



Pareto optimization of a five-degree of freedom vehicle vibration model using a multi-objective uniform-diversity genetic algorithm (MUGA)

N. Nariman-Zadeh^{a,b,c,*}, M. Salehpour^b, A. Jamali^b, E. Haghgoo^a

^a Faculty of Mechanical Engineering, Islamic Azad University, Takestan Branch, Takestan, Iran

^b Department of Mechanical Engineering, Faculty of Engineering, The University of Guilan, P.O. Box 3756, Rasht, Iran

^c Intelligent-based Experimental Mechanics Center of Excellence School of Mechanical Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran

ARTICLE INFO

Article history:

Received 23 January 2009

Accepted 31 August 2009

Available online 31 October 2009

Keywords:

Vehicle vibration model

Pareto

Genetic algorithm

Multi-objective optimization Diversity

MUGA

ABSTRACT

In this paper, a new multi-objective uniform-diversity genetic algorithm (MUGA) with a diversity preserving mechanism called the ϵ -elimination algorithm is used for Pareto optimization of a five-degree of freedom vehicle vibration model considering the five conflicting functions simultaneously. The important conflicting objective functions that have been considered in this work are, namely, seat acceleration, forward tire velocity, rear tire velocity, relative displacement between sprung mass and forward tire and relative displacement between sprung mass and rear tire. Further, different pairs of these objective functions have also been selected for 2-objective optimization processes. The comparison of the obtained results with those in the literature demonstrates the superiority of the results of this work. It is shown that the results of 5-objective optimization include those of 2-objective optimization and, therefore, provide more choices for optimal design of a vehicle vibration model.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

Vibration control of machines worked by engines has attracted great amount of research activities during last decades. In particular, vehicles' motions are influenced by the harmful effects of vibrations caused by engines and roads which have a pivotal role in driver's comfort. Griffin et al. (1982), Rakheja (1985), and Barak (1991) have shown that the interior vibration of a vehicle has a significant effect in comfort and road holding capability. In the case of reducing this type of vibration, manufacturers' efforts have led to produce a suspension system that is installed between road excitation and vehicle body. Bouazara (1997) studied the influence of suspension system parameters on the vibration of vehicle's model. In the same year, Hrovat (1991) used a three-dimensional vibration model instead of the two-dimensional model to get more exact results. Crolla (1992) applied a semi-active suspension model for improving the performance of vehicle. Bouazara and Richard (1996) presented their vibration model in three-dimensional space demonstrating that this model has a good estimation of the vehicle behavior. Bouazara and Richard (2001) also studied three types of suspension system (active, semi-active and passive) for an eight-degree of freedom vibration model. In his works, Bouazara combined all the

performance criteria to form an objective function for a single-objective optimization process. For this purpose, he used weighting coefficients to adjust comfort and road holding capability criteria in the single-optimization design process. Gündoğdu (2007) presented an optimization of a four-degree of freedom quarter car seat and suspension system using genetic algorithms to determine a set of parameters to achieve the best performance of the driver's seat. The desired objective was proposed as the minimization of a multi-objective function formed by the combination of not only suspension displacement and tire deflection but also the head acceleration and crest factor (CF), which is not practiced as usual by designers. Alkhatib et al. (2004) applied genetic algorithm (GA) to the optimization problem of a linear one-degree of freedom (1-DOF) vibration isolator mount and the method was extended to the optimization of a linear quarter car suspension model. The optimum solution was obtained numerically by utilizing GA and employing a cost function that sought minimizing absolute acceleration RMS (root mean square) sensitivity to changes in RMS of relative displacement.

In fact, optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. In multi-objective optimization problems, there are several objectives or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that as one objective function improves, another deteriorates. Therefore, there is no single optimal solution that is best with

* Corresponding author at: Department of Mechanical Engineering, Faculty of Engineering, The University of Guilan, P.O. Box 3756, Rasht, Iran.

E-mail address: nnzadeh@guilan.ac.ir (N. Nariman-Zadeh).

respect to all the objective functions. Instead, there is a set of optimal solutions, well-known as Pareto optimal solutions (Srinivas and Deb, 1994; Fonseca and Fleming, 1993; Coello Coello and Christiansen, 2000; Coello Coello et al., 2002), which distinguishes significantly the inherent natures between single-objective and multi-objective optimization problems. V. Pareto (1848–1923) was the French–Italian economist who first developed the concept of multi-objective optimization in economics Pareto (1896). The concept of a Pareto front in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. Evidently, changing the vector of design variables in such a Pareto optimal solutions consisting of these non-dominated solutions would not lead to the improvement of all objectives simultaneously. Consequently, such change leads to a deterioration of at least one objective to an inferior one. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space. The inherent parallelism in evolutionary algorithms makes them suitably eligible for solving MOPs. The early use of evolutionary search is first reported in 1960s by Rosenberg (1967). Since then, there has been a growing interest in devising different evolutionary algorithms for MOPs. Among these methods, the vector-evaluated genetic algorithm (VEGA) proposed by Schaffer (1985), Fonseca and Fleming's genetic algorithm (FFGA) Fonseca and Fleming (1993), non-dominated sorting genetic algorithm (NSGA) by Srinivas and Deb (1994) and strength Pareto evolutionary algorithm (SPEA) by Zitzler and Thiele (1998) and the Pareto-archived evolution strategy (PAES) by Knowles and Corne (1999) are the most important ones. A very good and comprehensive survey of these methods has been presented in Coello Coello, 1999 and in Khare et al. (2003). Coello Coello <http://www.lania.mx/~ccoello/EMOO/> has also presented an Internet-based collection of many papers as very good and easily accessible literature resources. Basically, both NSGA and FFGA as Pareto-based approaches use the revolutionary non-dominated sorting procedure originally proposed by Goldberg (1989). There are two important issues that have to be considered in such evolutionary multi-objective optimization methods: driving the search towards the true Pareto optimal set or front and preventing premature convergence or maintaining the genetic diversity within the population Toffolo and Benini (2003). The lack of elitism was also a motivation for modification of that algorithm to NSGA-II Goldberg (1989), in which a direct elitist mechanism, instead of a sharing mechanism, has been introduced to enhance the population diversity. This modified algorithm represents the state-of-the-art in evolutionary MOPs Coello Coello and Becerra (2003). A comparison study among SPEA and other evolutionary algorithms on several problems and test functions showed that SPEA clearly outperforms the other multi-objective EAs (Zitzler et al., 2000). Some further investigations reported in reference (Toffolo and Benini, 2003) demonstrated, however, that the elitist variant of NSGA (NSGA-II) equals the performance of SPEA.

In this paper, a new multi-objective uniform-diversity genetic algorithm (MUGA) with a diversity preserving mechanism called the ε -elimination algorithm is used for multi-objective optimization of a five-degree of freedom vehicle vibration model. The conflicting objective functions that have been considered for minimization are, namely, seat acceleration (\ddot{z}_c), forward tire velocity (\dot{z}_1), rear tire velocity (\dot{z}_2), relative displacement between sprung mass and forward tire (d_1) and relative displacement between sprung mass and rear tire (d_2). The design variables used in the optimization of vibration are, namely, seat damping coefficient (c_{ss}), vehicle suspension damping coefficient (c_{s1} and

c_{s2}), seat stiffness coefficient (k_{ss}), vehicle suspension stiffness coefficient (k_{s1} and k_{s2}) (and seat position in relation to the center of mass (r). Various pair-wise 2-objective optimization and 5-objective optimization processes are performed. The inclusion of the results by 5-objective optimization is verified using the results of different 2-objective optimization processes through some overlay graphs of the Pareto fronts. Prominently, it is shown that a trade-off optimum design can be verified from those Pareto fronts obtained by multi-objective optimization process. Finally, the superiority of time domain vibration performance of such design point is shown in comparison with those given in the literature.

2. Multi-objective Pareto optimization

Multi-objective optimization, which is also called multi-criteria optimization or vector optimization, has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions (Coello Coello and Christiansen, 2000; Jamali et al., 2010). In general, it can be mathematically defined as

Find the vector $X^* = [x_1^*, x_2^*, \dots, x_n^*]^T$ to optimize

$$F(X) = [f_1(X), f_2(X), \dots, f_k(X)]^T, \quad (1)$$

subject to m inequality constraints

$$g_i(X) \leq 0, \quad i = 1 \text{ to } m, \quad (2)$$

and p equality constraints

$$h_j(X) = 0, \quad j = 1 \text{ to } p, \quad (3)$$

where $X^* \in \mathfrak{R}^n$ is the vector of decision or design variables, and $F(X) \in \mathfrak{R}^k$ is the vector of objective functions, which must each be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions.

2.1. Definition of Pareto dominance

A vector $U = [u_1, u_2, \dots, u_k] \in \mathfrak{R}^k$ is dominant to vector $V = [v_1, v_2, \dots, v_k] \in \mathfrak{R}^k$ (denoted by $U < V$) if and only if $\forall i \in \{1, 2, \dots, k\}$, $u_i \leq v_i \wedge \exists j \in \{1, 2, \dots, k\}: u_j < v_j$. In other words, there is at least one u_j which is smaller than v_j whilst the remaining u s are either smaller or equal to corresponding v s.

2.2. Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in \mathfrak{R}^n satisfying Eqs. (2) and (3)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) < F(X)$. Alternatively, it can be readily restated as

$\forall i \in \{1, 2, \dots, k\}, \forall X \in \Omega - \{X^*\} f_i(X^*) \leq f_i(X) \wedge \exists j \in \{1, 2, \dots, k\}: f_j(X^*) < f_j(X)$. In other words, the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

2.3. Definition of a Pareto set

For a given MOP, a Pareto set \mathcal{P}^* is a set in the decision variable space consisting of all the Pareto optimal vectors $\mathcal{P}^* = \{X \in \mathfrak{R}^n : \nexists X' \in \mathfrak{R}^n : F(X') < F(X)\}$. In other words, there is no other X' as a vector of decision variables in Ω that dominates any $X \in \mathcal{P}^*$.

2.4. Definition of a Pareto front

For a given MOP, the Pareto front \mathcal{PF}^* is a set of vector of objective functions, which are obtained using the vectors of decision variables in the Pareto set \mathcal{P}^* , that is $\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), \dots, f_k(X)) : X \in \mathcal{P}^*\}$. In other words, the Pareto front \mathcal{PF}^* is a set of the vectors of objective functions mapped from \mathcal{P}^* .

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most of the difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find the Pareto front or quantification of the importance of each objective using numerical weights. In this way, the original non-dominated sorting procedure given by Goldberg (1989) was the catalyst for several different versions of multi-objective optimization algorithms (Srinivas and Deb, 1994; Fonseca and Fleming, 1993). However, it is very important that the genetic diversity within the population be preserved sufficiently. This main issue in MOPs has been addressed by many related research works (Toffolo and Benini (2003)). Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II (Deb et al. (2002)) has been used recently in a wide area of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different levels of Pareto frontiers. However, the crowding approach in such state-of-the-art MOEA (Coello Coello and Becerra (2003)) is not efficient as a diversity preserving operator, particularly in problems with more than 2 objective functions (Nariman-Zadeh et al., 2005; Jamali et al., 2010).

In this work, a new multi-objective uniform-diversity genetic algorithm method called MUGA is presented so that it can safely be used for any number of objective functions (particularly for more than 2 objectives) in MOPs.

2.5. Multi-objective uniform-diversity genetic algorithm (MUGA)

The multi-objective uniform-diversity genetic algorithm (MUGA) uses non-dominated sorting mechanism together with a ϵ -elimination diversity preserving algorithm to get Pareto optimal solutions of MOPs more precisely and uniformly.

2.5.1. The non-dominated sorting method

The basic idea of sorting of non-dominated solutions originally proposed by Goldberg (1989), which has been used in different evolutionary multi-objective optimization algorithms, as in NSGA-II (Deb et al. (2002)), has been adopted here. The algorithm simply compares each individual in the population with others to determine its non-dominancy. Once the first front has been found, all its non-dominated individuals are removed from the main population and the procedure is repeated for the subsequent fronts until the entire population is sorted and non-dominantly divided into different fronts.

A sorting procedure to constitute a front could be simply accomplished by comparing all the individuals of the population and including the non-dominated individuals in the front. Such procedure can be simply represented as following steps:

1. Get the population (pop)
2. Include the first individual {ind(1)} in the front P^* as $P^*(1)$, let $P^*_{size}=1$;

3. Compare other individuals {ind(j), $j=2, \dots, \text{Pop_size}$ } of the Pop with $\{P^*(K), K=1, P^*_{size}\}$ of the P^* ;
 If ind(j) < $P^*(K)$ replace the $P^*(K)$ with ind(j)
 If $P^*(K) < \text{ind}(j)$, $j=j+1$, continue comparison;
 Else include ind(j) in P^* , $P^*_{size}=P^*_{size}+1$, $j=j+1$, continue comparison;
4. End of front P^* ;

It can be easily seen that the number of non-dominated solutions in P^* grows until no further one is found. At this stage, all the non-dominated individuals so far found in P^* are removed from the main population and the whole procedure of finding another front may be accomplished again. This procedure is repeated until the whole population is divided into different ranked fronts. It should be noted that the first rank front of the final generation constitute the final Pareto optimal solution of the multi-objective optimization problem.

2.5.2. The ϵ -elimination diversity preserving approach

In the ϵ -elimination diversity approach that is used to replace the crowding distance assignment approach in NSGA-II (Deb et al. (2002)), all the clones and ϵ -similar individuals are recognized and simply eliminated from the current population. Therefore, based on a value of ϵ as the elimination threshold, all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ϵ -similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ϵ -similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ϵ -elimination approach is depicted in Fig. 1. Evidently, the clones and ϵ -similar individuals are replaced from the population by the same number of new randomly generated individuals. Meanwhile, this will additionally help to explore the search space of the given MOP more effectively. It is clear that such replacement does not appear when a front rather than the entire population is truncated for ϵ -similar individual.

2.5.3. The main algorithm of MUGA

It is now possible to present the main algorithm of MUGA which uses both non-dominated sorting procedure and ϵ -elimination diversity preserving approach, which is given in

```

ε-elim= ε-elimination(pop) // pop includes design variables and
                             objective function
i=1; j=1;
get K (K=1 for the first front);
While i,j < pop_size

e(i,j)= || X(i,:),X(j,:) || / || X(i,:) || ; X(i),X(j) ∈ P*k ∪ P*k

//finding mean value of ε within pop.

end
ε=mean(e);
i=1;
until i+1<pop_size;
j=i+1
    until j<pop_size
        if e(i,j)<ε
            then {pop}={pop}/ {pop(j)} //remove the ε-similar individual
            j=j+1
        end
    end
i=i+1
end
    
```

Fig. 1. Pseudo-code of ϵ -elimination.

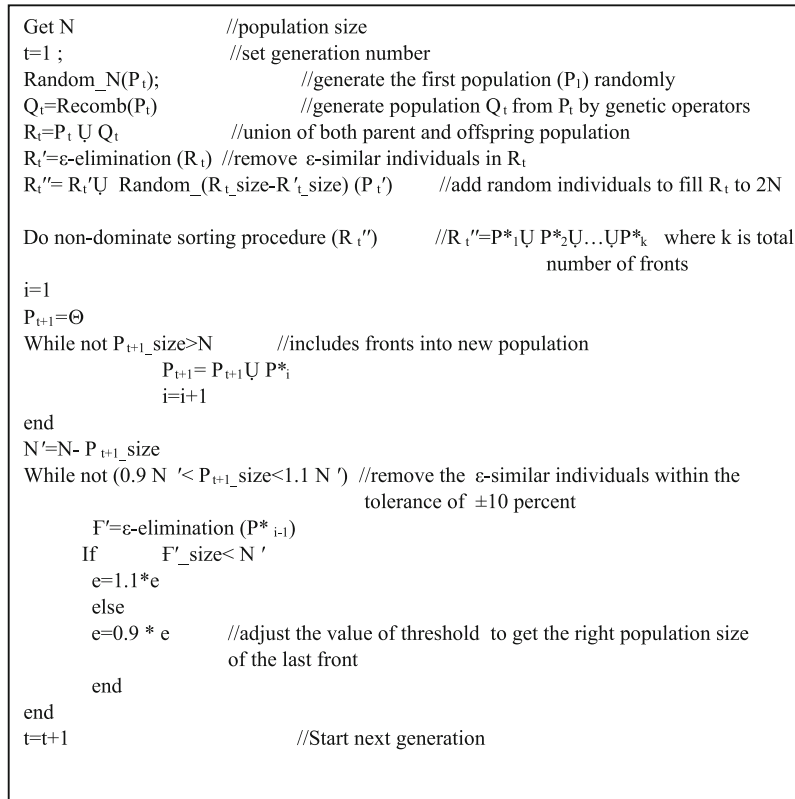


Fig. 2. The main algorithm of MUGA.

Fig. 2. It first initiates a population randomly. Using genetic operators, another same size population is then created. Based on the ε -elimination algorithm, the whole population is then reduced by removing ε -similar individuals. At this stage, the population is re-filled by randomly generated individuals which helps to explore the search space more effectively. The whole population is then sorted using non-dominated sorting procedure. The obtained fronts are then used to constitute the main population. It must be noted that the front, which must be truncated to match the size of the population is also evaluated by ε -elimination procedure to identify the ε -similar individuals. Such procedure is only performed to match the size of the population within ± 10 percent deviations to prevent excessive computational effort to population size adjustment. Finally, unless the number of individuals in the first rank front is changing in certain number of generations, randomly created individuals are inserted in the main population occasionally (e.g. every 20 generations of having non-varying first rank front).

3. Multi-objective optimization of vehicle vibration model

A five-degree of freedom vehicle with passive suspension, which is adopted from (Bouazara (1997)) is shown in Fig. 3. This model is composed of one sprung mass that joints to three unsprung masses (indicate tires and seat). Moreover, the effect of degrees of freedom, linear motion (in vertical direction for sprung and unsprung masses) and rotating motion (pitching motion) for sprung mass, in terms of acceleration, velocity and movement, are considered in formulation of motion equations. Parameters M_1 , m_2 , m_c , m_s , I_s , k_{p1} , k_{p2} , l_1 and l_2 , which denote the vehicle's fixed parameters are expressed as forward tire mass, rear tire mass, seat mass, sprung mass, momentum inertia of sprung mass, forward tire stiffness coefficient, rear tire stiffness coefficient, forward and

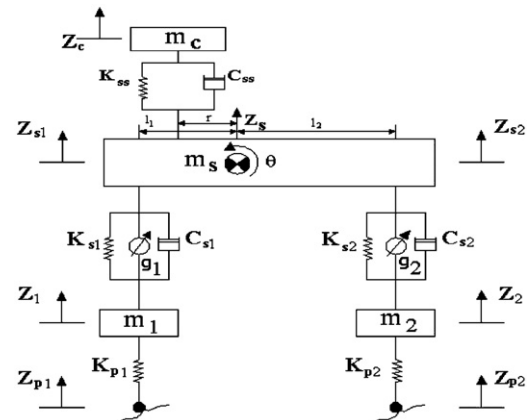


Fig. 3. Vehicle vibration five-degree of freedom model with passive suspension adopted from Ref. Bouazara (1997).

rear tires position in relation to the center of mass, respectively. Design variables k_{ss} , k_{s1} and k_{s2} , c_{ss} , c_{s1} and c_{s2} and r denote seat stiffness coefficient, stiffness coefficients for vehicle suspension, seat damping coefficient, damping coefficients for vehicle suspension and seat position in relation to the center of mass, respectively. Further, subscripts 1 and 2 indicate tire axes. It is also necessary to observe that in this case study, seat type is composed of a linear spring and damper. The dynamic of this model is excited by a double bump as shown in Fig. 4.

3.1. The governing dynamic differential equations of motion

The linearized differential equations of motion, with respect to the degrees of freedom and for small angle θ , are derived by the

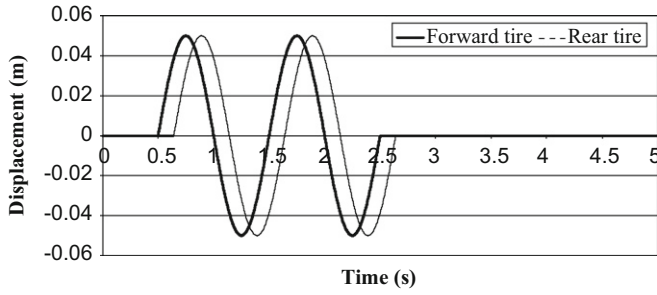


Fig. 4. Double bumps excitation.

Table 1

The values of fixed parameters of the model.

l_1	1.011 m
l_2	1.803 m
m_1	40 kg
m_2	35.5 kg
m_c	75 kg
m_s	730 kg
I_s	130 kg m ²
k_{p1}	175,500 N/m
k_{p2}	175,500 N/m

use of Newton–Euler equations and can be written as follows Bouazara (1997):

$$z_{ps} = z_s - r\theta \quad (4)$$

$$z_{s1} = z_s - l_1\theta \quad (5)$$

$$z_{s2} = z_s + l_2\theta \quad (6)$$

$$F_{ss} = k_{ss}(z_c - z_{ps}) + c_{ss}(\dot{z}_c - \dot{z}_{ps}) \quad (7)$$

$$F_{s1} = k_{s1}(z_{s1} - z_1) + c_{s1}(\dot{z}_{s1} - \dot{z}_1) \quad (8)$$

$$F_{s2} = k_{s2}(z_{s2} - z_2) + c_{s2}(\dot{z}_{s2} - \dot{z}_2) \quad (9)$$

$$m_c \ddot{z}_c = -F_{ss} \quad (10)$$

$$m_s \ddot{z}_s = -F_{s1} - F_{s2} + F_{ss} \quad (11)$$

$$I_s \ddot{\theta} = l_1 F_{s1} - l_2 F_{s2} - r F_{ss} \quad (12)$$

$$m_s \ddot{z}_1 = F_{s1} - k_{p1}(z_1 - z_{p1}) \quad (13)$$

$$m_s \ddot{z}_2 = F_{s2} - k_{p2}(z_2 - z_{p2}) \quad (14)$$

where, z_c , z_s , z_{s1} and θ are vertical seat displacement, vertical displacement of the central gravity of the sprung mass, vertical displacement of the ends of the sprung mass and rotating motion (pitching motion), respectively. Further, \dot{z}_c , \dot{z}_i and \dot{z}_{si} represent vertical seat velocity, vertical tires velocity and vertical velocity of the ends of the sprung mass, respectively. \ddot{z}_c , \ddot{z}_s , \ddot{z}_i and $\ddot{\theta}$ denote vertical seat acceleration, vertical acceleration of the central gravity of the sprung mass, vertical tires acceleration and rotating acceleration (pitch acceleration), respectively. Lastly, z_{p1} and z_{p2} represent the excitation via road double bumps, as shown in Fig. 4.

It is supposed that the vehicle moves at constant velocity $v=20$ m/s over a double bump, and it is further assumed that the rear tire follows the same trajectory as the front tire with a delay of $\Delta t=(l_f+l_r)/v$. The input values of fixed parameters are presented at Table 1 Bouazara (1997).

In this paper, $50,000 \leq k_{ss} \leq 150,000$, $10,000 \leq k_{s1} \leq 20,000$, $10,000 \leq k_{s2} \leq 20,000$, $1000 \leq c_{ss} \leq 4000$, $500 \leq c_{s1} \leq 2000$, $500 \leq c_{s2} \leq 2000$ and $0 \leq r \leq 0.5$ are observed as 7 design variables to be

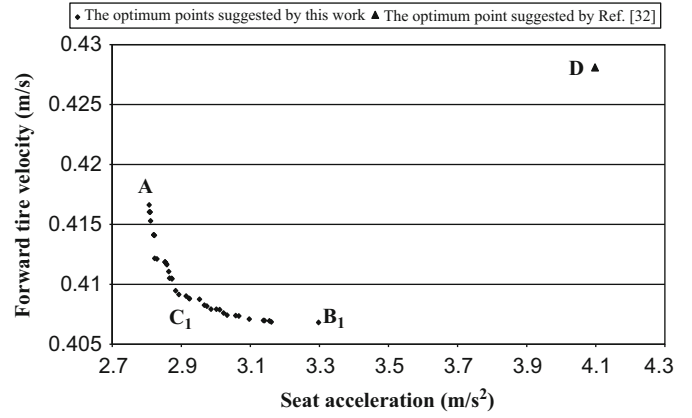


Fig. 5. Pareto front for seat acceleration and forward tire velocity in 2-objective optimization.

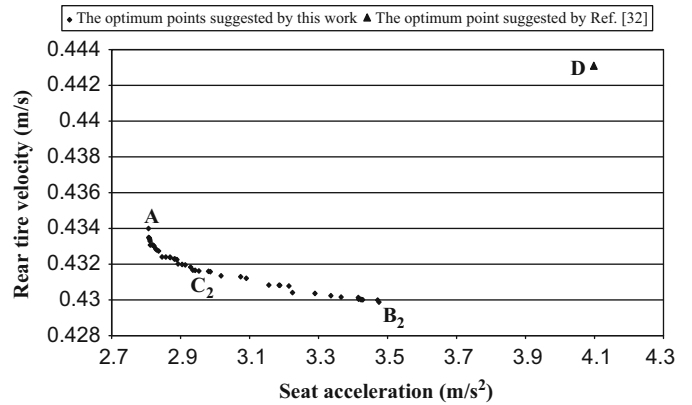


Fig. 6. Pareto front for seat acceleration and rear tire velocity in 2-objective optimization.

optimally found based on multi-objective optimization of 5 different objective functions, namely, seat acceleration (\ddot{z}_c), forward tire velocity (\dot{z}_1), rear tire velocity (\dot{z}_2), relative displacement between sprung mass and forward tire (\dot{z}_2) and relative displacement between sprung mass and rear tire (d_2).

3.2. Two-objective optimization of vehicle vibration model

In this section, the multi-objective uniform-diversity genetic algorithm (MUGA) (Jamali et al., 2010) presented in previous sections is used for multi-objective design of vehicle model which has been shown in Fig. 4. For this purpose, 4 different pairs out of 10 possible pairs of 5 objectives are considered in various 2-objective optimization processes. Such pairs of objectives to be optimized separately have been chosen as (\ddot{z}_c, \dot{z}_1) , (\ddot{z}_c, \dot{z}_2) , (\ddot{z}_c, d_1) and (\ddot{z}_c, d_2) , which stand for seat acceleration with forward tire velocity, rear tire velocity, relative displacement between sprung mass and forward tire and relative displacement between sprung mass and rear tire, respectively. Evidently, it can be observed that all of the objective functions are minimized in those sets of objective functions. A population of 80 individuals with a crossover probability of 0.9 and mutation probability of 0.1 has been used in 240 generations. Pareto fronts of each chosen pair of 2 objectives have been shown through Figs. 5–8. It is clear from all of the figures that obtaining a better value of one objective would normally cause a worse value of another objective. However, if the

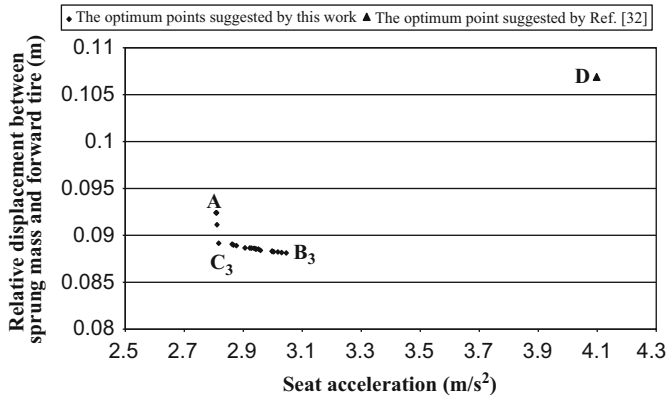


Fig. 7. Pareto front for seat acceleration and relative displacement between sprung mass and forward tire in 2-objective optimization.

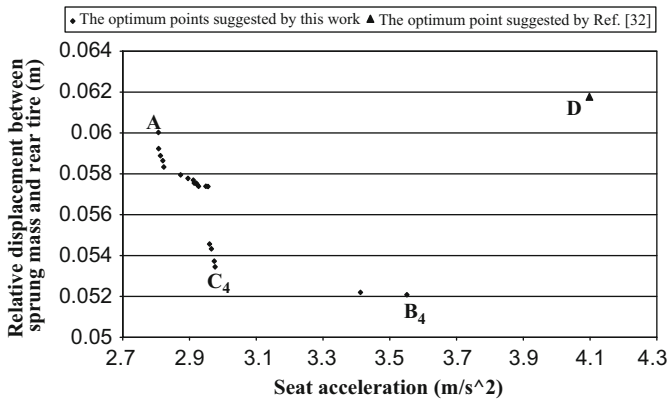


Fig. 8. Pareto front for seat acceleration and relative displacement between sprung mass and rear tire in 2-objective optimization.

set of decision variables is selected based on each of a Pareto front, it will lead to the best possible combination of that pair of objectives. In other words, if any other set of decision variables is chosen, the corresponding values of pair of objectives will locate a point inferior to the corresponding Pareto front. Such inferior area in the space of the objective functions for Figs. 5–8 are in fact top/right sides.

Fig. 5 depicts the Pareto front of seat acceleration and forward tire velocity representing different non-dominated optimum points with respect to the conflicting objectives. In this figure, points A and B₁ stand for the best seat acceleration and the best forward tire velocity, respectively. It should be noted that all the optimum design points in this Pareto fronts are non-dominated and could be chosen by a designer. It is clear from this figure that choosing a better value for any objective function in these Pareto fronts would cause a worse value of another objective function. Clearly, there are some important optimal design facts between these objective functions that can be readily observed in that Pareto front. In Fig. 5, point C₁ is the point which demonstrates an important optimal design fact. Optimum design point C₁ obtained in this paper exhibits a small increase in forward tire velocity in comparison with that of point B₁ (the design with the least forward tire velocity) whilst its seat acceleration improves about 13%. In fact, the trade-off design point, C₁, would not have been obtained without the use of the Pareto optimum approach presented in this paper.

Such non-dominated Pareto fronts of the other chosen sets of objective functions have been shown in Figs. 6–8. Design point A stands for the best seat acceleration whilst points B₂, B₃ and B₄ represent the best \dot{z}_2 , d_1 and d_2 , respectively. Similarly, the trade-

off designing points C₂, C₃ and C₄ are the design points which demonstrate the important optimal design fact. With more careful observation, it is found that the values of seat accelerations improve about 18%, 9% and 20% with a small increase in other objective functions from points B₂ to C₂, B₃ to C₃ and B₄ to C₄ in Figs. 6–8, respectively. In all these figures, point D represents the optimum design obtained in Bouazara (1997) which it is very evident that is drastically dominated by all Pareto fronts shown in these figures.

The time behavior of the seat acceleration of the trade-off design points of these figures and the one of the optimum point proposed in Bouazara (1997) are shown for comparison in Figs. 9–12. It is obvious from these figures that the values of seat accelerations of the design points obtained in this paper are better than those by the design point D given in Bouazara (1997). The corresponding values of objective functions and design variables of these optimum design points and the point one in Bouazara (1997) are also given in Table 2.

The Pareto optimum approach of this paper reveals some interesting and informative design aspects that may not have been found without multi-objective optimization. However, all such important and worthy information regarding the trade-off design point can also be simply discovered using a 5-objective Pareto optimization instead of 4 (or more) separate 2-objective optimization processes.

3.3. Five-objective optimization of vehicle vibration model

A multi-objective optimization design of vehicle model including all 5 objectives simultaneously can offer more choices

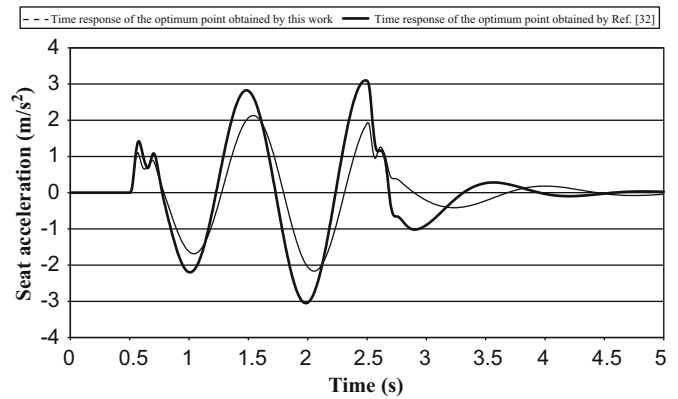


Fig. 9. Time responses of seat acceleration of design points C₁ and D.

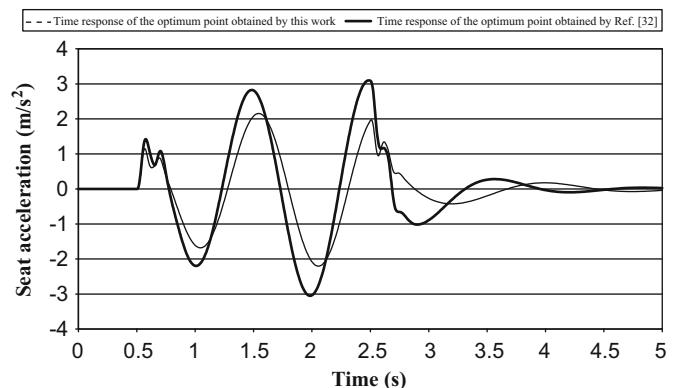


Fig. 10. Time responses of seat acceleration of points C₂ and D.

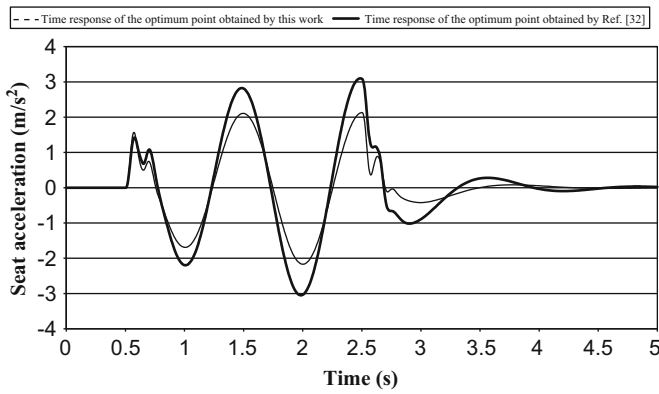


Fig. 11. Time responses of seat acceleration of points C₃ and D.

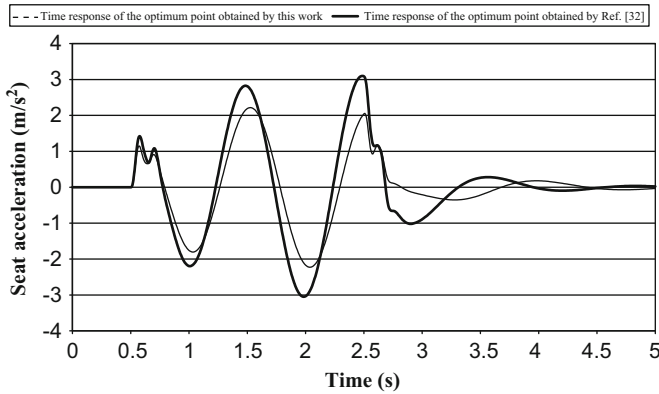


Fig. 12. Time responses of seat acceleration of points C₄ and D.

for a designer. Moreover, such 5-objective optimization can subsume all the 2-objective optimization results presented in the previous section. This will allow finding a trade-off optimum design point from the view of all 5 objective functions simultaneously. Therefore, in this section, 5 objective functions, namely, seat acceleration (\ddot{z}_c), forward tire velocity (\dot{z}_1), rear tire velocity (\dot{z}_2), relative displacement between sprung mass and forward tire (d_1) and relative displacement between sprung mass and rear tire (d_2) are chosen for multi-objective optimization in which all of them are minimized simultaneously. A population of 80 individuals with a crossover probability of 0.9 and mutation probability of 0.1 has been used in 240 generations.

Fig. 13 depicts the non-dominated individuals of 5-objective optimization in the plane of ($\ddot{z}_c - \dot{z}_1$) together with the results to 2-objective optimization found in previous section. Such non-dominated individuals of both 5- and 2-objective optimizations have alternatively been shown in the plane of ($\ddot{z}_c - \dot{z}_2$), ($\ddot{z}_c - d_1$) and ($\ddot{z}_c - d_2$) in Figs. 14–16, respectively. It should be noted that there is a single set of individuals as a result of 5-objective optimization of \ddot{z}_c , \dot{z}_1 , \dot{z}_2 , d_1 and d_2 that are shown in different planes together with the corresponding 2-objective optimization results obtained in previous section. Therefore, there are some points in each plane that may dominate others in the case of 5-objective optimization. However, these individuals are all non-dominated when considering all 5 objectives simultaneously. By careful investigation of the results of 5-objective optimization in each plane, the Pareto fronts of the corresponding 2-objective optimization previously found can now be verified in these figures. It can be readily observed that the results of such 2-objective optimization include the Pareto fronts of each 2-objective optimization and provide, therefore, more optimal choices for the designer.

Table 2
The values of objective functions and their associated design variables of the optimum points of this work and the one of Ref. Bouazara (1997).

	A	B ₁	C ₁	B ₂	C ₂	B ₃	C ₃	B ₄	C ₄	F	D
k_{ss} (N/m)	146,863	57,451	111,177	52,745.1	131,961	57,058.8	134,706	92,745.1	98,235.3	144,902	105,630
c_{ss} (Ns/m)	3941.18	2517.65	3858.82	3635.29	3400	3588.24	2505.88	2941.18	2917.65	2788.24	2548
k_{s1} (N/m)	10,039.2	10,039.2	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	14,958
k_{s1} (Ns/m)	1917.65	964.706	1264.71	670.588	1282.35	2000	2000	576.471	1311.77	1294.12	1956
k_{s2} (N/m)	10,313.7	18,196.1	10,117.7	10,000	10,000	10,000	10,078.4	18,313.7	18,313.7	10,196.1	14,985
k_{s2} (Ns/m)	1958.82	1476.47	1852.94	1658.82	1511.77	1676.47	1511.77	1994.12	1994.12	1982.35	1967
r (m)	0.5	0.3412	0.49,804	0.05,883	0.46,863	0.11,373	0.5	0.48,431	0.5	0.49,608	0.288
\ddot{z}_c (m/s ²)	2.80,701	3.29,747	2.92,268	3.4751	2.95,293	3.04,553	2.8165	3.5512	2.97,674	2.89,291	4.09,800
\dot{z}_1 (m/s)	0.41,663	0.40,682	0.40,883	0.41,082	0.40,922	0.41,800	0.41,754	0.41,187	0.40,914	0.40,92	0.4281
\dot{z}_2 (m/s)	0.43,399	0.45,235	0.43,223	0.42,988	0.43,162	0.43,302	0.43,327	0.44,511	0.44,637	0.43,282	0.44,309
d_1 (m)	0.09,241	0.14,567	0.12,095	0.17,393	0.11,961	0.08,813	0.08,915	0.1861	0.12,033	0.11,942	0.10,689
d_2 (m)	0.06,003	0.06,915	0.06,17	0.06,841	0.07,515	0.07,247	0.07,734	0.05,208	0.05,345	0.05,795	0.06,177

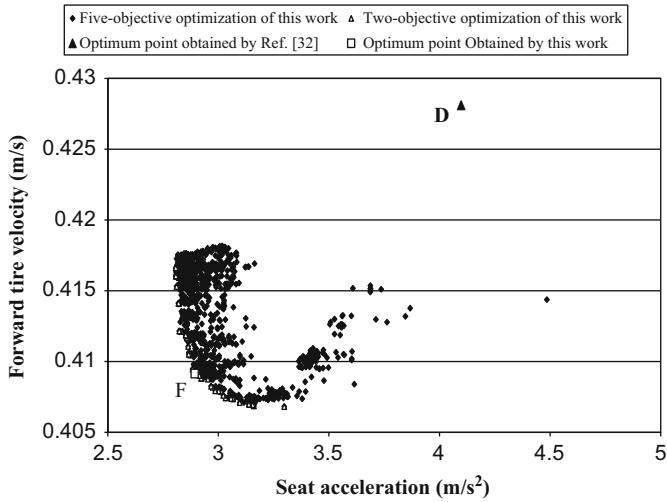


Fig. 13. Seat acceleration with forward tire velocity in both 5-objective and 2-objective optimizations.

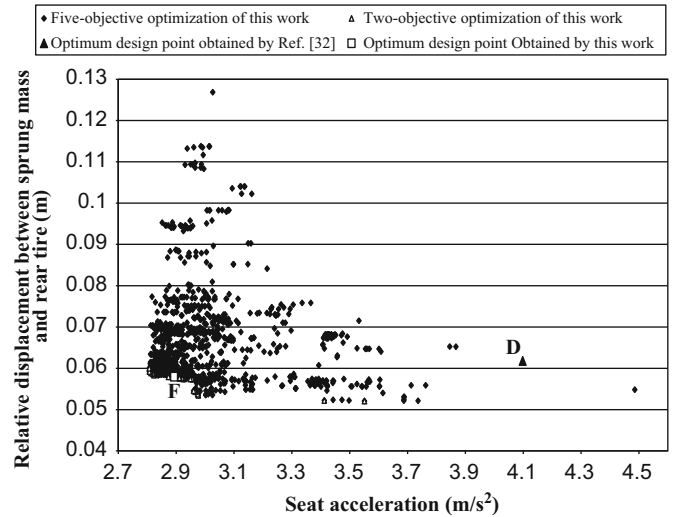


Fig. 16. Seat acceleration with relative displacement between sprung mass and rear tire in both 5-objective and 2-objective optimizations.

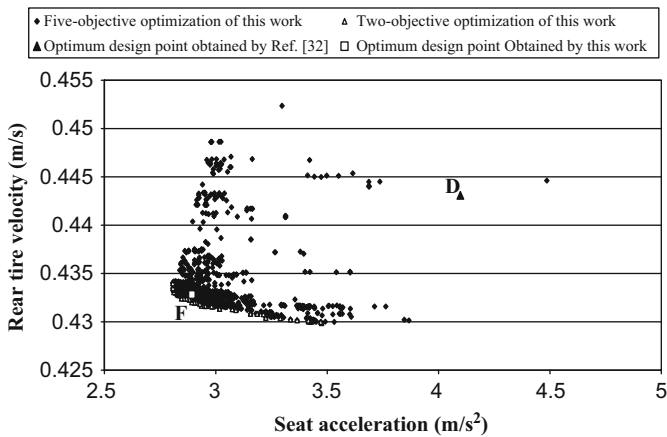


Fig. 14. Seat acceleration with rear tire velocity in both 5-objective and 2-objective optimizations.

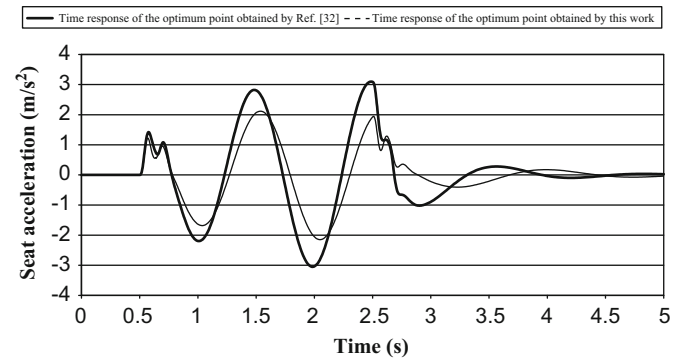


Fig. 17. Time response of seat acceleration of points F and D.

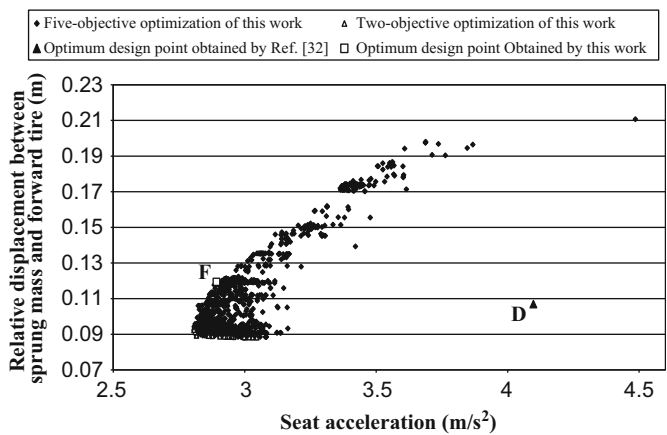


Fig. 15. Seat acceleration with relative displacement between sprung mass and forward tire in both 5-objective and 2-objective optimizations.

It may now be desired to obtain an optimum design point out of all non-dominated 5-objective optimization process somehow satisfying all 5 objective functions. In other words, each of the obtained design points given in previous section is acceptable based on the pertinent 2 objective functions, but there is no reason that such an optimum design point existed in one of the

Pareto fronts (e.g. plane of $(\ddot{z}_c - \dot{z}_1)$) is located in the other Pareto fronts too (e.g. plane of $(\ddot{z}_c - \dot{z}_2)$).

It is now possible to seek an optimum design point which is located almost on all Pareto fronts of Figs. 13–16. This can be simply achieved by mapping of the values of objective functions of all non-dominated points into interval 0 and 1. Using the sum of these values for each non-dominated points, the design point F simply represents the minimum of those values. It can be seen that the design point F is located on all Pareto fronts approximately. Moreover, it can be seen that point F significantly dominates point D proposed in Bouazara (1997). In the plane of $(\ddot{z}_c - d_1)$ both point F and D are non-dominated, but it is clear that with a small increase in d_1 from point D to F, \ddot{z}_c improves about 42%. The time responses of the seat acceleration of the proposed optimum point of this work and the one given in Bouazara (1997) are shown in Fig. 17. It is clear that the time response of point F is superior to that of point D. The values of objective functions and their associated design variables of F are shown in Table 2. The comparison of the values of objective functions associated with the optimum design point F obtained from the 5-objective functions optimization process with those of the 2-objective functions optimization processes of design points C_1 , C_2 and C_3 given in Table 2 also demonstrates the relative superiority of design point F.

Therefore, such multi-objective optimization of seat acceleration (\ddot{z}_c), forward tire velocity (\dot{z}_1), rear tire velocity (\dot{z}_2), relative

displacement between sprung mass and forward tire (d_1) and relative displacement between sprung mass and rear tire (d_2) provide optimal choices of design variables based on Pareto non-dominated points.

4. Conclusion

A multi-objective genetic algorithm with a recently developed diversity preserving mechanism has been used to optimally design vehicle vibration model. The objective functions which conflict with each other were selected as seat acceleration (\ddot{z}_c), forward tire velocity (\dot{z}_1), rear tire velocity (\dot{z}_2), relative displacement between sprung mass and forward tire (d_1) and relative displacement between sprung mass and rear tire (d_2). The multi-objective optimization of vehicle model led to the discovering of some important trade-offs among those objective functions. The superiority of the obtained optimum design points was shown in comparison with those reported in the literature. Such multi-objective optimization of vehicle model could unveil very important design trade-offs between conflicting objective functions which would not have been found otherwise. Further, it has been shown that the results of 5-objective optimization include those of 2-objective optimization in terms of Pareto frontiers and provide, consequently, more choices for optimal design.

References

- Alkhatib, R., Nakhaie Jazar, G., Golnaraghi, M.F., 2004. Optimal design of passive linear suspension using genetic algorithm. *Journal of Sound and Vibration* 275, 665–691.
- Barak, P., 1991. Magic numbers in design of suspensions for passenger cars. Passenger Car Meeting, Tennessee, SAE Paper 911921, pp. 53–88.
- Bouazara, M., 1997. Etude et Analyse de la Suspension Active et Semi-active des Vehicules Routiers. Ph.D. Thesis, Universite Laval, Canada.
- Bouazara, M., Richard, M.J., 1996. An optimal design method to control the vibrations of suspensions for passenger cars, in: International Mechanical Engineering Congress and Exposition: The Winter Annual Meeting of ASME, Atlanta, DSC 58, pp. 61–68.
- Bouazara, M., Richard, M.J., 2001. An Optimization Method Designed to Improve 3-D Vehicle Comfort and Road Holding Capability Through the Use of Active and Semi-active Suspensions, *European Journal of Mechanics-A/Solids* 20 (3), 509–520.
- Coello Coello, C.A., <http://www.lania.mx/~ccoello/EMOO/>.
- Coello Coello, C.A., 1999. A comprehensive survey of evolutionary based multi-objective optimization techniques, *Knowledge and Information Systems. An International Journal* 1 (3), 269–308.
- Coello Coello, C.A., Becerra, R.L., 2003. Evolutionary multiobjective optimization using a cultural algorithm. In: *Swarm, IEEE (Ed.), Intelligence Symposium, IEEE Service Center, Piscataway, pp. 6–13.*
- Coello Coello, C.A., Christiansen, A.D., 2000. Multiobjective optimization of trusses using genetic algorithms. *Computers & Structures* 75, 647–660.
- Coello Coello, C.A., Van Veldhuizen, D.A., Lamont, G.B., 2002. In: *Evolutionary Algorithms for Solving Multi-Objective Problems*. Kluwer Academic, Dordrecht.
- Crolla, D.A., 1992. Semi-active Suspension Control for a Full Vehicle Model, SAE Technical Paper Series 911904, pp. 45–51.
- Deb, K., Agrawal, S., Pratap, A., Meyarivan, T., 2002. A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6 (2), 182–197.
- Fonseca, C.M., Fleming, P.J., 1993. Genetic algorithms for multi-objective optimization: formulation, discussion and generalization. In: Forrester, S. (Ed.), *Proceedings of the Fifth International Conference on Genetic Algorithms*, Morgan Kaufmann, San Mateo, CA, pp. 416–423.
- Goldberg, D.E., 1989. *Genetic algorithms in search, Optimization, and Machine Learning*. Addison-Wesley, Reading, MA.
- Gündogdu, Ö., 2007. Optimal seat and suspension design for a quarter car with driver model using genetic algorithms. *International Journal of Industrial Ergonomics* 37 (4), 327–332.
- Griffin, M., Parsons, K., Whitham, E., 1982. Vibration and comfort. IV. Application of experimental results. *Ergonomics* 25, 721–739.
- Hrovat, D., 1991. Optimal active suspensions for 3d vehicle models. In: *Proceedings of the American Control Conference*, vol. 2, Arizona, USA, pp. 1534–1541.
- Jamali, A., Hajiloo, A., Nariman-zadeh, N., 2010. Reliability-based robust Pareto design of linear state feedback controllers using a multi-objective uniform-diversity genetic algorithm (MUGA). *Expert Systems with Applications* 37 (1), 401–413.
- Khare, V., Yao, X., Deb, K., 2003. Performance scaling of multi-objective evolutionary algorithms. In: *Proceedings of Second International Conference on Evolutionary Multi-Criterion Optimization, EMO '03, Portugal*.
- Knowles, J., Corne, D., 1999. The Pareto archived evolution strategy: a new baseline algorithm for multiobjective optimization. In: *Proceedings of the 1999 Congress on Evolutionary Computation*. IEEE Service Center, Piscataway, NJ, pp. 98–105.
- Nariman-Zadeh, N., Atashkari, K., Jamali, A., Pilechi, A., Yao, X., 2005. Inverse modeling of multi-objective thermodynamically optimized turbojet engine using GMDH-type neural networks and evolutionary algorithms. *Engineering Optimization* 37 (26), 437–462.
- Pareto, V., 1896. In: *Cours D'economie Politique*. Rouge, Lausanne, Switzerland.
- Rakheja, S., 1985. Computer-aided Dynamic Analysis and Optimal Design of Suspension Systems for Off-road Tractors. PhD Thesis, Concordia University, Canada.
- Rosenberg, R.S., 1967. Simulation of Genetic Populations with Biochemical Properties, PhD Thesis, University of Michigan, Ann Harbor, MI.
- Schaffer, J.D., 1985. Multiple objective optimization with vector evaluated genetic algorithms. In: Grefenstette, J.J. (Ed.), *Proceedings of the First International Conference on Genetic Algorithms and Their Applications*, Lawrence Erlbaum, London, pp. 93–100.
- Srinivas, N., Deb, K., 1994. Multi-objective optimization using non-dominated sorting in genetic algorithms. *Evolutionary Computation* 2 (3), 221–248.
- Toffolo, A., Benini, E., 2003. Genetic diversity as an objective in multiobjective evolutionary algorithms. *Evolutionary Computation* 11 (2), 151–167.
- Zitzler, E., Thiele, L., 1998. An evolutionary algorithm for multiobjective optimization: the strength Pareto approach. Technical Report 43, Computer engineering and communication network Lab, Swiss Federal Institute Of Technology, Zurich.
- Zitzler, E., Deb, K., Thiele, L., 2000. Comparison of multi objective evolutionary algorithms: empirical results. *Evolutionary Computation* 8 (2), 173–175.