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Analysis of Multiple Jets in a Cross-Flow

Introduction

The growing awareness of environmental pollution and the attempts to curb it have made the study of jets exhausting into a cross-flow of great practical interest. Chimney stacks exhausting smoke into a wind, cooling tower plumes, discharges of warm water from pipes laid out on the ocean bed, and pollutant discharges into a river are a few such examples. Other examples of gaseous jets in a cross-flow are the lift jets of V/STOL aircraft taking off and landing in strong winds, the injection of fuel into combustion chambers, and the cooling jets on turbine blades. Campbell and Schetz [1], Keffer and Baines [2], Abramovich [3], and others have given results from experimental and theoretical studies of single jets in a cross-flow. They employed momentum integral methods to predict gross properties such as the jet trajectory, the growth of the jet cross-sectional area, mass entrainment, the mean temperature of the jet, etc. and then compared the results with experimental data. These theoretical studies model the jet as a cylinder in a cross-flow, taking into account the "drag" force due to the blockage of the external flow, mass entrainment, and buoyancy. Good agreement with data have been found. (cf. reference [1]). Chien and Schetz [4] obtained exact numerical solutions for a three-dimensional buoyant jet in a cross-flow, using the steady state Navier-Stokes equations written in terms of velocity, vorticity, and temperature with a Boussinesq approximation for eddy viscosity. Patankar, Basu, and Alpay [5] numerically solved the elliptic equations for a deflected turbulent jet with the three velocity components and the pressure as the dependent variables. They used a two-equation turbulence model for the Reynolds stresses.

In many situations, however, one encounters more than one jet in proximity to another. Ziegler and Wooler [6] have analyzed multiple jets in a cross-flow, the jet induced velocity being determined by a combination of sinks and doublets. They assumed that the leading jet is not influenced by the presence of the rear jet; the rear jet is modified by the reduced dynamic pressure behind the leading jet. The present paper extends the analysis of Campbell and Schetz [1] to the study of multiple jets in a cross-flow. The interaction of the two jets is taken into account by a modification of the drag coefficient sensed by each jet.

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Analysis

The present analysis is similar to that of reference [1] and interested readers may find details reported there. The governing equations from reference [1] for a jet with average exit conditions are given below.

Mass continuity:

$$E=\frac{\partial}{\partial s}(\rho A v) \tag{1}$$

Entrainment function:

$$E=(A/C)\rho_\infty E^*(v-U_\infty) \tag{2}$$

The entrainment coefficient E^* is given by an empirical expression based on the experimental data for single jets (reference [1]).

$$E^*=0.2(s/d)^{1.37}/(v/U_\infty)^{0.6} \tag{3}$$

It is realized that this expression may not truly represent the entrainment mechanism in the case of multiple jet cases such as the present; its use is nevertheless partially justified by the subsequent good agreement obtained between the theory and the available data.

s-Momentum:

$$\frac{\partial}{\partial s}(\rho A v^2)=-A\frac{\partial \bar{p}}{\partial s}-gA(\rho-\rho_\infty)\sin\alpha+EU_\infty\cos\alpha-\Pi h\tau \tag{4}$$

This equation represents the balance of forces along the jet trajectory. The contributing terms are due to the rate of change of tangential momentum, the pressure gradient along the trajectory, the buoyancy, the mass entrainment and the shear stress. In order to evaluate the pressure gradient term, the assumption is made that the static pressure field around a solid cylinder imposes itself on the jet flow. This is a rather simplified model for the pressure field arising from the complicated process of turbulent jet injection into a cross-flow. However, the procedure has been found to be adequate for use in the present mathematical model. Differentiating the expression for freestream static pressure yields

$$\frac{\partial \bar{p}}{\partial s} = -q_\infty \sin\alpha\cos\alpha \frac{d\alpha}{ds} \qquad (5)$$

The viscous shear stress in the s-direction is proportional to the velocity gradient in that direction and can be expressed by

$$\tau = \rho(\nu + \epsilon)\frac{\partial u}{\partial n} \qquad (6)$$

The kinematic viscosity, ν, is neglected in the present study since, for turbulent mixing flows, it is small compared to the eddy viscosity, ϵ. The eddy viscosity is estimated using Prandtl's hypothesis (reference [7]) for free turbulent flows. The eddy viscosity is represented by

$$\epsilon = \beta b(u_{\max} - u_{\min}) \qquad (7)$$

The maximum velocity in this expression is defined as the mean jet velocity in order to be compatible with the mean flow assumptions of the present study, and the minimum velocity is the freestream velocity component in the s-direction. Thus the expression for the shear stress becomes

$$\tau = \rho\beta(v - U_\infty \cos\alpha)^2 \qquad (8)$$

The buoyancy term is significant only in cases where the jets are nonisothermal and/or the jets and the free stream consist of gases of different densities. Substituting for the pressure gradient term, equation (5), and the shear stress term, equation (8), into equation (4), the final s-momentum equation becomes

$$\frac{\partial}{\partial s}(\rho A v^2) = q_\infty A \sin\alpha\cos\alpha\frac{d\alpha}{ds} - gA(\rho - \rho_\infty)\sin\alpha + EU_\infty\cos\alpha - \Pi h\rho\beta(v - U_\infty\cos\alpha)^2 \qquad (9)$$

Similarly, the n-momentum equation may be written as

$$-\rho A v^2 \frac{d\alpha}{ds} = C_D q_\infty h \sin^2\alpha + gA(\rho - \rho_\infty)\cos\alpha + EU_\infty \sin\alpha \qquad (10)$$

The n-momentum equation represents the balance of forces acting perpendicular to the jet trajectory. The term on the left-hand side represents the centrifugal force resulting from the curvature of the jet trajectory. This centrifugal force is balanced by the components of the drag force, the buoyancy, and the mass entrainment. The drag term arises due to the blockage of the free stream; this is postulated to be the drag on an equivalent cylindrical shape inclined at an angle to the free stream flow.

Energy:

$$\frac{\partial}{\partial s}(\rho A v C_p T) = EC_p T_\infty + \bar{h}C(T_\infty - T) \qquad (11)$$

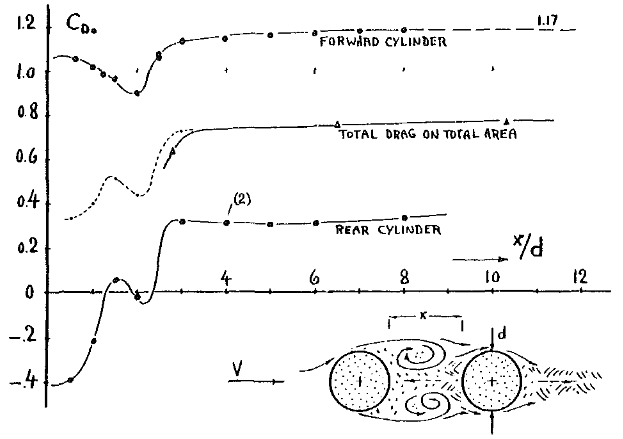

Fig. 1 Drag coefficient of two circular cylinders one placed behind the other. From reference [8].

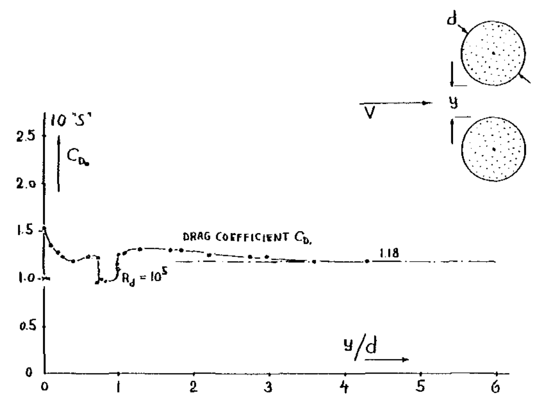

Fig. 2 Drag coefficient of a pair of circular cylinders placed side-by-side. From reference [8].

The energy equation represents the energy balance of the jet fluid due to temperature increase, mass entrainment, and convection at the jet boundary.

When there are two or more jets, their influence on each other may be represented through changes in the effective drag coefficient, C_D, sensed by each jet. Hoerner [8] gives drag coefficients of two circular cylinders when they are placed one behind the other and also side-by-side for Reynolds number of 10^5, as shown in Figs. 1 and 2. This value of Reynolds number is reasonable in many jet problems. In Fig. 1 the drag coefficient, C_D, tends to an asymptotic value

Nomenclature

A = cross-sectional area of the jet
C = effective jet circumference
C_D = drag coefficient
C_p = specific heat at constant pressure
D = jet exit diameter
d = effective jet diameter
ds = infinitesimal length of jet control volume
E = entrained mass per unit length of jet control volume
E^* = entrainment coefficient
g = gravity
h = width of jet
$\bar{h}$ = average film heat transfer coefficient
k = thermal conductivity of jet fluid
Nu_d = Nusselt number ($\bar{h}d/k$)
$\bar{p}$ = average static pressure of jet flow
Pr = Prandtl number
q_∞ = free stream dynamic pressure ($\frac{1}{2}\rho_\infty U_\infty^2$)
R = radius of curvature of jet trajectory
Re = Reynolds number based on d ($U_\infty d/\nu$)
s,n = natural coordinates along and normal to the trajectory
T = mean cross-sectional temperature of jet fluid
VR = effective velocity ratio $[(\rho v^2)/(\rho_\infty U_\infty^2)]^{1/2}$
u = general expression for velocity
v = mean cross-sectional velocity of jet fluid
U_∞ = free stream velocity
x,z = cartesian coordinates
α = inclination of the jet axis to the cross-flow direction
β = constant in expression for eddy viscosity
ρ = mean cross-sectional density of jet fluid
τ = shear stress in s-direction acting on jet control volume
ν = kinematic viscosity
ϵ = eddy viscosity

Subscripts

i = jet flow conditions at injection point
∞ = free stream conditions

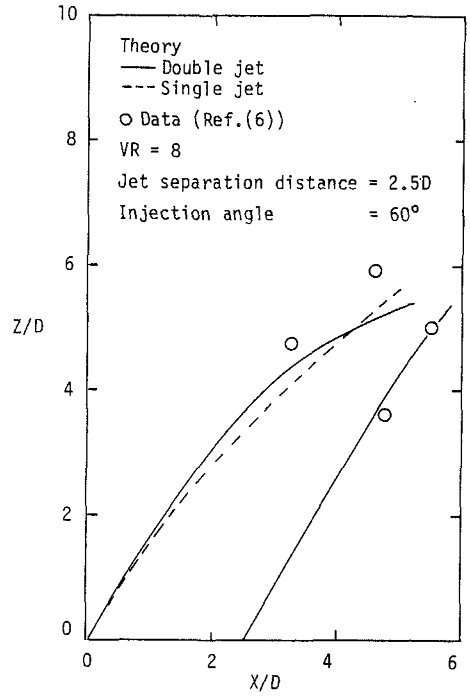

Fig. 3 Single and tandem jet trajectories

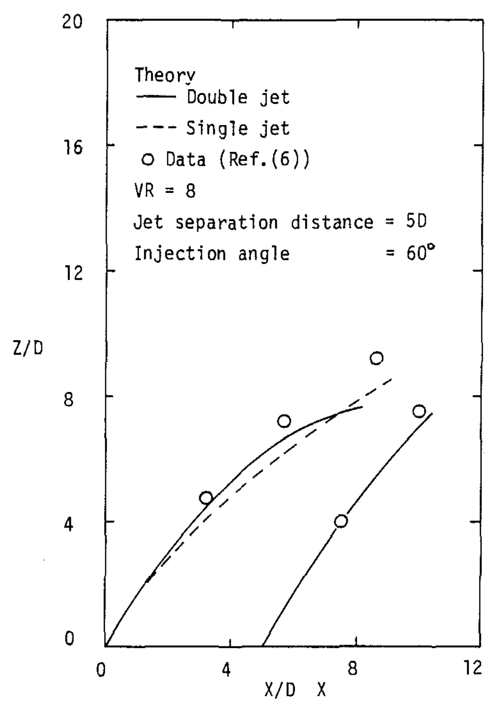

Fig. 4 Single and tandem jet trajectories

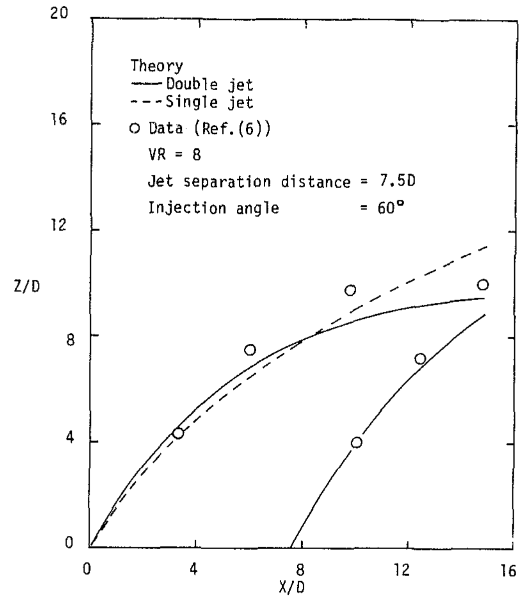

Fig. 5 Single and tandem jet trajectories

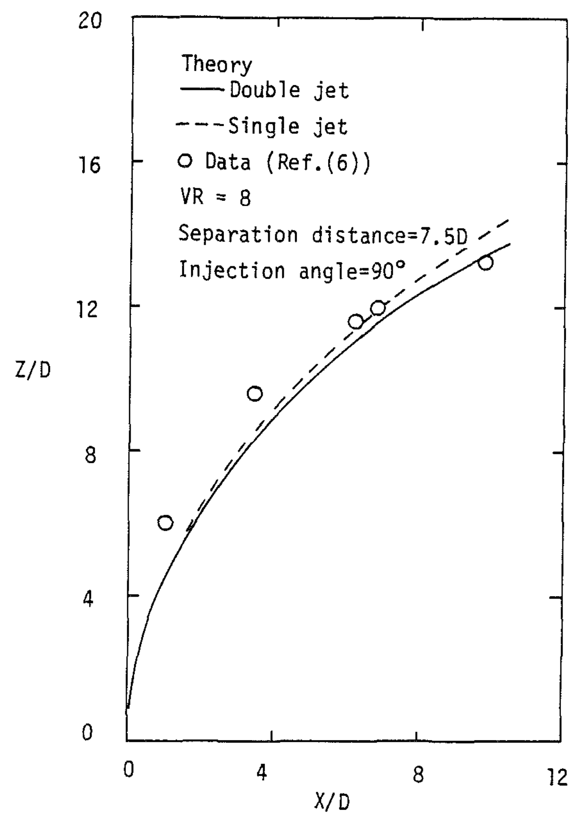

Fig. 6 Trajectories for single and side-by-side jets

of 1.18 for large separation distances. Note that the drag on the rear cylinder (and, hence rear jet) can actually be counter to the main flow.

Equations (1), (9), (10), and (11) form a set of coupled, nonlinear ordinary differential equations which may be solved by using standard routines available. Haming's Predictor-Corrector Method with a fourth order Runge-Kutta starter, as given in reference [9], has been employed in the present case. The perpendicular distance between the two jets is calculated at each step in the integration and the drag coefficient C_D is computed from Figs. 1 and 2 using a cubic spline interpolating routine. The computations are terminated when the jet cross-sections grow and coalesce into each other.

Results

Figures 3 to 6 show data of reference [6] along with the present calculations for a 60 deg injection angle for tandem jets and a 90 deg injection angle for side-by-side jets. The single jet trajectory is also shown for reference. The sparseness of the data prevents any definitive conclusions from being drawn from the comparison of the data and the present analysis. The agreement is good in the case of the rear jet trajectory. The most important result of the present analysis is that the rear jet trajectory is significantly modified by the presence of the front one even when the jets are spaced far apart. As expected, the jet in front is influenced less by the presence of the rear one than vice versa. For the case in Fig. 5, the trajectory of the front jet is significantly affected, and the analysis correctly predicts the observed influence.

The data of reference [6] indicate that, for the side-by-side case, the out of plane deflection of the jets is considerable for small separation distance (2.5D); hence no attempt is made to compare the data and the present analysis for this separation distance. However, for a separation distance of 7.5D, it is seen that the sideways deflection of jets is not signficant. It is, therefore, concluded that the analysis is valid when the jets are not too close to each other.

The main purpose of this study has been to show the significant results which could be obtained by a simple extension of the earlier approach, and the attempt seems to have been fruitful. A logical next step in the study would be to extend the analysis to the region of the merged jets, the initial conditions for which may be obtained by taking the mean properties of the two jets.

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