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Self-Excited Chatter and its Marks in Turning

Investigations were made on self-excited chatter of the work which is held at one end on a lathe machine. Analysis was carried out by a two-degrees-of-freedom system. It was verified that introduction of the multiple regenerative effect governs the finite amplitude of the vibration after it is excited. It was also shown that behavior of the work displacement rotating around the origin during the vibration could be explained by taking into account a resistive force which is inversely proportional to the cutting speed and is proportional to the velocity of the vibration. Phase difference of the vibration occurring for each turn of the work was measured by making use of a microcomputer system. Correlation of the pattern left on the work with the phase difference and with the multiple regenerative effect was studied. A slight change of the vibration frequency varies the pattern even though the phase difference is kept constant. This was also demonstrated by a simulation which generated the pattern of surface shape.

1 Introduction

A number of investigations on self-excited chatter of machine tools have been made with the objectives that the limit of the stability would be identified as the machine tool performance, that potential cutting conditions for stable machining would be investigated, and that an effective method to prevent occurrence of chatter would be developed. The role of the cutting mechanism, the regenerative effect, and the structural dynamics of a machine tool, should be made obvious to identify the stability limit for the chatter. A stability chart is presented on the basis of methods developed by Tlusty [1], Tobias [2], and Merritt [3].

The stability limit, which gives a critical threshold at which the self-excited chatter occurs or not, is one feature of the total characteristic of the chatter. These characteristics include the mechanism to suppress the amplitude after the onset of the chatter, the process to generate the chatter marks left on the machined work, and correlation of these. Analysis of such behavior after the excitation has not been considered necessary from the viewpoint of manufacturing because the onset of the vibration causes very rough machined surface and the accuracy cannot be kept to a sufficient extent. However, investigations on this behavior is required for a better understanding of the phenomena of the chatter. This might make it possible to develop an effective method for damping the chatter. One practical point at the moment is that, if the amplitude after the excitation can be predicted, it becomes possible to judge whether the machining may be continued even though it is exposed to vibrations.

Kondo and others pointed out the existence of a new phenomenon and named it the multiple regenerative effect: The chip cut by present tool motion has the trace of the tool motion not only one turn before, but two or more turns before [4]. This effect plays an important role in the continuation of the vibration with finite amplitude. The paper showed how it works for a basic system where a single-degree-of-freedom system was assumed. Investigations which tried to analyze the behavior by introducing some nonlinearities instead of this effect did not seem to be successful [5, 6]. The analysis of the behavior as a time series made it possible to explain the characteristic that the vibration amplitude stays finite after the onset [7, 8], although the effect itself was not presented so clearly as it was in [4].

The experiment to investigate the behavior after the onset was conducted in detail for a turning procedure which suggested a two-degrees-of-freedom system [9, 10]. However, the analysis in these were concerned with the stability limit, not with the chatter behavior. In this paper the investigation simulating the behavior of the work for turning will be performed by assuming a two-degrees-of-freedom system and taking the multiple regenerative effect into account. The experiment to be compared with this will be also carried out to obtain adequate parameters to the analysis of the system. The effect of a resistive force which looks like the penetration effect [11] will be studied for the two-degrees-of-freedom system. The existence of this force which is inversely proportional to the cutting speed and is proportional to the velocity of the vibration, was experimentally confirmed in the previous paper [4]; this was analytically introduced, as the penetration effect in [11]. Introduction of this force as a term in the equation of motion made it possible to find better agreement of the analysis with the phenomenon of the stability limit. This also gives good agreement to the chatter behavior of the work in the following study.

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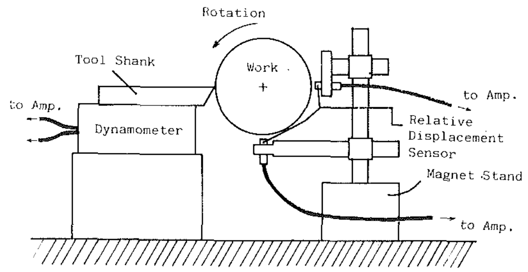

Fig. 1 Schematic view of the measurement system

One of the characteristics of the behavior after the onset of the self-excited vibration is that the trace of the vibration is left on the work as the chatter marks. Some previous studies have analyzed the chatter marks after the onset of the excitation [12, 13]. In these investigations a new apparatus was developed which made it possible to instrument the surface shape of the marks with an isometric view, and this led to an understanding correlation of the behavior of the vibration with the chatter marks in terms of the multiple regenerative effect. Those who observed chatter marks in the factory have noticed that the pattern of the chatter marks changes the winding direction around the work at random. A study of how this occurred was made [14]; however, it looked at the phenomena only from a geometrical approach. In this paper it will be systematically shown how the marks are generated by taking into account such parameters as the phase lag for the respective turns, the frequency of the chatter, the multiple regenerative effect, and the feed of the cutting tool.

The marks show a clear pattern which is generated in close correlation with the phase differences between the vibration at an angular position of a work revolution and that at the same position of the previous revolution. The phase difference is instrumented by an apparatus using a microcomputer. The characteristic of the pattern can be described as winding the work clockwise or counterclockwise along the feed direction, viewing from the tail stock side. These will be named clockwise or counterclockwise in the following. An investigation identifying the correlation of these characteristics with the phase difference was carried out. This makes it obvious that the characteristic of the pattern is caused by a slight change of the frequency of the vibration. This is also confirmed by the generation of the surface by simulation analysis.

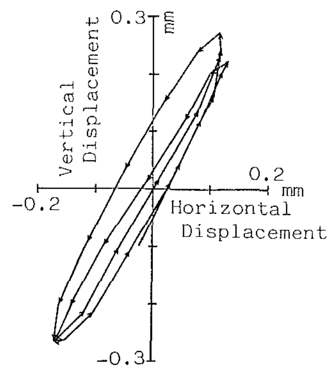

Fig. 2 Behavior of the work motion during the self-excited vibration

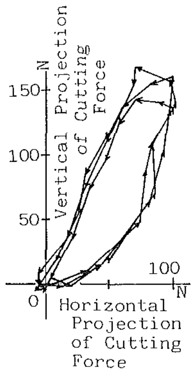

Fig. 3 Behavior of the cutting force during the vibration

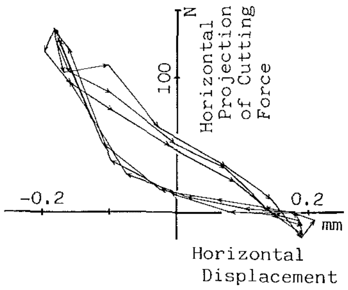

Fig. 4 Behavior of the horizontal displacement versus horizontal component of the cutting force

2 Analysis for Two-Degrees-of-Freedom System

2.1 Behavior of the Work After the Occurrence of the Vibration. Figure 1 shows a schematic view of the instrumentation for motion of the work and the cutting force in vertical and horizontal projection. Figures 2 and 3 show the instrumented motion of the work and the cutting force for the self-excited vibration caused for the work held by a chuck at

Nomenclature

m = equivalent mass of work
c = equivalent viscous damping coefficient of work
k = equivalent stiffness of work
x = horizontal displacement of work
y = vertical displacement of work
F_x = horizontal component of cutting force
F_y = vertical component of cutting force
K_x = coefficient for cutting area giving horizontal component of cutting force
K_y = coefficient for cutting area giving vertical component of cutting force
q = cutting area
w = cutting depth vertical to side cutting edge
s = average cutting width for side cutting edge
d = cutting depth in radial direction of work
f = feed
θ = angle for side cutting edge
R = nose radius of tool tip
Ω = rotational speed of work
t = time
D = diameter of work
ω_0 = natural circular frequency of work
ζ = equivalent damping ratio of work
f_s = frequency of self-excited vibration
T = force working resistively to cutting force in inverse proportion to cutting speed
α = coefficient giving resistive force
v = cutting speed
ϕ = phase difference of self-excited vibration for each revolution of work

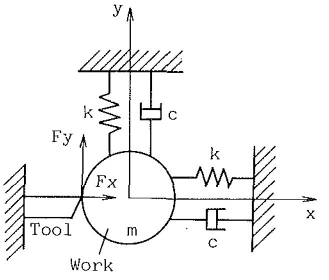

Fig. 5 Vibration model of two-degrees-of-freedom system

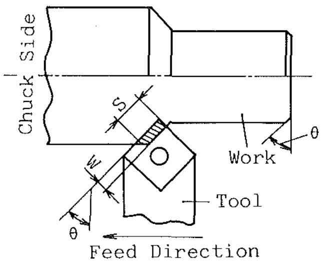

Fig. 6 Schematic view of the tool and the work

one end. The horizontal displacement is taken positive for the direction along which the work extends from the tool. Figure 4 takes the horizontal displacement and the horizontal component of the cutting force as the horizontal and the vertical axis, respectively. The following remarks summarize these investigations:

1 The behavior of the work motion to the cutting force after the excitation draws a loop rotating counterclockwise, whose average inclination is positive. This means that phase of the vertical component slightly lags behind that of the horizontal one.

2 The cutting force becomes zero for some duration of time while the vibration amplitude grows large enough. This leaves part of the trace of the tool of two or more turns before on the work and causes the variation of the uncut chip thickness. This results in keeping the amplitude finite after the excitation: the multiple regenerative effect.

3 The relation of the horizontal displacement to the horizontal component of the cutting force depicts a loop rotating clockwise, with a negative slope as the average inclination. This implies that the cutting force is larger while the relative displacement between the tool and the work increases, that is, for the time when the work leaves the tool, than for the time during which the work attacks the tool. This causes the vibration energy to be supplied during the motion and causes the vibration to be self-excitedly maintained. A similar relation can be observed between the vertical displacement and the vertical component of the cutting force.

These remarks agree with those by Doi and others [9, 10]; they are reconfirmed as the characteristics to be compared with the analysis in the following. Recognition of the multiple regenerative effect for the phenomenon pointed out in [2] was made possible by the previous study [4].

2.2 Analytical Description of the System. To identify the role of the multiple regenerative effect which governs the behavior after the excitation, the analysis by a single-degree-of-freedom system was made by Kondo and others [4]. It proved that the amplitude of the vibration was limited to be finite; however, the system was not sufficient to explain the property observed in the figures aforementioned.

In this investigation the analysis using the model shown in Fig. 5 could be carried out. The equations of motion can be written as

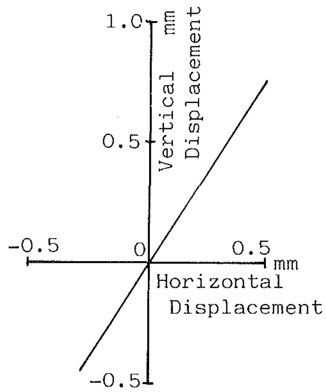

Fig. 7 Analytic behavior of the work motion without the resistive force

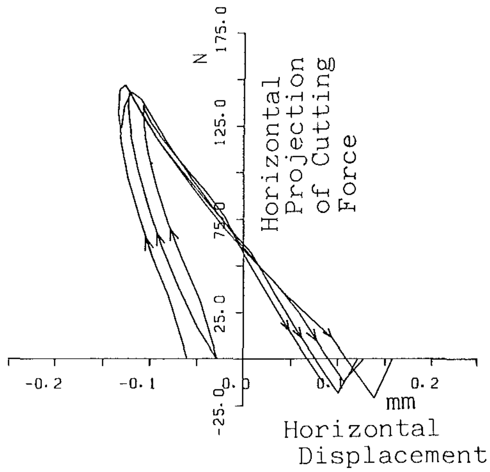

Fig. 8 Analytic behavior of the horizontal displacement versus horizontal component of the cutting force

$$m\ddot{x}+c\dot{x}+kx=F_x \tag{1}$$

$$m\ddot{y}+c\dot{y}+ky=F_y \tag{2}$$

x and y are taken so that the position of the center of gravity for the work at stand-still may be taken as zero. x is made positive for the direction that the work leaves from the tool, and y is made positive for the vertical upper direction. The damping ratio and the spring constant for both directions are taken equal for simplicity.

F_x and F_y are assumed in proportion to q as

$$F_x=K_xq \tag{3}$$

$$F_y=K_yq \tag{4}$$

q can be given as follows by taking into account the assumptions that the tool shape is as shown in Fig. 6 and that the trace of the horizontal displacement of the work for one, two, and more turns before is left on the uncut chip thickness. The nose radius of the tool is considered to be zero; then

$$q=ws \tag{5}$$

where

$$w=\frac{\Omega f\cos\theta}{60}t-x(t)\sin\theta \quad : \quad 0\le t<60/\Omega \tag{6}$$

$$w=f\cos\theta+\min\left\{x\left(t-\frac{60}{\Omega}\right)\sin\theta,\right.$$

$$f\cos\theta+x\left(t-2\frac{60}{\Omega}\right)\sin\theta,\ .\ .\ .\ ,(n-1)f\cos\theta$$

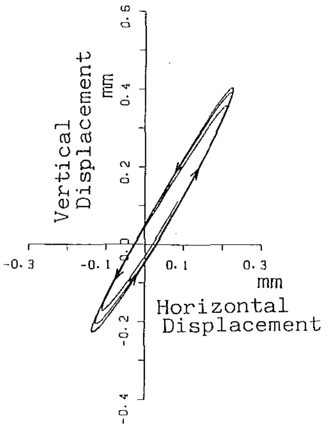

Fig. 9 Analytic behavior of the work motion with the resistive force

$$+x\left(t-n\frac{60}{\Omega}\right)\sin\theta\Big\}-x(t)\sin\theta \quad : \quad 60/\Omega\leq t \tag{7}$$

$$s=\left\{\frac{d-x(t)}{\cos\theta}-\frac{w}{2}\tan\theta\right\} \tag{8}$$

Equations (6) and (7) are given for the interval of the first turn and after the second turn, respectively. It is assumed that the face of the work is chamfered by the same attack angle of the tool, so that the same depth of cut may be provided from the first revolution of the cutting. When the tool is separated from the work, w is to be negative. However, it is taken as zero in the simulation. The relations described above are effective for $f<d$. These relations do not take the thread cutting as objective; the inequality is not applicable in this case.

Figures 7 and 8 show the results of solving equations (1) and (2) by the Runge-Kutta-Gill method; they are comparable with Fig. 2 and Fig. 4. The computer was a HITAC M-200H. Figure 8 depicts similar characteristics to Fig. 4 in that the loop rotates clockwise and the slope of the average inclination is negative. The characteristic of Fig. 7 is different from that of Fig. 2 in that the former does not draw a loop; however, the two inclinations are similar. This means that dynamic characteristics of the cutting force are not represented sufficiently by the equations described so far.

The existence of a resistive force which works in the horizontal direction was experimentally confirmed and the introduction of this into the analysis for the single-degree-of-freedom system allowed us to explain the enlargement of the stable region in the low cutting speed [4]. The characteristics of this force are similar to those of the penetration effect [11]. However, the penetration effect was analytically introduced in the study, so that the constants to be used in the analytical approach could not be given. These are derived by the characteristics obtained in the investigation related to the single-degree-of-freedom system [4]. The introduction of this force into the present analysis will be made. The force shows the following fundamental properties: It is 1) inversely proportional to the cutting speed, 2) proportional to horizontal relative velocity between the tool and the work, and 3) not dependent on the projected area of the uncut chip onto the tool tip.

So equation (3) is used by deforming into

$$F_x=K_xq-T \tag{9}$$

where

$$T=\alpha\frac{\dot{x}(t)\cos^2\theta}{v} \tag{10}$$

$$v=\frac{\pi D\Omega}{60}-\dot{x}(t) \tag{11}$$

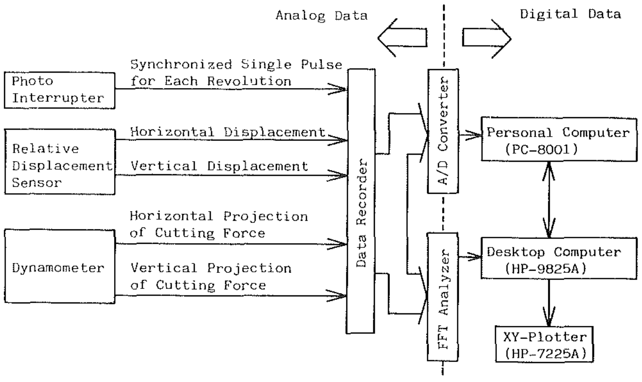

Fig. 10 Block diagram of the measurement system of the phase difference for the respective work revolutions

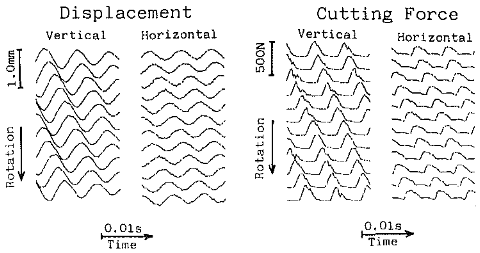

Fig. 11 The phase difference for the respective work revolutions

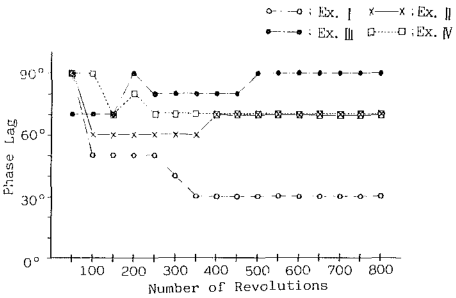

Fig. 12 Time-dependent characteristic of the phase difference

Figure 9 shows the solution as the motion of center of the work, which well simulates Fig. 2. It depicts a counterclockwise loop. This was impossible to obtain by the solution neglecting this force, as shown in Fig. 7. This also tells us that the introduction of the force makes it possible to give a good explanation to the work motion after the onset of the self-excited vibration.

3 Phase Difference for Respective Revolutions of the Work

The analysis of the stability showed that a phase difference occurs for successive revolutions of the work by the regenerative effect. However, few investigations describing the experimental characteristics of the phase difference have been published. One exceptional description of grinding [15] estimated the characteristics from the chatter marks on the work; here the wave form of the vibration for the respective revolutions was not instrumented quantitatively.

The phase difference has been considered to be directly related with the pattern of the chatter mark, which is likely to be clockwise [16]. However, the pattern is not limited to this, and it varies from clockwise to counterclockwise along the feed. The phase difference for the respective revolutions of the work was instrumented by an apparatus using a microcomputer and the correlation of this with the pattern of

Table 1 Cutting conditions as for Fig. 12

Tool (By Toshiba Tungaloy)
Shank : PSDNN 2525
Side Cutting Edge Angle ---------- 45°
Back Rake Angle ------------------ 6°
Tip : SNMR 431C (Nose Radius 0.4mm)

Work : Brass
Overhang Bit Length ---------------- 300mm

Rotational Frequency of Main Spindle --- 600rpm

Exp.	I	II	III	IV
Diameter (mm)	43.2	53.7	53.3	53.4
Depth of Cut (mm)	1.5	1.5	1.5	0.15
Feed (mm/rev)	0.05	0.05	0.15	0.05

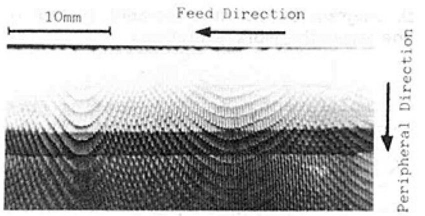

Fig. 13 An example of the chatter marks

the chatter marks is identified in the following. In this process it is required to observe the surface shape of the marks.

Figure 10 is a block diagram of the measurement system. Motion of the work in terms of the displacement and the cutting force in horizontal and vertical components is recorded into a data recorder. The signal from the photo interrupter which shows a definite rotating position of the spindle is simultaneously recorded. A fixed length of the signal starting from the pulse position of the photo interrupter for each revolution was stored in a personal computer PC-8001 made by Nippon Electric Co.

Figure 11 shows the wave forms by XY-plotter driven by a desk top computer HP-9825 after they are transferred from the PC – 8001. A uniform phase difference: Lag occurs for each revolution of the work for both the displacement and the cutting force. The same behavior can be expressed by the results of the simulation made in section 2. Figure 12 shows the variation of the phase difference after the early stage of the excitation for several examples based on the cutting conditions in Table 1.

No systematic relation between the cutting conditions and the phase difference has been investigated so far. Figure 12 shows that the phase differences become constant at a value between 30 and 90 deg following fluctuations after the excitation. The phase difference may increase or decrease during the fluctuation period. The reason why the phase difference becomes a fixed quantity after sufficient growth of the amplitude of the chatter is not obvious at present and should be studied in the future.

4 Measurement of the Chatter Marks

The chatter marks are generated in relation to the phase difference and the pattern of the marks winding clockwise. The analysis of how the pattern was generated was conducted in [14], but the correlation with the multiple regenerative effect was not made clear enough. The pattern changes greatly according to the direction of the winding. The measurement method of two-dimensional surface roughness [12, 13] was applied to observe the characteristic of this change. The pattern characteristics in relation with the phase difference and the multiple regenerative effect are identified in the following.

Figure 13 shows an example of the chatter marks whose characteristic pattern varies as the feed proceeds. Figures 14, 15, and 16 display equicontour maps of the two-dimensional surfaces of the part. These are the parts for counterclockwise, cross of counterclockwise and clockwise, and clockwise windings, respectively. In Fig. 14 hollow parts are located slightly downward along the feed direction: counterclockwise. In Fig. 16 the location of the hollow parts to the feed direction goes slightly upwards: clockwise; the tendency is symmetrically different from that in Fig. 14. The characteristic found in Fig. 15 is for another typical pattern where hollow parts are located in between one another.

The abstracted pattern in Fig. 17 is depicted from Fig. 14 by representing the ridges as lines. The dimension is given by taking the cutting conditions into account, too. The feed pitch corresponds to the horizontal length between F and G. The inclination between A and G can be given by the feed pitch and by the trace of the vibration after a revolution of the work. Following the trace of the vibration on the same

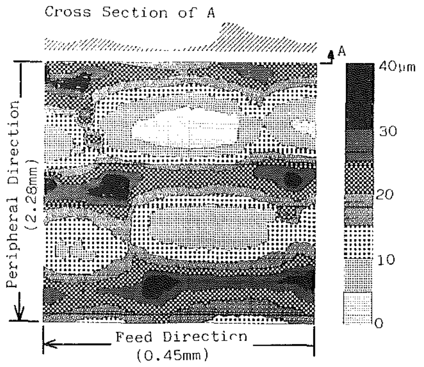

Fig. 14 Contour map of the chatter marks for pattern of counterclockwise winding

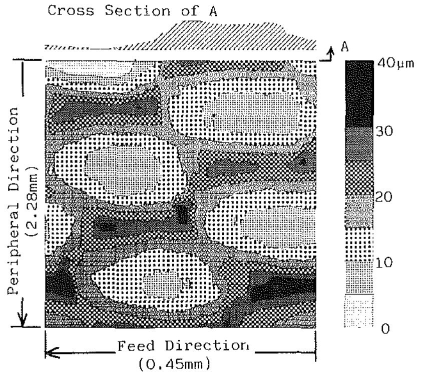

Fig. 15 Contour map of the chatter marks for pattern crossing clockwise and counterclockwise winding

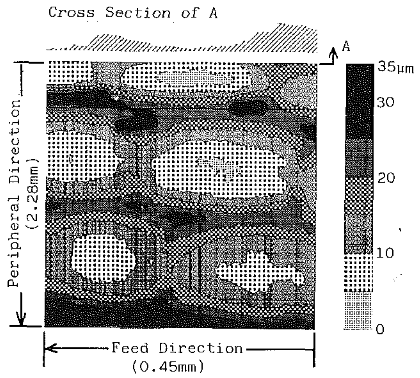

Fig. 16 Contour map of the chatter marks for pattern of clockwise winding

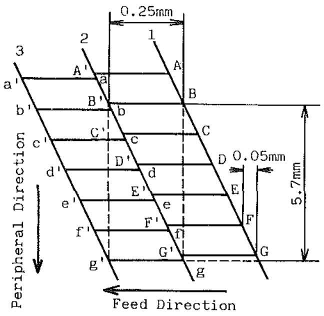

Fig. 17 Relation between the phase difference and the pattern of the chatter marks

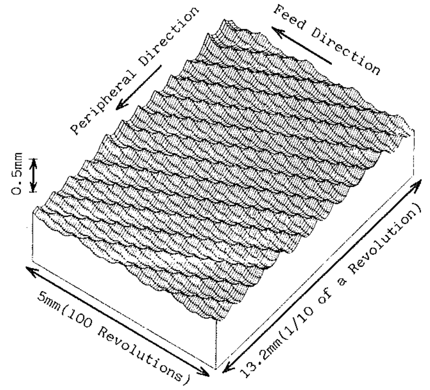

Fig. 18 Generation of the chatter marks (f_s = 198.40 Hz)

revolution, □ $AA'B'B$ is generated after □ $ff'g'g$ and $\overline{gB}$ is the wave length of the vibration which can be given by the diameter of the work, the rotational speed, and the frequency of the vibration.

The trace of one revolution after the generation of □ $ff'g'g$ is □ $ee'f'f$. The phase difference observed in Fig. 12 is that between the vibration for the present revolution and that after one revolution, looking at the position $\overline{gg'}$ for example. Then the characteristic of the counterclockwise pattern which results when g is located lower than G is indifferent to this phase difference. For the case shown in Fig. 17, the broken line which starts from $\overline{gg'}$ covers the trace which would be generated by the vibration for the following revolutions of the work. This means that uncut chip thickness for □ $bb'c'c$ may be affected by the trace of the vibration which generated the mark from □ $ff'g'g$ to □ $cc'd'd$, that is, the multiple regenerative effect is occurring as soon as the vibration is excited.

However, the discussions on Fig. 17 illustrate that a slight change of the frequency of the self-excited vibration may easily bring g higher than G which changes the pattern from counterclockwise to clockwise, without significant change of the phase difference. This indicates that the pattern of the chatter marks may change from that winding counterclockwise to that clockwise. The change of the frequency for Figures 14, 15, and 16 could not be obtained directly by the conventional frequency analysis for lack of resolution. A comparative method using digital oscillator and synchroscope made it possible to confirm that the change occurs from 210.2 to 209.5 Hz as the tool is fed. This variation may be caused by the fact that the work is held only by a chuck and the tool moves keeping contact with the work.

5 Generation of the Chatter Marks by Simulation

As mentioned above, the role of the multiple regenerative effect in a two-degrees-of-freedom system was confirmed, the phase difference due to the regenerative effect was measured, and characteristic of the pattern of the chatter marks was identified. In this section the relation between the frequency and the pattern is verified by assuming the wave form as sinusoidal and by generating the cut surface. In this analysis the wave form which could be obtained by the simulation due to two-degrees-of-freedom was not used, since it is time-consuming for the computation, and it is still not easy to predict the behavior after the excitation by setting the adequate parameters. In the following, the trace of the vibration left on the surface is obtained by assuming these terms:

1 Only horizontal projection is taken as the component of the vibration.

2 Wave form is replaced by sinusoidal one.

3 Shape of the cut surface is obtained by combining the traces which are closest to the center of the work.

Ω = 600 rmp, f = 0.05 mm/rev, d = 1.5 mm, D = 42 mm, R = 0.4 mm, and θ = 45 deg are assumed as the cutting conditions, which are almost the same as those of the previous experiment. The frequency of the vibration was taken as f_s = 198 Hz for a similar reason; this satisfies the condition causing the phase difference:

$$10n-5<f_s<10n \tag{12}$$

The shapes of the surface shown in Figures 18, 19, and 20 are for f_s = 198.40, 198.34, and 198.27 Hz. The change of the frequency only covers 0.13 Hz and the pattern of the marks varies with inclination. Phase differences for each revolution were almost constant: 57.6 deg, 59.8 deg, and 62.3 deg. The change of the pattern of the chatter marks which is caused by a slight change of the frequency could be demonstrated by the simulation.

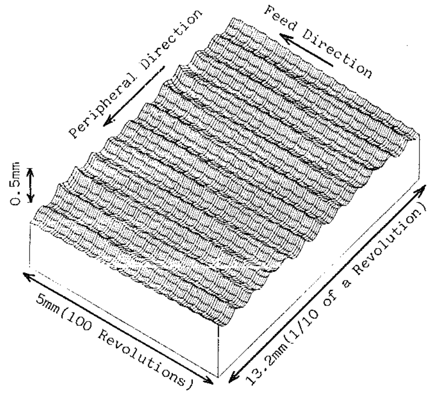

Fig. 19 Generation of the chatter marks (f_s = 198.34 Hz)

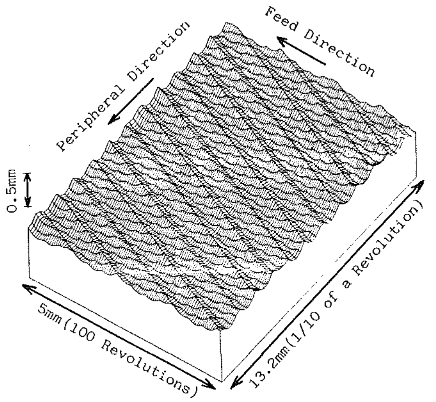

Fig. 20 Generation of the chatter marks (f_s = 198.27 Hz)

6 Conclusions

Analysis of the self-excited vibration which occurs in turning of the bar work held only by a chuck was made. The behavior after the excitation can be made to agree with that by using two-degrees-of-freedom system by introducing the multiple regenerative effect and a resistive cutting force which is inversely proportional to the cutting speed and is proportional to the relative velocity between the work and the tool. The phase difference of the vibration for each revolution of the work was instrumented by apparatus using a microcomputer and the relation with the pattern of the chatter marks was identified. The following conclusive remarks can be summarized from these investigations:

1 Experimental observation of the behavior after the excitation of the self-excited chatter confirms that the vertical component has a slight phase lag behind the horizontal component in both the displacement and the cutting force. It was shown that this motion which supplies the vibration energy and maintains a stationary amplitude could be well simulated by the analysis of a two-degrees-of-freedom system with the multiple regenerative effect.

2 The phase difference of the vibration occurring for each revolution of the work was instrumented between 30 and 90 deg depending on cutting conditions. It varies soon after the vibration starts; however, it stays constant after the excitation.

3 It was demonstrated that the pattern of the chatter marks varies significantly in spite of the constant phase difference. The characteristics of the pattern, winding clockwise or counterclockwise along the feed direction, were identified by observing the equicontour map of the two-dimensional surface roughness. The change of the pattern could be generated irrespective of the phase difference.

4 It was shown that a significant variation of the pattern could be generated by a slight change of the frequency by assuming the wave form as sinusoidal and by considering the horizontal motion. The change of the frequency which caused the pattern winding from counterclockwise to clockwise was 0.13 Hz for 193 Hz.

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