Unsymmetrical Short-Circuit Fault Analysis for Low Voltage Distribution Networks

M. Najafi, R. Ebrahimi, A. Babaee and M. Hoseynpoor
Department of Engineering, Bushehr Branch, Islamic Azad University, Bushehr, Iran

Abstract: Protective devices in distribution systems are utilized to optimize the network reliability and to avoid or minimize the damaging in the circuits and the instruments. The considered method and the level of the protection, is under various factors. One of the most important tools in selecting the type of the distributed system's equipment is evaluation of the short-circuit current correctly. In addition, this analysis is very noticeable in allocation and setting the protective devices in any networks. In this study, a novel method is presented in order to analysis the unsymmetrical fault occurs in the low voltage distributed networks. Initially, regarded to a complete model, the line impedances such as overhead and underground line impedances, in the network are computed and the methodology and modeling associated with two relationship matrices BIBC and BCBV is described. Then, the calculations method for the fault's current is evaluated by defining a new model consists of four matrices. Therefore, a program is provided for determination of short-circuit current, identifying the branches which carry short-circuit current and the results of it on the voltage profile, according to presented equations and Delphi software. At the end, all connection types and their results on the network are evaluated on a real low voltage network.

Key words: Four impedances model, unbalanced distribution network, unsymmetrical faults, unsymmetrical short-circuit fault

INTRODUCTION

The low voltage networks are the final and the only part of the vast set of power system cycle, which is related to the energy consumers, directly. Due to simple instructions used in the low voltage networks, economic reasons and in contrast with other voltage levels, it seems to be fair to say that this part is under high inattention and it has backwardness, which this provides many problems such as the voltage drops, vide rang of losses and faults. In the most recent years, regarding to increasing the problems and the importance of quality among consumers, number of companies offered and obtained the solutions.

The short-circuit current is an important parameter in designing such a network, choosing a desired conductor size for its lines, the type and the nominal current of fuses. One of the most obvious properties of the low voltage networks is using underground lines (cables) to transmission electric power, because this method would be safer and more economical. The necessity of the fault currents calculations for designing the networks are highlighted by considering the high vulnerability of cables in LV distribution networks. Therefore, the desirable designing of the network can eliminate the high cost in which spending for troubleshooting and replacing the cables.

In the past few years, the single-phase modeled and symmetric elements were used in order to calculate the fault current in distributed networks for both symmetric and unsymmetrical faults (Roy, 1979; Brandwajn and Tinney, 1985). Selecting these methods, regarding to unbalanced loads and unsymmetrical status in recent distributed networks, brings vast range of errors. In recent years, using compensation method is very common on short circuit calculation beside of using in load flow calculation (Chen *et al.*, 1992; Gross and Hong, 1982; Alvarado *et al.*, 1985).

Nevertheless, the system can use the equivalent part for loads and the network equipment to increase the speed of this method (Miu and Yiming, 2002). The compensation hybrid method for determining the short-circuit current is presented in reference (Zhang *et al.*, 1995), where also calculates the fault current in networks with more than three-wires (Ciric *et al.*, 2005).

The BIBC (Bus Current Injection to Branch Current) and the BCBV (Branch Current to Bus Voltage) relationship matrices and all types of short connections in average voltage network are introduced and debated for radial and weakly meshed distribution networks in references (Teng, 2005, 2010), respectively.

In this study, a novel methodology based on both relationship matrices, mentioned in Teng (2005, 2010), is discussed to analysis all types of all unsymmetrical short-circuits in the low voltage networks. Firstly, a complete

Corresponding Author: M. Najafi, Department of Engineering, Bushehr Branch, Islamic Azad University, Bushehr, Iran
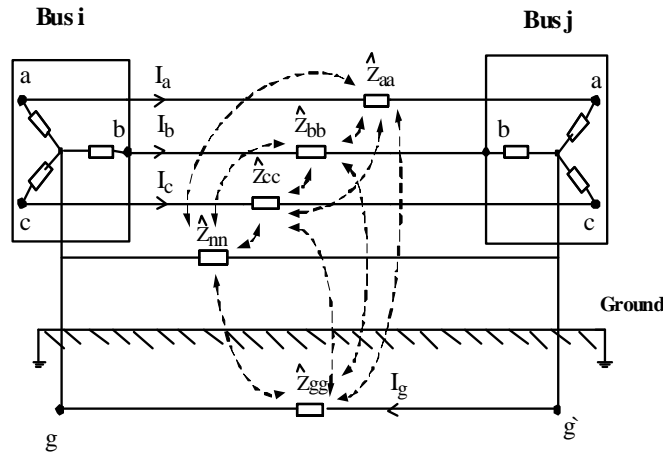
Fig. 1: The impedance model of four-wired lines in the low voltage

model of lines impedances such as overhead lines and underground lines is presented for modeling the network, after that the above matrices are introduced.

In the fallowing, the technique in calculation of all types of unsymmetrical short circuits in the network is demonstrated by using four impedances and displaying a simple method. Finally, all kinds of short-circuits and the results of them on the real network associated with the Delphi software are then evaluated.

ANALYZING THE LINES IMPEDANCE OF THE DISTRIBUTION NETWORKS

In addition to self-impedance for the unsymmetrical distributed network such as three-line, double-line and single-line, the mutual-impedance between phases and the ground effect on them should be considered in order to analysis the lines real model. It is impossible to use phase impedance and symmetric elements to calculate line impedance, because the distributed networks have no transposition and no balanced loads. Therefore, determining the self-impedances and mutual-impedances individually between phases is needed.

The low distributed networks, is the composite of overhead lines and underground cables that the way to calculation them is presented in fallowing. The calculation detail is illustrated in reference (Kersting, 2002).

Overhead lines: The overhead lines in low distributed networks, are shaped in four-wired frame, involve three phases (a, b, c) and the null wire. Figure 1 shows an example of these lines.

In this study, Carson equations are used in order to compute the self-impedances and the mutual-impedances of each conductor with the earth return effect.

So, for self-impedance and mutual impedance, the bellow equations are written:

$$\hat{Z}_{ii} = ri + 9.869 \times 10^{-4} \cdot f + j1.256 \times 10^{-3}$$
$$f \cdot (In\frac{1}{GMR_i} + 6.4905 + \frac{1}{2}In\frac{\rho}{f}) \tag{1}$$

$$\hat{Z}_{ii} = r_i + 9.86963 \times 10^{-4} \cdot f + j1.25664 \times 10^{-3}$$
$$f(In\frac{1}{D_{ij}} + 6.4905 + In\frac{\rho}{f}) \tag{2}$$

where,

r_r : The resistor value of conductor per the length unit (Ω/km)
f : The frequency for power system
GMR_i : Conductor geometric mea radius (m)
ρ : Earth resistivity (generally 100 Ωm)
d_{ji} : The distance between two wires (m)

Regarding to above equations, the initial matrix, is a 4×4 matrix, which applying the Kron reduction and considering the null effect of phase impedance, a 3×3 is required:

$$\begin{bmatrix} \hat{Z}_{aa} & \hat{Z}_{ab} & \hat{Z}_{ac} & \hat{Z}_{an} \\ \hat{Z}_{ba} & \hat{Z}_{bb} & \hat{Z}_{bc} & \hat{Z}_{bn} \\ \hat{Z}_{ca} & \hat{Z}_{cb} & \hat{Z}_{cc} & \hat{Z}_{cn} \\ \hat{Z}_{na} & \hat{Z}_{nb} & \hat{Z}_{nc} & \hat{Z}_{nn} \end{bmatrix} \xrightarrow{\textit{Kron reduction}}$$

$$\tag{3}$$

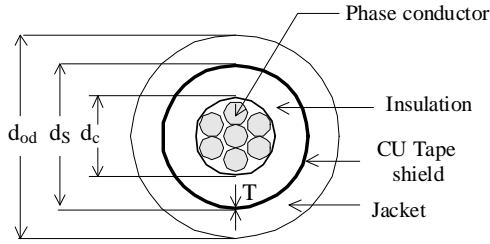$$\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}$$

Fig. 2: Sample of tape-shielded cable details

Fig. 3: Modeling of types of faults with four impedances

The underground lines (cable): In this section, tape-shielded cables are selected for underground lines. Figure 2 displays a sample of shielded cable within details.

where,
d_c : Diameter of phase conductor (mm)
d_S : Outside diameter of the tape shield (mm)
d_{od} : Outside diameter over jacket (mm).
T : Thickness of copper tape-shield (mm)

Once again, the modified Carson's equations will be applied to calculate the self impedances of phase conductor and the tape shield, as well as the mutual impedance between the phase conductor and the tape shield. The resistance and GMR of the phase conductor are found in a standard table of conductor data. The resistance of tape shield is given by:

$$r_{shield} = \frac{10^9}{\pi} \cdot \frac{\rho}{T.d_s} \qquad (4)$$

According to above explanations, it is possible to determine both self and mutual impedances of shielded cable using Eq. (1) and (2). Regarding to conductors, shields and null wire for an underground three-phase-line with the null wire, the initial impedance of cable is a 7×7 matrix in which it is converted to a 3×3 phased matrix, by reducing Cron's equations. Also, the complexity of impedance computations are decreased for cables with no shields, thus the overhead impedance calculations method is recommended.

Therefore, the total lines impedances of the low voltage network can be calculated to determine the short circuit current of the network.

Introducing the BIVC and BCBV matrices: In conventional methods, the networks are evaluated with both impedance and admittance matrices. Although, driving the impedance matrix is much harder than impedance matrix, it contains more data. In this study, according to radial distributed networks configuration, the two relationship ma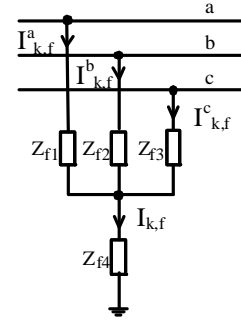trices are called the bus-current-injection-to-branch-current matrix (BIBC) and the branch-current-to-bus-voltage matrix (BCBV) were derived and utilized to analysis the network. The BIBC matrix is an upper-triangular matrix where the name of it defines the relationship between the bus-current injections and branch currents, and the equation can be written as:

$$[B] = [BIBC][I] \qquad (5)$$

where, [B] and [I] are the vectors of branch currents and bus current injections, respectively. Thus the currents flows in the network lines are founded and the bus voltage changes due to this current is called BCBV and written as:

$$[\Delta V] = [BCBV][B] \qquad (6)$$

where, ΔV known as the bus voltage changes and BCBV calculated from matrices determined in pervious section.

The introduction and providing these proposed matrices are completely discussed in reference (Teng, 2005). The bus voltage changes and the short circuit current can be analysis within driving these two matrices.

Short-circuit fault analysis: The unsymmetrical faults are the most existence faults in the network; consist of the single-line-to-ground fault, the double-line-to-ground fault and the line-to-line fault. Regarding to unbalanced loads and unsymmetrical lines in distributed networks, the both the three-line fault and the three-line-to-ground fault can be joined to this group. Among faults mentioned above, the single-line-to-ground is the most conventional type of the faults occurs in the network. In addition, the three-line fault which the power of circuit breakers are estimated by it, is the worth type of faults and the occurrence probability is also minimum.

In the presented method, the analysis of these faults is evaluated from the combination of matrices associated

with prefault and postfault conditions. In this study, only one model is used in order to analysis types of unsymmetrical faults. This model is simple but it's complete. This model consists of four different variable impedances and covers all the fault modes within flexibility. Figure 3 shows the mentioned model.

According to Figure 3, the variety types of the faults mode can be demonstrated by selecting the minimal value of its impedances (zero) and the maximal value of 10^{12}. As an example, if the Z_{f4} select a value of 10^{12}, then the short-circuit without the earth return will be shaped. Nevertheless, the both Z_{f4} and Z_{f1} impedances take the value of zero and impedances Z_{f2} and Z_{f3} take 10^{12} in order to survey about the single-phase-on-phase-a fault.

Therefore, all faults can be considered using this model. Considering the Fig. 3, when a fault occurs due to all above impedances in K_{th} bus then:

$$I_k^a = I_{k,f}^a, \ I_k^b = I_{k,f}^b \quad I_k^c = I_{k,f}^c \qquad (7)$$

And regarding to KCL law it can be written as:

$$I_{k,f} = I_{k,f}^a + I_{k,f}^b + I_{k,f}^c I \qquad (8)$$

where, the I_{kf}^a, I_{kf}^b, and I_{kf}^c are currents of phase a, b, c respectively, that occur regarded to the faults. Because the load current is negligible in comparison with short-circuit-current, the only assumed lines currents are the fault currents. Meanwhile, the bus voltage of fault is changed by following the currents in the network, voltage of bus K illustrated as:

$$\begin{aligned} V_{k,f}^a &= Z_{f1}I_{k,f}^a + Z_{f4}I_{k,f}^a \\ V_{k,f}^a &= Z_{f2}I_{k,f}^a + Z_{f4}I_{k,f}^a \\ V_{k,f}^a &= Z_{f3}I_{k,f+}^a \quad Z_{f4}I_{k,f}^a \end{aligned} \qquad (9)$$

Therefore, the variation of the K_{th} bus postfault voltage can be calculated by distribution load results and the voltage of bus known as $V_{k,0}^{abc}$ before the fault occurs. The equation illustrated the voltage changes of K_{th} bus after the fault occurred:

$$\begin{aligned} \Delta V_{k,f}^a &= V_{k,0}^a - Z_{f1}I_{k,f}^a - Z_{f4}I_{k,f} \\ \Delta V_{k,f}^b &= V_{k,0}^b - Z_{f2}I_{k,f}^b - Z_{f4}I_{k,f} \\ \Delta V_{k,f}^c &= V_{k,0}^c - Z_{f3}I_{k,f}^c - Z_{f4}I_{k,f} \end{aligned} \qquad (10)$$

According to Eq. (7) which shows the currents in network associated with Eq. (5) we would have:

$$\left[B_f \right] = \left[BIBC \right] \left[0 ... I_{k,f}^a \ I_{k,f}^b \ I_{k,f}^c \ 0 \right]^T \qquad (11)$$

Then, considered BIBC matrix, the Eq. (11) can be written as fallowing:

$$\left[B_f \right] = \begin{bmatrix} BIBC_k^a & BIBC_k^b & BIBC_k^c \end{bmatrix} \begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix} \qquad (12)$$

where, $BIBC_k^{abc}$ is the column vectors of BIBC matrix based on the fault related bus and phase. After identifying the branch currents, the changes of bus voltages can be computed by substituting the Eq. (12) into (6), as fallowing:

$$\left[\Delta V \right] = \left[BCBV \right] \begin{bmatrix} BIBC_k^a \\ BIBC_k^b \\ BIBC_k^c \end{bmatrix}^T \begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix} \qquad (13)$$

and, for the bus voltage variation, Eq. (13) can be expressed as:

$$\begin{bmatrix} \Delta V_{k,f}^a \\ \Delta V_{k,f}^b \\ \Delta V_{k,f}^c \end{bmatrix} = \begin{bmatrix} BCBV_k^a \\ BCBV_k^b \\ BCBV_k^c \end{bmatrix} \begin{bmatrix} BIBC_k^a \\ BIBC_k^b \\ BIBC_k^c \end{bmatrix}^T \begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix} \qquad (14)$$

By substituting the Eq. (8) into (10), the Eq. (15) is presented:

$$\begin{bmatrix} V_{k,0}^a - Z_{f1}I_{k,f}^a - (I_{k,f}^a + I_{k,f}^b + I_{k,f}^c)Z_{f4} \\ V_{k,0}^b - Z_{f2}I_{k,f}^b - (I_{k,f}^a + I_{k,f}^b + I_{k,f}^c)Z_{f4} \\ V_{k,0}^c - Z_{f3}I_{k,f}^c - (I_{k,f}^a + I_{k,f}^b + I_{k,f}^c)Z_{f4} \end{bmatrix}$$
$$= \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix} \qquad (15)$$

where, L matrix is determined by multiplying the both BIBC and BCBV matrices.

By rewriting the mentioned equations, displays as fallowing:

$$\begin{bmatrix} V_{k,0}^{a} \\ V_{k,0}^{b} \\ V_{k,0}^{c} \end{bmatrix} = \left[[L] + [Z_f] \right] \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} \qquad (16)$$

where, Z_f is the combination of the fault matrices, as shows in fallowing:

$$[Z_f] = \begin{bmatrix} Z_{f1} + Z_{f4} & Z_{f4} & Z_{f4} \\ Z_{f4} & Z_{f2} + Z_{f4} & Z_{f4} \\ Z_{f4} & Z_{f4} & Z_{f3} + Z_{f4} \end{bmatrix} \qquad (17)$$

The fault currents of the K point can be evaluated by fallowing equation:

$$\begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} = \left[[L] + [Z_f] \right]^{-1} \begin{bmatrix} V_{k,0}^{a} \\ V_{k,0}^{b} \\ V_{k,0}^{c} \end{bmatrix} \qquad (18)$$

where, the summation of Z_f and L, from the fault point of view, is known as the impedance matrix. Therefore associated with above equations, it is easy to evaluate all kinds of the short circuit currents occur in the network. Increased the value of impedance for each phase means the disconnection in the network where the current presented in Eq. (17).

One of the properties of BIBC and BCBV is branch currents which are generated by the fault currents can be found directly by Eq. (11) and (13).

RESULTS

In this study, a program using Delphi software is provided in order to the fault currents calculation. In this program, first, impedances calculate by considering the technical characters of the overhead and underground lines and regarding to the network structure an algorithm is provided from BIBC and BCBV matrices. After providing these relationship matrices the all kinds of unsymmetrical faults in the network due to the fault impedance can be calculated associated with mentioned model and equations in previous sections. Nevertheless, calculations of the voltage drops of buses due to the fault current and identifying the branches fault current are other properties of this program.The low voltage network of Tehran - Ebrahim Abad is discussed in the field of the fault currents calculations. Figure 4 shows this LV distribution network. The formation of overhead/ underlines of network is shown in Fig. 5.
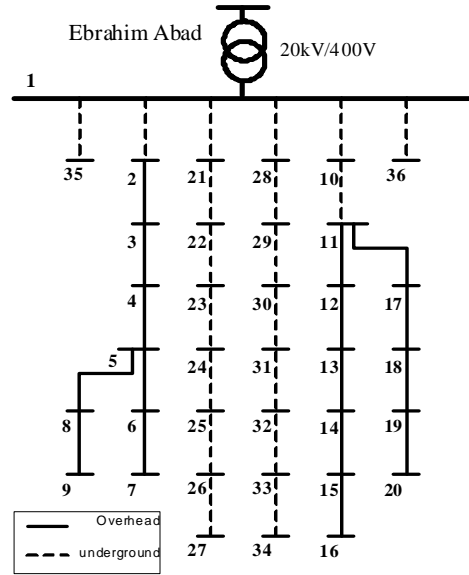

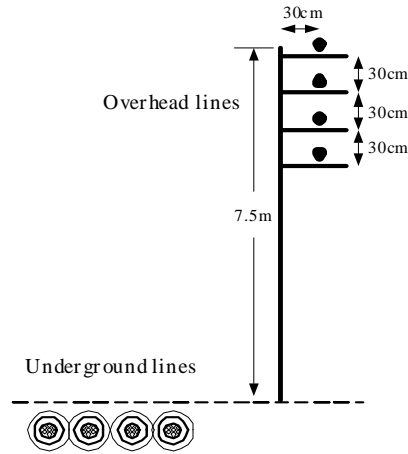Fig. 4: The low voltage network of Ebrahim-Abad

Fig. 5: The formation network lines

According to above figure, the network consists of six underground and overhead feeders which the underground lines increased the importance of the fault currents determination. In this part, the fault currents and the results of them on profile of network voltage is discussed.

The single-line-to-ground and the three-line-to-ground are known as conventional and dangerous types of faults, respectively and that is why their short-circuit currents analysis are the most common type among them. In this section, the current of these both faults analysis in points of network and the result illustrated in Table 1.

Table 1: The short-circuit current in some points of network

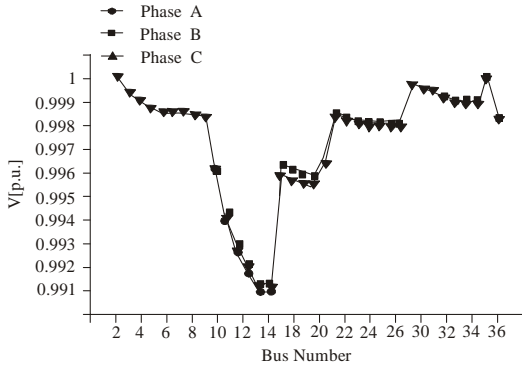|	Type and fault current (KA)					
	Single line	Double lines to ground		Three lines to ground		
Bus No.	Ia	Ib	Ic	Ia	Ib	Ic
8	1.762	2.136	2.436	2.351	2.510	2.526
13	2.410	3.042	3.223	3.134	3.499	3.359
25	4.370	15.782	4.710	4.963	5.711	5.208
29	13.457	15.355	13.071	14.068	14.398	13.775

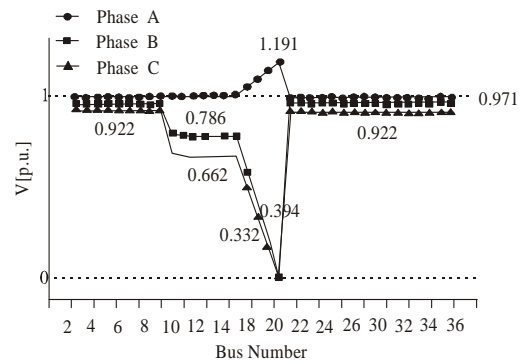Fig. 6: The voltage profile of the network before the fault occurrence

Fig. 7: The voltage profile after the double line to ground fault to bus number 20

According to the values in table 1, it can be seen that the rate of the three-line-to-ground fault in the all four points are more than the single-line-to-ground fault. Therefore, according to above figure and the networks parameters, buses 29 and 8 are selected as the farthest and nearest fault buses among the 4 chosen points, respectively and as the matter of fact, they both has the maximum and the minimum values of the fault currents, respectively.

After identifying the rate of the fault the BCBV matrix is used to analysis the result of these currents on the buses voltages of the network.

In the previous fault mode, the profile voltage of the network is illustrated in Figure 6. The step shape of Fig. 6 is because of the numbering buses and configuration of network. Thereby, the voltage profile of network is presented in the Fig. 7 and 8 due to the assumption of the double lines to ground fault on phases a and b occurs to the bus numbers 20 and 28 as nearest and farthest buses, respectively.

Regarding to achieved results, in addition to the zero voltage phases of busses located in the fault branch, the serious drops occurs in the whole network on phases a and b where the rate of this drop related to the power of short-circuit and location of fault.

According to Fig. 4 and 7, distance of bus 20 from the substation bus, reduced the short circuit power. Because of this reduction, this short circuit affected only
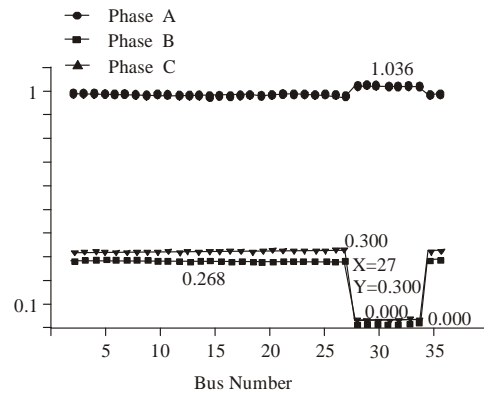

Fig. 8: The voltage profile after the occurrence of the double-line-to-ground fault in bus number 28

buses 16 to 20 seriously. Whereas Fig. 8 shows that by occurrence similar fault on bus 28, the voltage of all buses on this feeder would be zero. Therefore the difference of distance fault location from substation is very important in short circuit analysis.

It should be note that, the double line to ground fault occurrence reduces the voltage magnitude of fault phases but increases the voltage of phase without fault, in which this statement is shown in Fig. 7 and 8 clearly. So, this problem should be considered in distribution system planning beside of short circuit analysis.

CONCLUSION

In this study, a new model is provided to analysis the unsymmetrical faults where all the types of short circuit can be evaluated by only one model. This model dramatically reduces the volume of calculations and required memory to provide software. One of the clear properties of this model is the high capability for simulating all types of the fault impedance Z_f.

By surveying about simulation results, it can be seen that, the only factor to avoid the fault current, is the rate of the impedance between substation bus and the short-circuit location.

Therefore, it is very significant to use certain model to evaluate the line impedances, which is considered in this study.

Also, another property of the presented method is to identify the fault current carrying branches. Regarding to the expand usage of the underground lines in low voltage networks, this property is very useful to have an optimize selection of cable, type and the size of fuses.

ACKNOWLEDGMENT

REFERENCES

Alvarado, F.L., K.M. Sao and M.K. Enns, 1985. A fault program with macros, monitors and direct compensation in mutual groups. IEEE Trans. PAS, 104(5): 1109-1120.

Brandwajn, V.W. and F. Tinney, 1985. Generalized method of fault analysis. IEEE Trans. PAS, 104(6): 1301-1306.

Chen, T.H., M.S. Chen, W.J. Lee, P. Kotas and P.V. Olinda, 1992. Distribution system short circuit analysis-rigid approach. IEEE T. Power Syst., 7(1): 444 450.

Gross, G. and H.W. Hong, 1982. A two step compensation method for solving short circuit problems, IEEE Trans. Power App. Syst. PAS, 101(6): 1322-1331.

Kersting, W.H., 2002. Distribution System Modeling and Analysis. CRC Press, New York.

Miu, N.K. and M. Yiming, 2002. Network equivalent models for short circuit. IEEE Conf., 2: 862-865.

Ciric, R.M., L.F. Ochoa, A. Padilla-Feltrin and H. Nouri, 2005. Fault analysis in four-wire distribution networks. Proc. IEE. Gen. Trans. Distr., 152(6): 977-985.

Roy, L., 1979. Generalized polyphase fault-analysis program: Calculation of cross-country fault. Proc. IEE., 126(10): 995-1001.

Teng, J., 2005. Systematic short-circuit-analysis method for unbalanced distribution systems. IEE Proc. Gene. Trans. Distr., 152(4): 549-555.

Teng, J., 2010. Unsymmetrical short-circuit fault analysis for weakly meshed distribution systems. IEEE T. Power Syst., 25(1): 96-105.

Zhang, X., F. Soudi, D. Shirmohamdi and C. Cheng, 1995. A distribution short circuit analysis approach using hybrid compensation method. IEEE T. Power Syst., 10(4): 2053-2059.