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Analytical Solution and Scaled Model of a Unipolar HVDC Transmission Line

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Abstract

Existing analytical solutions for the ionized field of a unipolar HVDC transmission line are reviewed. The analytical relations indicate that the corona current is directly proportional to the square of the corona onset field strength if the ratio of the applied voltage to the corona onset value and that of the conductor height to its radius remain fixed. This is confirmed by numerical results. It is pointed out that this observation makes it possible to derive the basis of a scaled model to evaluate the corona current and hence the resulting power loss of a full-scale unipolar dc line.

1. Introduction

The corona loss of a dc power transmission line may be evaluated through solution of its ionized field. However, the nonlinearity of the field makes the problem very difficult to solve. An analytical solution is possible only for a coaxial cylindrical geometry since the field can be formulated as a one dimensional problem. Based on Townsend's work [1] on a coaxial cylindrical geometry and Deutsch's assumption [2] which states that the presence of ions only affects the magnitude of the field but not the direction, Popkov [2] derived an approximate formula for the voltage-current (V-I) characteristic of a line-plane geometry. This formula takes a form similar to that for the coaxial cylindrical geometry, but an empirical constant P is introduced. Reasonable agreement has been shown in Sarma's work [3] between the results generated by Popkov's formula and numerical methods for a properly chosen value of P. Although it has been pointed out that the constant P is dependent upon line parameters, no theoretical guideline is available for choosing this constant. On the basis of Deutsch's assumption, Ciric *et al.* [4] presented a rigorous analytical expression for the V-I characteristic of the line-plane configuration. This formula, unlike Popkov's, does not include any empirical constant. The only possible source of error in this method arises due to using Deutsch's assumption. Studies [5] show that this formula generates reasonably accurate results for a unipolar line in the absence of wind. Although a number of efficient numerical methods have now been developed, the analytical expression for the V-I characteristic in [4] may be used to estimate the corona current as well as its corresponding power loss.

It would be of benefit to practising engineers to relate the experimental results obtained from a scaled model to an actual dc line. However, due to the nonlinearity of the ionized field, the construction of the scaled model of an ionized field is not as simple as that of a charge free field. To the authors' knowledge, there are no theoretical guidelines on how to construct a scaled model for the ionized field. In [6] it is pointed out that although the dimensions of a scaled-down model may not be fully representative of actual dc lines the trends in the results are more sensitive to the ratio of conductor height to radius than to actual dimensions.

In this paper it is pointed out that scaling of a unipolar line is governed by certain general relations which are suggested by examination of available analytical expressions for the corona current characteristics of unipolar configurations. Furthermore, a firm basis is proposed for the construction of scaled models, which makes it possible to experimentally derive the corona characteristics of a full scale unipolar dc line from experiments conducted on a small model. The validity of the basis has been verified by application of a numerical method, devised by the authors, to two different line configurations.

2. Analytical Solutions for Unipolar Ionized Fields

2.1. Equations and Boundary Conditions

A unipolar ionized field in the absence of wind is described by Poisson's equation coupled with the cur-

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rent continuity equation, i.e.,

$$\nabla^2 u = -\rho/\varepsilon_0 \tag{1}$$

$$\nabla \cdot (\rho k \nabla u) = 0 \tag{2}$$

where u is the electric potential, ρ the space charge density, ε_0 the permittivity of free space, and k the ionic mobility which is usually treated as a constant over the whole solution domain.

The boundary conditions include: (i) the electric potential on the boundary, and (ii) the electric field strength on the coronating conductor surface. The latter condition is determined by using Kapzow's assumption, i.e. the electric field on the coronating conductor surface remains at its corona onset level.

For the past several decades, considerable efforts have been made, both analytically and numerically to solve this difficult nonlinear problem. The end result of an analytical solution is usually an expression for the V-I characteristic, while a numerical solution yields a distribution of the ionized field in the whole solution domain. In this paper, interest is focused on the analytical solution of the ionized field.

2.2. V-I Characteristic of Coaxial Cylindrical Geometry

In this case, the ionized field reduces to a one dimensional problem along any radial direction. The V-I characteristic [3] is obtained as

$$\frac{V-V_c}{V_c}\ln\frac{R}{r_0} = \sqrt{1+Y_c} - 1 + \ln\frac{2}{1+\sqrt{1+Y_c}} \tag{3}$$

with

$$Y_c = \frac{I}{2\pi\varepsilon_0 k}\left(\frac{R}{E_c r_0}\right)^2 \tag{4}$$

where V is the applied voltage, V_c the corona onset voltage, I the corona current, r_0 the conductor radius, and k the ionic mobility.

2.3. V-I Characteristic of Line-Plane Geometry

Popkov [2] extended the expression for the V-I characteristic of the coaxial cylindrical geometry to the case of a line-plane configuration with the error being corrected by an empirical constant P. The V-I relation takes the form

$$0.41P\frac{V-V_c}{V_c}\ln\frac{2H}{r_0} = \sqrt{1+Y_c} - 1$$
$$-1 + \ln\frac{2}{1+\sqrt{1+Y_c}} \tag{5}$$

with

$$Y_c = \frac{PI}{2\pi\varepsilon_0 k}\left(\frac{2H}{E_c r_0}\right)^2 \tag{6}$$

where H is the conductor height.

Sarma [3] made detailed numerical investigations for the validity of Popkov's formula for different geometric parameters and concluded that this formula gives reasonably accurate results, but the constant P is a function of line parameters.

2.4. Ciric-Kuffel Formula for Line-Plane Geometry

Using Deutsch's assumption, Ciric *at al.* [4] derived the following corona V-I characteristic:

$$\frac{V}{V_c}\xi_1 = \sqrt{(1+Y_c)}\int_0^{\xi_1}\sqrt{1-\frac{2Y_c}{\sqrt{3}C(1+Y_c)}f(\xi)}\,d\xi \tag{7}$$

with

$$f(\xi) = \ln\frac{\sqrt{3}+\tanh\xi}{\sqrt{3}-\tanh\xi} \tag{8}$$

$$\xi_1 = \ln[H/r_0 + \sqrt{(H/r_0)^2 - 1}] \tag{9}$$

$$Y_c = C\frac{I}{2\pi\varepsilon_0 k}\left(\frac{H}{E_c r_0}\right)^2 \tag{10}$$

$$C = \frac{2}{\sqrt{3}}\ln\frac{\sqrt{3}+\sqrt{1-(r_0/H)^2}}{\sqrt{3}-\sqrt{1-(r_0/H)^2}} \tag{11}$$

For practical HVDC transmission lines, $r_0/H \ll 1$, and therefore equations (9) and (11) may be simplified to $\xi_1 = \ln(2H/r_0)$ and $C = 1.520692$, respectively. This simplification results in negligible error.

Several simplified forms of equation (7) have been derived in [4]. Alternatively, equation (7) can be easily evaluated using a numerical integration method.

Numerical results [5] show this formula is reasonably accurate from an engineering point of view.

3. An Important Observation

From Eqs. (3) and (4), we have the following relation for the coaxial cylindrical geometry:

$$I = E_c^2 f(V/V_c, R/r_0) \quad (12)$$

where $f(\ )$ is a function of V/V_c and R/r_0. It indicates that the corona current is directly proportional to the square of the corona onset field strength for fixed V/V_c and R/r_0

Similarly, for the line-plane geometry, from Eqs. (5)-(11), the corona current can be explicitly expressed as

$$I = E_c^2 f(V/V_c, H/r_0) \quad (13)$$

where $f(\ )$ is a function of V/V_c and H/r_0. This observation is further confirmed by numerical results, which is shown in the next section.

4. Numerical Solution

In this section, the observation made in Eq. (13) is verified by means of an iterative algorithm developed by the authors [7], in which Poisson's equation and the current equation are solved by the finite element (FE) method and the upwind finite volume method, respectively. A relaxation technique is introduced to enhance the stability of the iterative process.

4.1. Geometry and FE Mesh

The line-plane geometry consists of a line situated at a distance of $H = 2$ m above a grounded plane. The artificial boundary is placed at a distance of $8H$ from the conductor. The ionic mobility is taken to be $k = 1.4 \times 10^{-4}$ m²/V.s. The ionized field is solved for two values of conductor radii such that $r_0 = H/800$ and $H/400$. The corona onset field strength is determined by Peck's law. The FE mesh (Fig. 1) for the former geometry includes 2612 nodes, and that for the latter one includes 2516 nodes. The meshes are generated by using the program in [8].

4.2. Numerical Results

For each geometry, the ionized field is solved corresponding to different corona onset field strengths which are realized by changing the surface factor of the conductor(i.e. $m = 1$, 0.8, 0.6, and 0.4). The ratio of the applied voltage to the corona onset value is $V/V_c = 2$ in all cases. Using the calculated nodal potentials and charge densities, the corona currents are found, which are listed in Table 1. From this table, it is seen that the quantity I/E_c^2 is constant for fixed V/V_c and H/r_0.

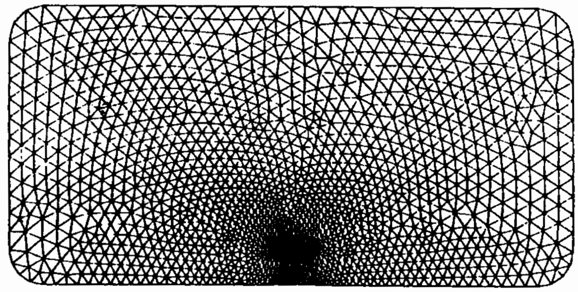

Figure 1. FE mesh for the line-plane geometry with $H = 2$, $r_0 = H/800$, and $n_p = 2612$.

Table 1. Numerical results for the corona current I as a function of the corona onset field strength E_c for a line-plane geometry with $H = 2$ m, and $V/V_c = 2$.

H/r_0	E_c (V/m)	I (A/m)	I/E_c^2
400	1.71081E+6	3.30865E-6	1.1304E-42
	2.56622E+6	7.44469E-6	1.1305E-42
	3.42163E+6	13.2350E-6	1.1305E-42
	4.27704E+6	20.6800E-6	1.1305E-42
800	1.92240E+6	1.23635E-6	3.3454E-43
	2.88360E+6	2.78182E-6	3.3455E-43
	3.84480E+6	4.94539E-6	3.3454E-43
	4.80600E+6	7.72726E-6	3.3455E-43

5. Scaled Model of a Unipolar Ionized Field

The quantities and parameters associated with the scaled model are indicated by a superscript "s". Let the parameters of the scaled model and the practical dc line satisfy the following relations:

$$V^s/V_c^s = V/V_c \quad (14)$$

$$H^s/r_0^s = H/r_0 \quad (15)$$

According to the observations in section 3, we have

$$I/I^s = (E_c/E_c^s)^2 \quad (16)$$

or

$$I = (E_c/E_c^s)^2 I^s \quad (17)$$

Using this equation, the corona current of a full-scale dc line can be easily evaluated based on the measured corona current of its scaled model.

6. Conclusions

The corona current of a line-plane geometry is directly proportional to the square of the corona onset field strength for fixed V/V_c and H/r_0. This observation makes it possible to derive a scaled model to predict the corona currents and hence the corresponding power loss of a full-scale unipolar dc line.

Acknowledgements

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