

PID feedback controller used as a tactical asset allocation technique: The G.A.M. model[☆]

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Abstract

The objective of this paper is to illustrate a tactical asset allocation technique utilizing the PID controller. The proportional–integral–derivative (PID) controller is widely applied in most industrial processes; it has been successfully used for over 50 years and it is used by more than 95% of the plants processes. It is a robust and easily understood algorithm that can provide excellent control performance in spite of the diverse dynamic characteristics of the process plant.

In finance, the process plant, controlled by the PID controller, can be represented by financial market assets forming a portfolio. More specifically, in the present work, the plant is represented by a risk-adjusted return variable. Money and portfolio managers' main target is to achieve a relevant risk-adjusted return in their managing activities. In literature and in the financial industry business, numerous kinds of return/risk ratios are commonly studied and used.

The aim of this work is to perform a tactical asset allocation technique consisting in the optimization of risk adjusted return by means of asset allocation methodologies based on the PID model-free feedback control modeling procedure. The process plant does not need to be mathematically modeled: the PID control action lies in altering the portfolio asset weights, according to the PID algorithm and its parameters, Ziegler-and-Nichols-tuned, in order to approach the desired portfolio risk-adjusted return efficiently.

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1. Objective

The aim of this work is to verify the efficiency of proportional–integral–derivative (PID) controller application to financial management portfolio activity. The PID controller is widely applied in most industrial

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processes; it has been successfully used for over 50 years and it is used by more than 95% of the plant processes. It is a robust and easily understood algorithm that can provide excellent control performance in spite of the diverse dynamic characteristics of the process plant [1]. In particular, the aim of this paper is to illustrate a new application of a controller system, the G.A.M. model; in this model, a PID controller is applied to a portfolio in order to perform a tactical asset allocation on such portfolio. This work verifies the efficacy and efficiency of PID model-free controlling methods as an asset allocation technique. A PID model-free approach requires the PID controller to get tuned in loop with the given plant to be controlled; whereas, other approaches build a model of the plant and, accordingly, decide the parameters of the controller by deterministic or optimization methods [2]. The controller provides signals to dynamically manage the asset mix of a portfolio. This paper aims to verify the efficiency of the PID controlling action in order to achieve a specific performance measured by a risk-adjusted return indicator. Following the G.A.M. model description, the adopted methodology, results and conclusions will be presented.

2. The G.A.M. model

The G.A.M. model is a technique for allocating a plurality of assets in a portfolio, via tactical asset allocation in order to achieve a long-term target over a desired time horizon. More particularly, the present work relates to a method and system for asset allocation of assets having different degrees of risk and return consisting in the optimization of portfolio risk-adjusted return based on the PID free-model feedback control modeling procedure [3].

In industrial environments such as chemical plants, power plants, and engineering industries, numerous processes need to be tightly controlled to comply with the required specifications for the resulting products. The control of processes in the plant is provided by a process control methodology and apparatus, which typically senses input/output variables. The process control apparatus then compares these variables against desired predetermined values (set-point). If unexpected differences exist or get formed during the plant process, changes are made to the input variables to return the output variables to a predetermined desired range (set-point). Most commonly, the control of a process is provided by a PID controller. PID controllers provide satisfactory control behavior for many single input/single output systems.

The general concept of proportional plus integral plus derivative (PID) control is well known in the art. This type of control was first described in a mathematical context early in the 20th century. In the process control system, a measured process variable is compared with a set point or desired value in a controller to generate an error signal. A control signal generated in the controller as a function of the error signal is applied to a final control element, which regulates the flow of energy into or out of the process.

The four basic types of controllers are (1) the off-on, (2) the integral, (3) the proportional, and (4) the derivative. The off-on type, as the name implies, applies a full-on or full-off signal to the final control element in order to maintain the measured variable near the set point. The proportional controller generates a control signal, which is directly proportional to the magnitude of the error signal. In both the off-on and the proportional controller, the control signal applied to the final control element is zero when the error signal is zero. The integral controller generates a control signal, which is the integral of the error signal. Thus, the rate of change of the control signal is directly proportional to the magnitude of the error signal and the control signal is equal to the area under the time–error signal curve. Since the integral action controller generates a signal, which is a function of the history of the error signal, it maintains a new level after the error returns to zero following a disturbance. This gives the integral controller the capability of eliminating off-set caused by load changes, and for this reason integral control is often referred to as automatic reset control. The not yet fully appreciated advantage of integral control, in addition to the automatic reset feature, is that abrupt changes in the set point are applied gradually to the process through the integral action. In other words, the control signal lags the set point change. The derivative controller generates a control signal, which is the derivative of the error signal. This means that the control signal is directly proportional to the rate of change of the error signal. It is clear then that the control signal generated by this type of controller is equal to zero except when the error is changing, and thus will remain equal to zero in the presence of a constant error signal. The derivative mode generates a control signal, which leads the error signal and, for this reason, is useful in initiating a change in operating conditions in systems having prolonged time constants. In practice, two or

more modes of control may be combined to provide the desired control for a particular process. The combination of the proportional, integral, and derivative modes is now widely used to provide a control system, which is efficient under almost any condition. The relative contributions of the various control actions must be considered in adapting a three-mode controller to a particular process in such a manner as to optimize the response time and the stability of the system. In all of the above controllers, the control action or actions are applied to the error signal and is thus referred to as error control.

The G.A.M. model is a method and system for allocating a plurality of assets in portfolio, via tactical asset allocation in order to achieve a long-term target over a desired time horizon. More particularly, the present work relates to a method and system for asset allocation of assets having different degrees of risk and return consisting in the optimization of portfolio risk-adjusted return based on the PID feedback control modeling procedure. In finance, the process plant, controlled by PID, can be represented by financial market assets forming a portfolio. The assets mix of the portfolio determines a risk-adjusted return of the portfolio. Rebalancing the portfolio correspondingly alters its risk-adjusted return. In various aspects, this present work provides methods and systems as a novel approach to tactical asset allocation. The rebalancing is not performed upon the determination of a forecasted vector of expected returns [4] and the determination of a covariance matrix. The rebalancing is dictated by an asset selection technique consisting in the optimization of risk-adjusted return by means of asset allocation methodologies based on the PID feedback control modeling procedure. The controlled process plant, namely the risk-adjusted return variable, does not need to be modeled by mathematical closed form equations, nor assumptions, linearization, and simplification procedures on the dynamics of the plant are required. The PID control action lies in altering the portfolio asset weights, according to the PID algorithm and its parameters; the PID parameters are appropriately tuned by the largely used Ziegler-and-Nichols method [5]; such controlling system and apparatus is targeted to approach the desired portfolio risk-adjusted return. The selection and rebalancing of the various assets is performed dynamically at a predetermined frequency. The process is performed for the whole duration of the planned time horizon.

The methodology of rebalancing the portfolio following the PID controller is innovative and naturally fitting. In fact, the PID controller tends to take into consideration the natural phenomena occurring in the dynamics of time series: for example, if too much push has been given towards more risky assets in order to increase return, then memory, inertia, and other dynamic effects cause overshoot and excessive acceleration; in that phase, the PID actuator attempts to anticipate such behavior by counteracting ahead of time in order to smooth out the previous effect. The innovation is exactly in the controlling action over the uncertain behavior of the plurality of assets comprising the portfolio. In addition, the innovative part outlines the possibility of avoiding modeling and formulating simplifying assumptions on the dynamics of financial assets underlying the controlled portfolio. The controller, despite the unavailability of the mathematical model for the plant [3], attempts to regulate the dynamics of the portfolio by rebalancing the weights of the different assets in such a way to force the portfolio risk-adjusted return to approach the set point. The same controlling mechanism that a temperature controller does in stabilizing and setting a desired temperature when thermal insulation is very poor and external agents act and modify the temperature. Fig. 1 depicts a block diagram illustrating a PID controller and the controlled process in feedback configuration in the continuous-time domain.

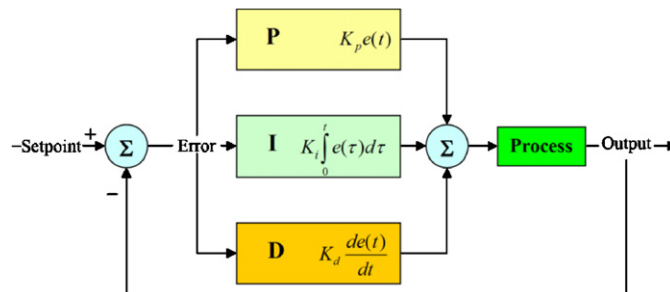


Fig. 1. PID controller block diagram.

The familiar continuous-time PID algorithm for system output, $u(t)$, is given by Ref. [3]

$$u(t) = k_p \left(e(t) + \frac{1}{K_i} \int e(\tau) d\tau + K_d \frac{de(t)}{dt} \right) \quad (1)$$

where $u(t)$ is output; $e(t)$ is error; k_p the proportional constant; K_i the integral constant; K_d the derivative constant;

The analogous PID controller algorithm in discrete-time, position form, can be easily derived by finite difference approximations of derivatives and integrals, yielding

$$u_n = k_p \left(e_n + \frac{1}{K_i} \sum_{m=0}^n e_m + K_d (e_n - e_{n-1}) \right), \quad (2)$$

where the derivative term is approximated by a backward difference and the integral term by a sum using rectangular integration. Index n refers to the time instant. For this present work, the spread-out industry standard simple-lag implementation for the integral term has been utilized as shown in the following recurrence relation [3]:

$$u_n = (k_p e_n + K_i (e_n - u_{n-1}) + u_{n-1} + K_d (e_n - e_{n-1})), \quad (3)$$

where the integration is performed by using a simple-lag. The more common discrete-time algorithm, used in the industry is the velocity form [3]

$$u_n = u_{n-1} + a e_n + b e_{n-1} + c e_{n-2},$$

where

$$a = k_p + k_i + k_d; \quad b = -[k_p + 2k_d]; \quad c = k_d. \quad (4)$$

The discrete time z -transfer function is [3]

$$D(z) = \frac{a + bz^{-1} + cz^{-2}}{1 - z^{-1}},$$

where

$$a = k_p + k_i + k_d; \quad b = -[k_p + 2k_d]; \quad c = k_d \quad (5)$$

The diagram of the PID controller (Fig. 1) shows the three major components of the PID controller: the proportional, the integral, and the derivative contributions to the can be easily derived by PID controller.

Input–output controllability is the ability to achieve acceptable control performance, that is, to keep the outputs within specified bounds or displacements from their set points, in spite of unknown variations such as disturbances and plant changes, using available inputs and available measurements. In summary, a plant is controllable if there exists a controller that yields acceptable performance for all expected plant variations [6,7,19].

Recalling that the use of feedback in a system serves to achieve the following [3]:

- reducing the effects of parameter variations;
- reducing the effects of disturbance inputs;
- improving transient response characteristics;
- reducing steady state errors;

setting the parameters appropriately produces satisfactory results and controllability. For example, large gains (large values of the parameter k_p) produce a decrease in errors and a greater speed of response of the system. On the other hand, a too large a gain value may bring system instability. If the forward loop gain (combination of controller and plant gain) is much greater than unity, variations in the plant gain or in the controller parameters do not affect the output/input relationship as much; hence, sensitivity to plant and parameter variations is decreased. In fact, a feedback system is less sensitive to plant variations and more sensitive to feedback sensor and forward loop gain than an open-loop system. Sensitivity to disturbances gets

also reduced, as the forward gain is much larger than unity. Feedback systems can be robust so that they are insensitive to changes in the plant and can maintain their stability and performance.

Remembering that the plant is represented by a risk-adjusted return variable of a financial portfolio comprising various risky assets, the plant obviously presents a non-linear process with low controllability. The main objective of this work is to improve the overall performance of the system. The great variety of processes and mechanisms underlying financial assets dynamics, uncertainty with the process, and the desire to avoid modeling the process plant by mathematical closed form equations, suggest the use of PID controllers to improve the overall performance and controllability of the system as a whole. The great advantage here, versus the typical PID feedback system is that there is no need to be concerned with the actual implementation and realization (hardware) of the system. Tuning helps achieve stability and controllability. In order to achieve more stability and controllability of systems where the plant is unknown, time-variant and non-linear, self-tuning and adaptive PID controllers may also be employed.

3. The method

This work originates from the wide literature debate about dynamic and static portfolio asset allocation. In particular, recent papers [8] underline the consequence of dynamic asset allocation compared to static asset allocation [9,10]. For this reason, the aim of this work is to present and to verify an innovative dynamic asset allocation technique.

To demonstrate the G.A.M. model efficacy we start by defining the input data. Nine asset classes have been chosen: EUR/USD, Nikkei225, DJIA, China 1-2 Cp Index, Hang Seng, 30Y Treasury Bond, SMI Swiss Index, Gold (\$/ounce), and USD/JPY. These asset classes were picked so that their mix allows achieving a good portfolio diversification. At the same time, the number of asset classes is small according to recent studies: in fact, good diversification is possible by using a limited number of asset classes [11].

Our methodology is based on the use of an 11-year monthly frequency time-series covering the years 1996–2006. The method is mathematically described by Eq. (3) defined in the previous paragraph. The analysis consists in commencing in year 1996 by defining a portfolio formed by nine assets equally weighted. Such portfolio, the Buy&Hold portfolio, which could be applied as a benchmark for Discretionary Mandates via mutual funds or hedge funds, is then kept constant for the whole duration of the observation period. The monthly returns of the Buy&Hold portfolio are calculated on the last day of every month together with their variances, correlations, and Sharpe Ratios [12]. In literature, we can find different portfolio performance indices [13]. In order to measure the portfolio efficiency, we adopt one of the more famous indices, namely, the Sharpe Ratio [14].

A second portfolio named the G.A.M. portfolio is subject to the PID control action and thus rebalanced on a monthly basis. Initially, the G.A.M. portfolio is defined identically to the Buy&Hold portfolio and positioned in year 1996. The PID methodology has been applied to such portfolio assuming no knowledge of future market conditions and dynamics. The methodology consists in rebalancing the G.A.M. portfolio assets at the end of each month. The rebalancing is a function of several variables: the PID action (parameters), the controlled variable (G.A.M. portfolio's Sharpe Ratio) dynamics and history, current and past market conditions, and the set-point value (desired Sharpe Ratio value). Rebalancing occurs at the end of each month. At each occurrence of the month-end time the PID outputs its required value, which is to be assigned to the controlled variable (Sharpe Ratio). Such value, given to the controlled variable, determines a set of asset weights such that, under current market conditions (present market values), the portfolio approaches the PID-returned Sharpe Ratio value. The portfolio, defined by such weights, is exposed to the market for the following month. The following month market conditions yield a new Sharpe Ratio which, in turn, is subject to the PID controller. This routine gets repeated for the whole observation period on a monthly frequency, at the end of each month, for 11 years from 1996 until 2006. At the end of each year and at the end of the observation period, the two portfolios, namely, the Buy&Hold and the G.A.M. portfolios, are observed and compared with the aim to verify the efficiency of the G.A.M. model compared to the Buy & Hold strategy. The comparison is carried out with an observation period of 11 years not taking into consideration fiscal and transaction costs.

Once the initial portfolio has been built, the target value, or set-point value (represented by the Sharpe Ratio) is to be defined; the G.A.M. model portfolio Sharpe Ratio will have to approach such set-point value during the considered investment period. The Sharpe Ratio set-point value is not defined at random; but, it is set according to the features of the initial portfolio, which, in turn, is chosen in compliance with the specificity of a hypothetical investor. The initial portfolio Sharpe Ratio was recorded to be equal to 1.0; thus, the Sharpe Ratio set-point value equals 1.0.

Each month [15,16], until the end of the investment period is reached, the weights of the asset classes in the G.A.M. portfolio get rebalanced according to the PID signal. The PID parameters utilized by the G.A.M. model were tuned set, via Ziegler-and-Nichols procedure [5], to 0.30, 0.30, and 0.45 for the proportional, integral, and derivative constants, respectively.

4. Tests and empirical results

The following table summarizes the tests executed on an 11-year period time horizon, monthly frequency data time series. The table serves to outline and compares the performance of the benchmark Buy & Hold

Table 1
G.A.M. model results

	B&H portfolio returns (%)	G.A.M. portfolio returns (%)	B&H portfolio σ (%)	G.A.M. portfolio σ (%)	B&H portfolio sharpe ratio	G.A.M. portfolio sharpe ratio
1996	9.30	9.52	4.84	4.77	1.30	1.37
1997	2.66	8.59	8.25	12.04	-0.04	0.46
1998	-1.42	-9.71	10.47	23.42	-0.42	-0.54
1999	16.69	58.99	8.27	27.86	1.65	2.01
2000	-0.51	16.79	7.64	17.32	-0.46	0.80
2001	-9.91	14.78	8.53	12.55	-1.51	0.94
2002	-8.88	28.79	8.05	17.54	-1.48	1.47
2003	17.80	0.87	5.61	16.06	2.64	-0.13
2004	2.46	20.32	4.49	11.89	-0.12	1.46
2005	7.75	-2.36	5.80	13.37	0.82	-0.40
2006	16.47	27.01	4.00	13.15	3.37	1.83

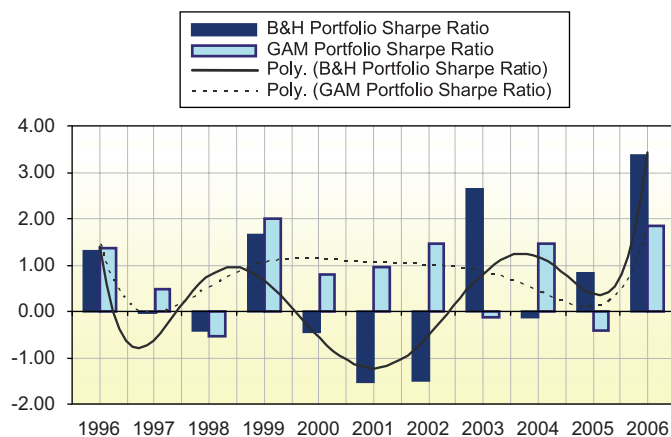


Fig. 2. Sharpe Ratio data.

portfolio, managed by the Buy & Hold strategy, and the portfolio initially identical to the benchmark portfolio, subsequently subject to the G.A.M. modeling technique. (Table 1).

The table depicts several columns: the first column, starting from the left, indicates the years. The second column contains the yearly returns of the Buy & Hold portfolio. The third column indicates the yearly returns of the G.A.M. model managed portfolio. The fourth and fifth columns illustrate the annual standard deviation (risk) of the two portfolios, respectively. Finally, the last two columns present the annualized Sharpe Ratios of the two portfolios, respectively. It is interesting to point out that the peculiarity of the G.A.M. model is to provide stability [17] and performance consistency. The following bar chart depicts annual Sharpe Ratio values for the two portfolios. The dark blue bars indicate the annual Sharpe Ratios of the Buy & Hold portfolio over the 11-year time horizon. The light blue bars illustrate the same risk-adjusted return parameter for the G.A.M. model managed portfolio. By looking at Fig. 2, it is possible to observe a stability effect produced by the PID controller on the portfolio's Sharpe Ratio compared to the Buy&Hold portfolio's Sharpe Ratio. By evaluating the two six-degree polynomial regression lines, calculated on the respective Sharpe Ratio values of the two portfolios, it is possible to outline a more evident stability in the G.A.M. portfolio Sharpe Ratio than in the Buy&Hold portfolio Sharpe Ratio. The aim of the G.A.M. model is not to have the controlled portfolio's Sharpe Ratio approach 1; its main objective is to stabilize the controlled variable in the neighborhood of such value. The controllability of the whole system depends also on the set-point value represented by the Sharpe Ratio: a set-point value equal to 1 is ambitious but attainable.

5. Conclusions

The innovation presented in this work consists in conveying to tactical financial portfolio asset allocation the well-known PID feedback controlling mechanism, widely used in engineering and industrial plant processes. No modeling or simplifying assumptions on the dynamics of portfolio financial assets is required. The goal is to achieve long-term performance stability over time by controlling the risk-adjusted return variable of portfolios. Thus, in this paper, the main feature to notice is the achieved stability and consistency of the Sharpe Ratio of the G.A.M. model portfolio in comparison with the benchmark Buy & Hold portfolio.

Future work and studies will include adopting many more and other asset classes. Using an optimal Markowitz portfolio [18] as a benchmark for the G.A.M. model, making assets time series vary in frequency and length, using other risk-adjusted returns or other indices (i.e., Sortino, Information Ratio, etc.). Furthermore, taking into account transaction and management fees will be part of our research as well as using parameter setting and constraints, stability, and controllability theory in support of our research. In addition, given the evident stability effect imparted by the G.A.M. model on financial portfolios behavior, current research is also devoted to applying the G.A.M. model action directly to time series returns and observing risk-adjusted return levels at the end of the desired time horizon or observation period, as an alternative application in order to further verify the efficacy of the model.

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