On the Probability of Finding Non-interfering Paths in Wireless Multihop Networks

David R. Cheriton School of Computer Science
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1
{spwaharte,rboutaba}@uwaterloo.ca

Abstract. Multipath routing can improve system performance of capacity-limited wireless networks through load balancing. However, even with a single source and destination, intra-flow and inter-flow interference can void any performance improvement. In this paper, we show that establishing non-interfering paths can, in theory, leverage this issue. In practice however, finding non-interfering paths can be quite complex. In fact, we demonstrate that the problem of finding two non-interfering paths for a single source-destination pair is NP-complete. Therefore, an interesting problem is to determine if, given a network topology, non-interfering multipath routing is appropriate. To address this issue, we provide an analytic approximation of the probability of finding two non-interfering paths. The correctness of the analysis is verified by simulations.

1 Introduction

The landscape of network communications has evolved over the last years with the emergence and deployment of protocols enabling more flexible and reliable means to communicate wirelessly over single or multiple hops. Deploying wireless backbone networks (also referred to as Wireless Mesh Networks) is becoming increasingly popular in view of the reduced initial investment cost and flexibility of deployment. Wireless sensor networks have also received great attention from the research community, which can be explained by their application potential in various areas such as military applications, environment monitoring, etc. [1]. However dealing with interference in a shared transmission environment still remains a challenging task. Environmental noise can be partly responsible for the quality degradation of data transmissions. In addition to that, the necessity to share the transmission channel among wireless nodes within the same vicinity significantly contributes to decreasing the nominal capacity available to wireless nodes. It becomes therefore crucial to develop appropriate mechanisms to alleviate the effect of these limitations on the system performance.

Multipath routing has been put forward as a solution to leverage the capacity limitations in wireless networks. Spreading traffic flows over multiple paths has been demonstrated to achieve a better load balancing than single path routing potentially leading to an increase in the nominal achievable throughput [3] [5]. But

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the resulting performance gain greatly depends on the choice of the routing paths. If the paths are at a distance such that the traffic on one path interferes with the traffic on the other path, the throughput improvement overall becomes negligible [7]. It is also important to factor in the choice of the routing approach the cost of the paths establishment. As the control overhead increases with the number of paths [6], it can potentially void the benefit of multipath routing. Therefore, estimating the probability of finding non-interfering paths given a network topology can help deciding whether for a particular network, multipath routing is an appropriate approach and if it can improve the system performance at all.

To support our analysis, we first show that routing over non-interfering paths can lead to a better network utilization. We then study the complexity of finding two non-interfering paths and show that the problem is actually NP-complete. Therefore, in order the reduce the cost of establishing multiple paths, we provide an analytic estimate of the probability of finding two non-interfering paths as a function of the network density. This result can subsequently be used to evaluate the suitability of implementing a non-interfering multipath routing algorithm for a given network topology and to adjust accordingly the routing strategy. We validate the correctness of our analysis through simulations.

The remainder of this paper is organized as follows. We describe our analysis of the complexity of 2-path routing in Section 2. The analytical derivation of the probability of finding two non-interfering paths is presented in Section 3. Section 4 summarizes our contributions and concludes this paper.

2 Complexity Analysis of Disjoint Path Routing with Interference Constraints

The problem we are studying consists in finding multiple non-interfering paths between a source-destination pair. To analyze the complexity of the problem, we restrict our analysis to the case in which only two paths are set up between a source and destination node. We refer to this problem as *2-path routing*. We prove the NP-completeness of this problem, i.e. that is, the polynomial time solution for this problem is unlikely to exist.

Definition 1. *Given a directed graph $G(V, E)$ and two nodes $(s, t) \in V$, 2-path routing consists in finding two paths P_1 and P_2 between s and t such that: 1/ all the nodes in P_1 and all the nodes in P_2 form a connected graph; and 2/ there exists no edge between a node in P_1 and a node in P_2.*

Theorem 1. *2-path routing is NP-complete.*

Proof. Omitted due to space restrictions.

3 Probability of Finding Two Non-interfering Paths

3.1 General Methodology

Given a source node and a destination node, two paths minimizing the interference shoud consist of a set of nodes such that the nodes on one path do not interfere with the nodes on the other path, except for the first-hop nodes and last-hop nodes. We assume that nodes locations are known. In order to avoid the choices of multiple paths with significantly different lengths (and consequently to avoid the burden of handling traffic flows with different end-to-end delays), the approach we considered is to define a guard band between the source and the destination and to set up paths outside this band but as close as possible to it (within a distance ϵ of the guard band). Therefore, the paths are guaranteed not to interfere. ϵ is a tunable parameter that can be adjusted depending on the network density. More details on the tuning of this parameter are given at the end of this section. We therefore need to compute:

1. The probability P_1 to find two non-interfering nodes at the first hop.
2. The probability P_2 that two paths exist after the first hop and before the last hop, with the constraint that the nodes along each path do not interfere with each other.

3.2 Computation of P_1

The first condition to satisfy is to find two non-interfering nodes in the transmission area of the source. We assume that for each path the next-hop nodes are located in the half-plane oriented towards the destination (i.e. no backward transmission). For the first hop, this constraint added to the physical transmission range limitation restricts the feasible geographic location of the first-hop nodes to half a disk. We need therefore to compute:

1. the probability $P_{11}(k)$ to have k nodes in half a disk;
2. the probability $P_{12}(k)$ that at least two of these k nodes are separated by a distance greater than R (transmission range).

P_1 can be determined by: $P_1 = \sum_{k=1}^{\infty} P_{11}(k) P_{12}(k)$

Computation of $P_{11}(k)$. We assume that the nodes are uniformly distributed over an area A with a density ρ. They have a transmission range R and their positions are independent of each other. For a large number of nodes, the probability that k nodes are located in a given area A can be approximated with a Poisson distribution (Eq. 1) [4].

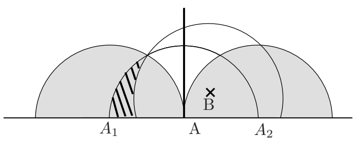
$$P(\text{number of nodes} = k) = \frac{(\rho A)^k}{k!} e^{-\rho A} \tag{1}$$

Computation of $P_{12}(k)$. Let A be the source node and B a node randomly located in the half-disk centered at A with respective polar coordinates $(0, 0)$ and (r, θ). We refer to the extreme points of the diameter of the half-disk as A_1 and A_2. In order not to interfere with B, a node should respect the following conditions:

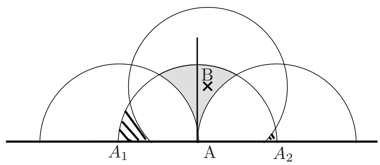
1. to be at transmission range of A
2. not to be at transmission range of B

To find the probability that at least two nodes randomly located in half a disk do not interfere, we adopt the following method. We choose a point B in the transmission area of node A and determine the probability that there exists at least one node non-interfering with B (therefore located in one of the dashed areas as shown in Fig 1) and this, for all possible positions of B. Depending on the position of B, two scenarios can occur:

- **Case 1:** B is located at a distance less than R from either A_1 or A_2 (Fig. 1 (a)). One single solution area exists. This corresponds to the case where $\cos(\theta) \geq \frac{R}{2r}$.
- **Case 2:** B is located at a distance greater than R from both A_1 and A_2 (Fig. 1 (b)). Two solution areas exist. This corresponds to the case where $\cos(\theta) \leq \frac{R}{2r}$.



(a) Case 1: 1 feasible solution area



(b) Case 2: two feasible solution areas

Fig. 1. Computation of $P_{12}(k)$ (the non-interfering zones are dashed)

A general formulation of the probability to find two nodes distanced by at least R can be expressed as follows.

Theorem 2. *Let us assume that there are N nodes at transmission distance of A. The probability P that at least two of these N nodes are at a distance greater than R is:*

$$P = 1 - \int_{r=0}^{R} \int_{\theta=0}^{\frac{\pi}{2}} \frac{2}{\pi R}(1 - (\frac{2(A_{inter}(r,\theta))}{\pi R^2})^{n-1})\partial r \partial \theta \tag{2}$$

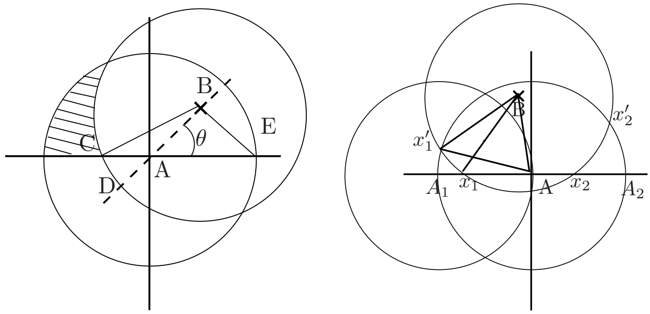
where $A_{inter}(r,\theta)$ is the interference area of a node with polar coordinate (r,θ) in the solution domain (half-disk).

In order to determine the actual value of the interference area $A_{inter}(r,\theta)$, we need to breakdown the computation into the two cases previously described.

Case 1: One feasible region Let N be the number of nodes in the half-disk area obtained by properly choosing the network density so that the probability to find at least 2 nodes tends to 1. If we consider one node (Node B) among these N nodes, the probability that at least one of the remaining $N-1$ nodes is at least at a distance R from B can be determined by computing the complement of the probability that all the nodes are at a distance less than R from B.

Let us first calculate the intersection between the coverage areas of A restricted to the half-disk oriented towards the destination node and the coverage of B. The intersection area between the disk centered at A and the disk centered at B form a lens whose area is referred to as A_{lens}. This area can be straightforwardly computed geometrically as follows:

$$A_{lens} = 2R^2 \arccos(\frac{r}{2R}) - \frac{r}{2}\sqrt{4R^2 - r^2} \tag{3}$$



(a) Case 1: 1 feasible region (b) Case 2: 2 feasible regions

Fig. 2. Computation of the non-interfering zone (dashed)

We can also observe that since A and B are at transmission range of each other, these points are necessarily located in the lens whose area has been previously computed. In particular, we can establish the following relation: $S_{ACD} = S_{BCD} - S_{ABC}$

The area formed by BCD consists of a disk section that can be directly computed:

$$S_{BCD} = \frac{\widehat{BCD} * R^2}{2} \tag{4}$$

To compute $\widehat{BCD}$, let us define $AC = x$. By construction, we have $\widehat{BAC} = \pi - \theta$, $AB = r$ and $BC = R$. Using the law of cosine, we determine x:

$$x = -r\cos(\theta) + \sqrt{R^2 - r^2(\sin(\theta))^2}$$

$\widehat{BCD}$ can therefore be deducted using the same method.

To obtain the area of ABC, we apply Heron's formula:

$$s = \frac{x + r + R}{2}$$

$$S_{ABC} = \sqrt{s(s-r)(s-x)(s-R)} \tag{5}$$

By combining Eq. 4 and Eq. 5, we obtain:

$$S_{ACD} = \arccos(\frac{R^2 + r^2 - x^2}{2Rr})\frac{R^2}{2} - \sqrt{s(s-r)(s-x)(s-R)} \tag{6}$$

We can therefore compute the intersection area:

$$S_{inter}(r, \theta) = \frac{A_{lens}}{2} - S_{ACD} + R^2\frac{\theta}{2} \tag{7}$$

Finally, the probability that at least one of these $N-1$ nodes does not fall in this area is:

$$P_{case1} = 1 - (\frac{S_{inter}(r, \theta)}{\frac{\pi R^2}{2}})^{N-1} \tag{8}$$

Case 2: Two feasible regions

In this case, node B is at a distance at least R away from A_1 and A_2. Without lack of generality, let us assume that B is located in the same quarter of disk as A_1 (Fig. 2 (b)).

The disk centered at B cut the x-axis in two points x_1 and x_2 such that $x_1 < x_2$. Obviously a solution zone exists in this area only if $|x_1| < R$. Let x'_1 be the intersection point with the smallest x-coordinate between the circle centered at A and the circle centered at B.

The solution area is therefore bounded by $A_1x_1x'_1$. By geometric considerations we can observe that:

$$S_{A_1x_1x'_1} = S_{ABx'_1A_1} - S_{ABx'_1x_1}$$

By calculating $S_{ABx'_1A_1}$ and $S_{ABx'_1x_1}$, we find the solution area.

Due to space limitations, we omit the details of the computation that follows similar steps as in the previous case. The probability to find two non-interfering nodes in this second case is:

$$P_{case2} = 1 - (\frac{\pi R^2}{2} - S_{A_1x_1x'_1} - S_{A_2x_2x'_2})^{N-1} \tag{9}$$

Evaluation. To evaluate the accuracy of the upper bound of our analysis, we compared the results obtained by our derivation with the ones obtained through simulations by computing the distance between two pairs of nodes in a random distribution. We ran the experiments 1000 times for various network densities. The results of the simulations are depicted in Fig. 3.

We can see that our analysis provides a close approximation of the probability of finding two non-interfering paths when compared with simulations.

3.3 Computation of P_2

For the subsequent hops along each path, it is sufficient to determine the probability that each node has at least one neighbor towards the destination while respecting the interference constraint, that is to say that the chosen node should

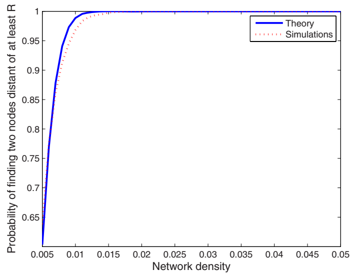


Fig. 3. Probability of finding two non-interfering nodes

not interfere with the nodes on the other path [2]. In addition, for quality of service purposes, it is necessary to limit the number of hops along each path. This can be achieved by constraining the possible location of the next-hop node. Considering that each node has a fixed transmission range, we can adjust the width ϵ of the band depending on the network density and quality-of-service constraints (Eq. 10). For a network density of 8e-4 nodes/m^2 and a transmission range of 250m, the probability to find a next-hop node with a probability of 95% is achieved with a value of epsilon of $15m$.

$$P(\text{at least 1 neighbor}) = 1 - P(\text{no neighbor})$$

$$P(\text{at least 1 neighbor}) = 1 - e^{-\rho R \epsilon}$$

$$\epsilon = \frac{-\ln(1 - P(\text{at least 1 neighbor})))}{\rho R} \tag{10}$$

Let h be the number of hops between the source and the destination, $h \geq 2$. The probability to find 2 totally-disjoint paths can be expressed as:

$$P_2 = (1 - e^{-\rho R \epsilon})^{2(h-2)} \tag{11}$$

4 Conclusion

With the increasing deployment of wireless networks, the necessity to cope with limited link capacity becomes more and more stringent. This problem is further exacerbated when transmission occurs over multiple hops. The gain in flexibility in network deployment is counterbalanced by a reduction in available throughput. This is directly linked to the problem of interference. All devices in the same vicinity have to share the transmission medium, which consequently reduces the capacity available to each. To tackle this issue, some research works have focused on developing routing protocols that could account for interference and improve the network performance. Multipath routing has been proposed as an alternate solution to single path routing due to its potential to improve the network throughput by balancing the load more evenly. However, to take full advantage of this routing method, interference has to be accounted for during the choice of the routing paths.

The contributions of this work are the followings. First we studied the complexity of finding two paths between a given source and destination and we proved the NP-completeness of this problem. Then, we analytically derived the probability of finding two non-interfering paths given a certain network density.

The results obtained are noteworthy as they can be directly applied to the choice of a routing strategy. The network density and therefore the probability of finding non-interfering paths can lead to different routing decisions. The results derived in this paper can consequently enable adaptive routing strategies depending on the network characteristics.

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