defined as the distance by which the solid surface would have to be displaced to maintain the same mass flow rate in a uniform frictionless flow ($\Delta\Phi_{\text{uniform}}$).)

(b) Two-Dimensional Model. It is not readily apparent how one would achieve the "square-wave" weight vector distribution that was used in (5) to obtain the displacement thickness result of (9). A more plausible weight vector can be derived using the EBL probe in Fig. 2. It consists of two parallel infinite line electrodes (these eliminate end effects) with an infinitely long magnetic field between the electrodes. (Here infinite means that both c and d in Fig. 1 are much greater than a.) As before, it is assumed that there is a steady two-dimensional boundary layer flow (thickness δ) occurring in the x-direction, given as $u = u(y)$ for $y < \delta$ and $u = u_\infty$ (a constant) for $y > \delta$, with no (or small) change of δ in the x-direction.

It is assumed for the purpose of this analysis that the magnetic field has component B_y (a constant) only, as shown in Fig. 2. The magnetic field will eventually arch over and curve back to the south pole of the magnet (or coil), but it is assumed here that this will occur far from the electrodes and contribute little to **W**. Equation (5) then becomes

$$\Delta\phi = \int_{-\infty}^{\infty} \int_0^{\infty} \int_{-a}^{a} u(y) B_y J_{v_z}\, dz\,dy\,dx \qquad (10)$$

The virtual current component in the z-direction, J_{vz}, can be obtained from a classic solution for the voltage field around a line sink and a line source pair. This is given by Skitek and Marshall (1982)

$$\phi_v(y, z) = \frac{i_v}{4\pi\sigma} \ln\left[\frac{y^2 + (z+a)^2}{y^2 + (z-a)^2}\right] \qquad (11)$$

where Φ_v is the voltage due the virtual current (as defined by Bevir, 1970) I_y, and i_v is the virtual current per unit length of electrode, (amps/m). Lines of constant Φ_y and constant virtual

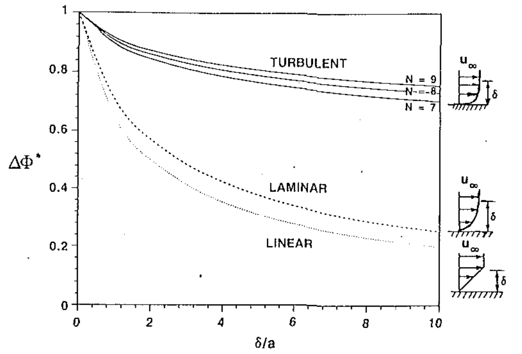

Fig. 3 Nondimensional voltage output ($\Delta\Phi^* = \Delta\Phi/(-u_\infty) B_y a$) as a function of boundary layer thickness divided by half electrode spacing, δ/a, for various boundary layer velocity profile shapes.

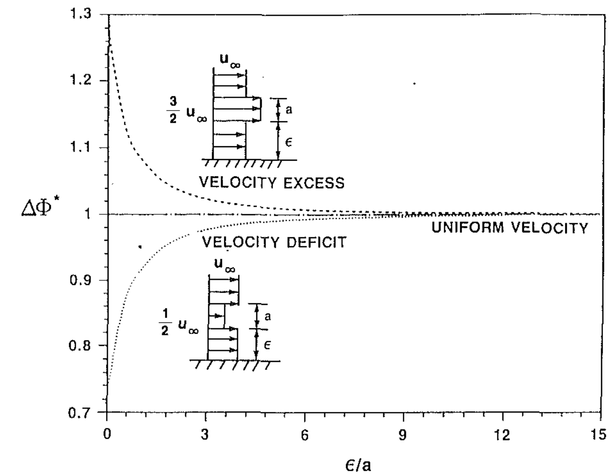

Fig. 4 Nondimensional voltage response of the two-dimensional model of the EBL probe to the position of imposed velocity discontinuities. Note that each curve represents a constant volume flow rate.

current are shown in Fig. 2. From Ohm's law and the definition of virtual current, J_{vz} in (10) is given by

$$J_{v_z} = -\frac{\sigma}{I_v}\frac{\partial \phi_v}{\partial z}, \tag{12}$$

having the units of amperes per ampere of virtual current per unit area normal to the virtual current.

Using (11) and (12) in the SB equation as given in (10), the EBL probe voltage for this two dimensional model is

$$\Delta\phi = -\frac{B_y}{2\pi}\int_0^\infty u(y)\ln\left(\frac{4a^2+y^2}{y^2}\right)dy \tag{13}$$

where B_y is the magnetic flux density (a constant as shown in Fig. 2), and $2a$ is the electrode spacing.

Using Eq. (13), various forms of the velocity $u(y)$ can be assumed to calculate the voltage output $\Delta\Phi$. For a uniform flow, $u(y) = u_\infty$, (13) yields

$$\Delta\phi_{\text{uniform}} = -u_\infty B_y a \tag{14}$$

which shows very simply that the voltage output of this two-dimensional EBL probe depends on the flow velocity, the magnetic flux strength and the electrode spacing.

For boundary flow, as shown in Fig. 2, consider three boundary layer profiles as given by

$$\left.\begin{aligned} &\text{linear} \quad u = u_\infty\left(\frac{y}{\delta}\right) \\ &\text{laminar} \quad u = u_\infty\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right) \\ &\text{turbulent} \quad u = u_\infty\left(\frac{y}{\delta}\right)^{1/n} \end{aligned}\right\} \quad 0 \le y \le \delta \tag{15}$$

all with $u = u_\infty$ for $y > \delta$. Equation (13) was evaluated using (15) for various boundary-layer thicknesses. The linear case and laminar case (cubic laminar flow velocity profile) can each be integrated directly. The turbulent cases were integrated numerically for a range of values of n. The results are shown in Fig. 3, which is a plot of nondimensional $\Delta\Phi^*$ (Eq. (13) divided by (14)) as a function of boundary layer thickness, δ, divided by the electrode half-spacing, a (Fig. 2).

In Fig. 3, a uniform velocity profile ($\delta = 0$) yields a probe voltage output of $\Delta\Phi^* = 1$ as a standard of comparison. All profiles in Fig. 3 start at $\Delta\Phi^* = 1$ for $\delta = 0$, but then the voltage output decreases as δ/a is increased, compared to the uniform velocity ($\delta = 0$) case. Most important, given a δ/a value, the output voltage values show that the EBL probe should be able to discriminate between a linear, laminar and turbulent boundary layer with the same free stream velocity, u_∞. In fact, with the latter, discrimination between turbulent (time-averaged) velocity profiles at different Reynolds numbers (different values of n) is possible as shown by the voltage curves calculated for $n = 7$, 8, and 9.

All of the curves in Fig. 3 show that as the volume flow rate across a yz plane decreases (i.e., with increasing δ/a) the output voltage decreases. The same is true for different velocity profiles at the same δ/a, with the linear profile having the lowest mass flow rate and lowest $\Delta\Phi^*$.

To show that the EBL probe is not just a volume flow measuring device (such as commercial units that measure volume flow in a pipe) consider the results of an analysis using the probe model of Fig. 2 and Eq. (11) at a constant flow rate. A uniform velocity profile that has a moveable "notch" (velocity deficit or wake) in it was analyzed. This velocity profile is given by,

$$\left.\begin{aligned} &u = u_\infty, \quad 0 \le y \le \epsilon \\ &u = \frac{1}{2}u_\infty, \quad \epsilon \le y \le a+\epsilon \\ &u = u_\infty, \quad a+\epsilon \le y \le \infty \end{aligned}\right\} \tag{16}$$

where a is the electrode half spacing and ϵ is the location of the notch. Putting Eq. (16) into Eq. (13) yields a closed-form solution

$$\Delta\phi^* = \frac{\Delta\phi}{-u_\infty B_y a} = 2\pi a + a\left[\frac{\epsilon}{2a}\ln\left[1+4\left(\frac{a}{\epsilon}\right)^2\right] + 2\tan^{-1}\left(\frac{\epsilon}{2a}\right) - \frac{1}{2}\left(\frac{\epsilon}{a}+1\right)\ln\left[1+4\left(\frac{a}{\epsilon+a}\right)^2\right] - 2\tan^{-1}\left(1+\frac{\epsilon/a}{2}\right)\right] \tag{17}$$

and the evaluation of this is shown in Fig. 4.

The lower curve in Fig. 4 is a plot of the nondimensional voltage output as a function of ϵ, the location in the y-direction of the notch in the velocity distribution. This curve shows that, at a constant flow rate, the relatively simple two-dimensional EBL probe model is able to detect the position of the notch up to a value of ϵ/a of about 10. If the notch is above 10, the probe in effect senses a uniform flow.

The major finding here is that the EBL probe can detect details in a velocity profile within about five electrode spacings. The upper curve in Fig. 4 shows the case of a velocity overshoot (a jet or velocity notch excess of $3/2\ u_\infty$), a distance ϵ away from the wall and a in extent. The probe output voltage is now higher than the uniform flow case (since some of the flow field is moving at a higher velocity than u_∞). Again, when the velocity discontinuity position is below an ϵ/a value of 10 the probe output voltage is sensitive to its position.

In summary, the model of Fig. 2 gives results that are shown in Figs. 3 and 4, and demonstrates what the EBL probe is measuring in terms of fluid flow velocities over a range of parameters. Also, all of the above analysis was based on steady-state or time-averaged fluid flows. What needs to be done now is to extend the analysis to a three-dimensional sensor (i.e., finite length electrodes) such as the one shown in Fig. 1.

(c) Three-Dimensional Model. By using a two-dimensional model (Fig. 2) the complicating end effects of the magnet and the electrodes (electrode half length c and **B** field half length d in Fig. 1) were eliminated. These can be taken into account by using a three-dimensional line source and sink pair in place of the two-dimensional virtual voltage field of Eq. (11). Using an expression for a finite line source (or sink) and a sink image, this virtual voltage field Φ_v for the electrode pair shown in Fig. 1 is given by (e.g., see Weber (1950))

$$\phi_v(x, y, z) = \frac{i_v}{4\pi\sigma}\left[\ln\frac{[(x+c)^2+(z-a)^2+y^2]^{1/2}+(x+c)}{[(x-c)^2+(z-a)^2+y^2]^{1/2}+(x-c)} - \ln\frac{[(x+c)^2+(z+a)^2+y^2]^{1/2}+(x+c)}{[(x-c)^2+(z+a)^2+y^2]^{1/2}+(x-c)}\right] \quad (18)$$

where $i_v = I_v/2c$, I_v is the virtual current (amps) and a and c are the electrode pair dimensions as shown in Fig. 1.

As in the two-dimensional case, a simple form is assumed for the magnetic field, given by $B_y(x, z) = B_y =$ constant for $-d \le x \le d$ and $-b \le z \le b$, and $B_y = 0$ for $|x| > d$ and $|z| > b$. As before the **B** field has been simplified with a y-component ($B_y =$ const) only, assuming that the other components of **B** necessary to form closed flux lines occur far enough away from the magnet so that their contribution to **W** is small. Using B_y as defined and Eq. (18), the SB equation, (5), can be normalized by the two-dimensional uniform flow result (14), to get

$$\Delta\phi^* = \frac{\Delta\phi}{-u_\infty B_y a} = \frac{1}{4\pi ac}\int_0^\infty \frac{u(y)}{u_\infty}\int_{-d}^{d} G(x, y)\,dx\,dy \quad (19)$$

where $u(y)$ has the same restrictions that were placed on the boundary layer for the two-dimensional probe case, and $G(x, y)$ (from the source-sink model, (18)) is

$$G(x, y) = \ln[[(x+c)^2+(b-a)^2+y^2]^{1/2}+(x+c)][[(x-c)^2+(b+a)^2+y^2]^{1/2}+(x-c)] - \ln[[(x+c)^2+(b+a)^2+y^2]^{1/2}+(x+c)][[(x-c)^2+(b-a)^2+y^2]^{1/2}+(x-c)] \quad (20)$$

Equation (19) was evaluated analytically for the case of $u(y)$ as a linear profile boundary layer given in (15). The results for the nondimensional output voltage $\Delta\Phi^*$, are shown in Fig. 5 as a function of b/a for a magnet length and electrode length equal to electrode spacing ($d = c = a$). Each curve in Fig. 5 represents a linear profile boundary layer thickness, δ/a, ranging from uniform flow ($\delta/a = 0$) to $\delta/a = 3$. The electrode half spacing, a, has been chosen as the normalizing length dimension since it would be an easily measured or specified quantity.

The analytical results (from (19)) in Fig. 5 show key features of the EBL probe. The $\Delta\Phi^*$ curve for uniform flow ($\delta/a = 0$) has the largest values of output voltage, with a very pronounced maximum of $\Delta\Phi^* = 0.5$ occurring at $b/a = 1.0$, where the magnetic field width is equal to the electrode spacing. This maximum is half that of the equivalent two-dimensional model (Eq. (14)), which shows that end effects and a finite length magnet induce a smaller voltage. For this $\delta/a = 0$ curve, the output voltage rises steeply in a linear fashion as b/a values are increased from 0 to 1.0. The voltage then falls off in an exponential fashion for b/a values greater than 1.0.

The voltage output curves for each $\delta/a > 0$ linear profile boundary layer thickness in Fig. 5 have similar shapes, but each falls below the uniform flow ($\delta/a = 0$) curve (the latter is a limiting case for any boundary layer shape) as the boundary layer thickness increases and less of the flow is within the

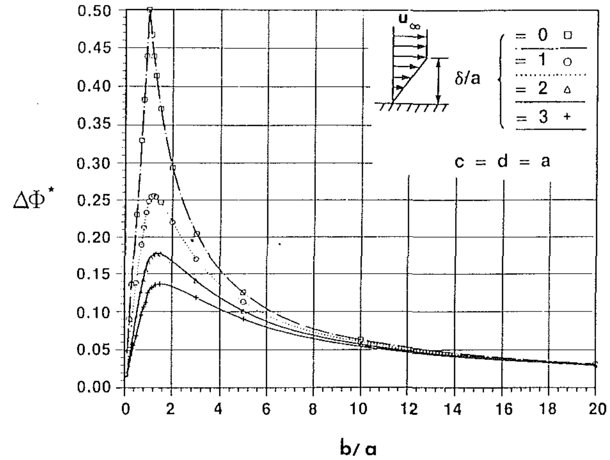

Fig. 5 Nondimensional voltage response of the three-dimensional model of the EBL probe as a function of magnetic field width divided by electrode spacing for four linear profile boundary layer thicknesses

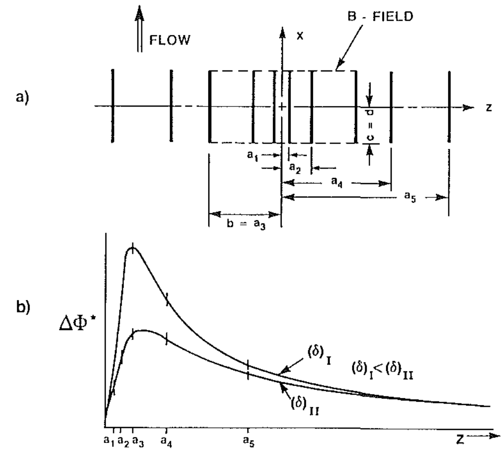

Fig. 6 Multiple paired electrode EBL probe. (a) Top view of array of five pairs of electrodes, for $b = a$ and $d = c$. (b) Voltage output of (a) for two boundary layer thicknesses, corresponding to results of Fig. 5.

"effective range" of the EBL probe (see Fig. 4). The maximum $\Delta\Phi^*$ value for each δ/a curve is slightly shifted to the right (e.g., the maximum $\Delta\Phi^*$ (0.5) for $\delta/a = 0$ occurs at $b/a = 1.0$ while the maximum $\Delta\Phi^*$ (0.14) for $\delta/a = 3$, occurs at $b/a = 1.5$).

What is striking about the family of curves in Fig. 5 is how clearly the effect of boundary layer thickness is demonstrated. It shows that at the very least, the EBL probe could be used to identify an unknown boundary layer if it were calibrated with a known boundary layer family. Similar curves (with higher $\Delta\Phi^*$ values) will result when laminar and turbulent boundary layer profiles (Eq. (15)) are used in (19). They will fall between the linear profile curves and the uniform flow curve of Fig. 5, as shown by the results of Fig. 3 for the two-dimensional model.

In Fig. 5, suppose that b/a is varied by holding the magnetic field width b constant and varying the electrode spacing, a (see (19) and (20)). Then, for a given δ/a, the voltage output curve can be thought of as being traced out by discrete voltages measured from an array of paired, parallel electrodes, for a fixed magnetic field.

An example of such a multiple paired electrode arrangement is shown in Fig. 6(a) as a sketch of a top view of an EBL probe with five pairs of electrodes, where the magnetic field is equal in width to the spacing of the third pair of electrodes and a_3 is taken to be equal to b. Based on Fig. 5 results, the voltage outputs from the five pairs of electrodes would produce a $\Delta\Phi^*$ versus z curve as sketched in Fig. 6(b) for two boundary layer thicknesses, δ_{I} and δ_{II}, where $\delta_{II} > \delta_{I}$. The inner two electrode pairs (a_1 and a_2, mounted well within the magnetic field) define the linear part of each curve. The electrode pair mounted at half spacing a_3 at the edge of the field, yields a voltage that is at or near the maximum $\Delta\Phi^*$ for each boundary layer flow.

Thus, an EBL probe with an array of parallel paired electrodes (such as in Fig. 6(a)) could be used to unobtrusively measure the entire boundary-layer flow by recording the voltages for each pair of electrodes.

The three-dimensional model given by Eq. (19) can further be used as a "design tool" to investigate the variation of other parameters. Voltage characteristics curves can be fairly easily generated for other combinations of u_∞, B_y, δ, electrode length c, and magnetic field length d and width b.

Summary and Conclusions

The goal of this paper has been to gain better understanding of what the proposed electromagnetic boundary layer probe (Fig. 1) actually measures in the boundary-layer flow of an electrically conducting fluid over a solid insulated surface. To this end the following was found:

1. The Shercliff-Bevir equation, (5), much used in the design and understanding of electromagnetic volume flow rate meters, was applied to the EBL probe. It was shown that the voltage output of a probe results in a boundary layer integral quantity, which under restricted weight vector conditions, can be used as a direct measure of the boundary-layer displacement thickness, δ_1 (Eq. (9)).
2. Using a two-dimensional line source-sink model for the virtual current part of the Shercliff-Bevir equation, some operating characteristics of the EBL probe were determined. One key result showed the range of sensitivity of the EBL probe to a parametric change in the location of a wake (or jet) embedded in an otherwise uniform two dimensional flow (about five electrode spacings from the wall for the case considered in Fig. 4). It was also shown that under controlled conditions, laminar and turbulent boundary layers could be differentiated (Fig. 3).
3. A three-dimensional line source-sink analysis used for calculating the virtual current provided a more comprehensive prediction of what an actual EBL probe would measure in a boundary layer flow. This analysis provided the means (Fig. 5) to reason that a multiple paired electrode design (Fig. 6) would be a very beneficial feature. The analysis also provides the means to predict the influence of any one of the ten independent variables that determine the output voltage of the EBL probe.

In conclusion, the analysis presented in this paper shows that the electromagnetic boundary layer probe holds promise for the nonobtrusive measurement of boundary-layer flows at low magnetic Reynolds numbers. The characterization it would provide is independent of such fluid properties as density and viscosity. The only fluid property of importance is the fluid electric conductivity, σ (Eq. (1)).

Results from the three-dimensional model (shown in Fig. 5) demonstrate that it should be possible to experimentally calibrate a multiple paired electrode EBL probe (with a non-idealized magnetic field) in a known boundary layer flow. The shape of voltage curves from such a calibration should be the same as those shown in Fig. 5 and 6(b). These curves are approximately fitted by equation

$$\Delta\phi = \frac{\alpha}{z}(1 - e^{-\beta z^2}) \tag{21}$$

where α and β are constants that depend on the linear slope and the location and value of the maximum value of $\Delta\Phi$. This equation could serve as a basis of a calibration curve for an EBL probe.

The electrode pairs have been treated as being parallel to the two-dimensional flow considered in this paper, so that the voltage output is maximized (Eq. (1)). The electrodes could also be mounted in a nonparallel position to get other voltage components of a given three-dimensional flow.

All of the analysis presented here was for steady-state or time-averaged flow velocities. Further work needs to be done on extending the EBL probe analysis to the measurement of time-dependent (e.g., turbulent) fluid flows.

Finally, the analysis presented here has answered the question of what the EBL probe actually measures in a boundary layer flow. The answer is the integral given on the right-hand side of the Shercliff-Bevir equation in (5). The EBL probe measures a voltage that is given by an integral or velocity over space, but not by the velocity itself. If the weight vector $\mathbf{W}$ (Eq. (6)) is chosen correctly, the voltage measured is directly proportional to the displacement thickness of the boundary layer (Eq. (9)), δ_1.

Thus, future work on the EBL probe might not only include experiments to verify that it can be calibrated as reasoned here, but also could include analytical efforts to find an electrode geometry and magnetic field combination that will yield a weight vector $\mathbf{W}$ which is independent of y. (This will satisfy the conditions that were used to arrive at Eq. (9)). The resulting EBL probe would then be a very effective boundary-layer displacement thickness meter.

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