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PATH PLANNING FOR AUTOMATED ROBOT PAINTING

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ABSTRACT

This paper describes the analysis work underlying the path-planning algorithm for a robotic painting system. The system requires no bespoke production tooling and fills an automation gap in rapid prototyping and manufacturing technology that is currently occupied by hand painting.

The system creates images by exposing individual pixels of a photographic coating with a robot-mounted laser. The painting process requires no physical contact so potentially images could be developed on any shape regardless of its complexity: As objects can only be "painted" when their surface can be "hit" (i.e. exposed) by the light beam the system requires six degrees of freedom to ensure all overhanging or reentrant areas can be exposed.

The accuracy of serial robots degrades with the length of the kinematic chain (in other words six axis robots cannot position themselves with the same accuracy as four axis ones). Consequently to ensure high precision in the location and orientation of the light source, the object being exposed is mounted on a rotary tilt table within the workspace of a four-axis robot. This gives a six-degree of freedom positioning system composed of two separate kinematic chains.

Although the resulting system is accurate the problems of constructing a coordinated path that allows the light beam to efficiently sweep (i.e. cover) the surface regardless of its geometry are challenging.

This paper describes the difficulties and, after reviewing existing path planning algorithms, a new algorithm is introduced firstly by describing the nature of the system's configuration space and then further developing this concept as an alternative to a previously described planning algorithm. Having outlined the approach the paper presents a kinematic model for the system and compares the configuration space approach to a purely Cartesian planning approach.

1 Introduction

The ever-increasing pace of product development is forcing continuous reductions in lead times and the development of many rapid manufacturing processes. This growth in rapid manufacturing has in turn made the development of secondary processes, such as automated painting, finishing and assembly technologies a necessity if all production bottlenecks are to be removed.

Motivated by this the Heriot-Watt's Digital Tools Group has developed a rapid painting unit which uses photosensitive paint, exposed by a tri-colour laser [1,2 Sung]. The laser unit is mounted on a 4-axis robot with an additional 2-axis rotary, tilt table (incorporating work piece holder) Fig 1. The system has produced high quality (300dpi) and accurate images from a high resolution VRML2 file that incorporates both shape and surface colour data (i.e. texture map). From this a file containing position, orientation and laser intensity is generated using a part programming system based on the *Hoops* modelling kernel. The system generates this file by extracting surface data (such as the global position and the normal vector) for individual points on the model's surface and then extracts colour information from the image mapped to the models surface. This information can be extracted at various resolutions (i.e. 150, 250, 300dpi) up to the stage where hardware limitations become the restricting factor. Once this "point/normal/color" (PNC) file has been created it is used to generate the path for the robot to follow. Prior to this work the path that was generated followed the default layer based strategy used to generate the points, meaning that the robot works down the object in small vertical increments towards the table and for each step down works "round" the perimeter of the model exposing the points to pin sized burst of light.

The use of this simple path generation routine initially benefited the development of the system as it reduced the variables involved in testing the proposed system however; having established the basic viability of the approach it became important to produce a more efficient path planning algorithm. Essentially the experimental system proved to be inefficient since it did not take into account the configuration time required for each robotic movement. In other words, although two points could be geometrically close on a part's surface (i.e. adjacent on a laminar slice) changes in the surfaces orientation could require large movements in the robot's attitude. This paper reports a path planning algorithm that uses configuration space (C-Space) to reduce robotic movement and so allows a much quicker and more efficient path.

The rest of this paper is structured as follows:

Section 2 reviews the literature and technology underpinning the proposed system. Section 3 describes the system's architecture (both hardware and software) and details the proposed configuration space based path planning algorithm. Section 4 constructs the kinematic model for the system. Section 5 discusses the experimental results before Section 6 draws conclusions and proposes improvements to the system.

2 Literature Review

Path planning has been an important aspect of many forms of automated manufacturing processes (e.g. robotics, CMM inspections, surface machining etc). The development of 5 and 6 axis systems for complex machining and inspection processes furthers the need for automatic generation of efficient and precise paths. This section will give a brief overview of the research that has been done into path planning for applications that share some of the challenges of the optical painting process. Particular consideration is given to configuration based approaches to path planning.

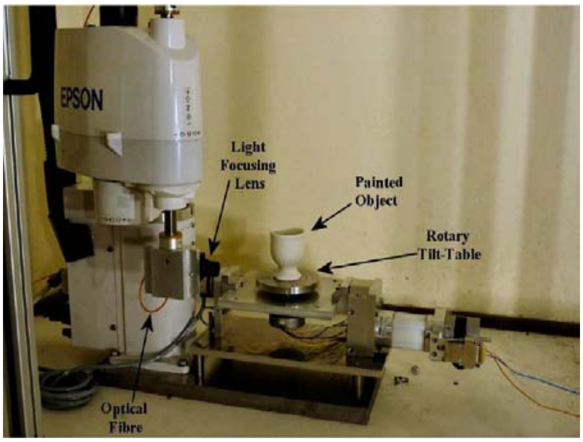

Figure 1:- Layout of Robotic Cell

2.1 Path Planning

The authors' application requires surface coverage (to ensure all areas are exposed), obstacle avoidance (to ensure the surface does not occlude the light) and efficiency (to ensure the highest possible speed).

Path planning research relevant to these areas has been carried out since the early eighties and can be broken down into two main categories: Parametric (typically for machining of complex surfaces) and point to point (for the avoidance of obstacles), both categories have been driven extensively by the automotive industry however for very different applications.

[3 Bi] recently introduced a paper investigating a framework for integrating Cad and sensor based automated painting motivated by the needs of the ship building industry where historically up to 23% of the workforce is used in painting Bi identifies three primary categories for subdividing path planning and then builds a theoretical model based on simple experiments for solving each scenario. The paper identifies a lack in automated painting of utilising modern robotic planning algorithms and the dependence on classical manual robotic teaching where parametric planning could be used.

Parametric surface path planning has been exploited in many different manufacturing procedures [4 Qiulin] discusses the basic concepts associated with contact machine path planning; building from dealing with a simple two-dimensional tool paths based on an offset surface to a more complex three-dimensional tool path with parametric surface descriptors (u, w). Using the differentials of these descriptors the tangential plane can be constructed and it can be shown that while using a round cutter a parametric offset surface can be used in a similar fashion as the two-dimensional case. Using these assumptions a simple theoretical path can be constructed using surface normals. Qiulin builds on this by demonstrating that the accuracy of a surface is dependant on a '*function of the cutter radii*'. [5 Fan] demonstrates the construction of a local machining co-ordinate system where the z-axis acts along the surface normal and the x-axis is dictated by the cutting direction once the surface has been defined. Fan then discusses the accuracy of the surface in terms of *Gouging*. Gouging has been investigated by many researchers and tends to be classified into three separate categories: local, rear and global, depending on the position of the gouging [6 Jun]. Local gouging refers to the removal of excess material in the vicinity of the cutter contact point and tends to be due to a curvature mismatch between the part surface and the parametric offset surface equation; this could be equated to a similar problem that occurs in the laser painting rather than surface damage when the surface gradient is mismatched, the circular laser beam deforms (when projected on the surface) to produce an irregular shape. Rear gouging occurs when the contact between the end mill and the surface causes interference with previously milled surfaces. However in the painting application this can be disregarded as surface contact is not required in laser painting. However in contrast "global" gouging is the most relevant to laser painting applications as this concerns the accessibility of a specific surface location. In surface machining the interference tends to be caused by the shaft or flute of the tool; in laser painting the problem can cause the wrong surface to be exposed with a burst of light and can be looked at in a similar fashion to the computer vision problem.

Parametric path planning has been implement in many different manufacturing situations. As previously mentioned, it can be used extensively in contact machining this is partly due to the move towards parametric modellers; non-contact manufacturing has also taken advantage of parametric path planners. One of the biggest applications for planners is in the automotive industry where complex curved surfaces have utilised the simple principles of offset planes to develop continuous spray paths for automated spray painting units. The use of automated spray paths has increased rapidly and now most automotive panels are sprayed automatically; many researchers have investigated optimizing the spray path by reducing the amount of overshoot required to maintain an even coat and the optimum trajectory to produce a clean and accurate finish.

The research that has been carried out in point to point planners differs significantly as the problems that need to be addressed and the applications of this form of planning are different. The use of parametric planners tends to be in situations where complete or partial surface operations need to be performed whereas in point to point planners singular or batch operations must occur at specific points on a surface (such as welding). The use of point to point planners requires knowledge about the environment and using this information a path must be generated. The path that is created may be in a similar form to a parametric path however it may also be a linear approach path. Point to point planners have been developed in a variety of different ways though the fundamental requirement is the acquisition of environmental data; once this data has been retrieved a collision free path is planned and a vast amount of work has been done in collecting and processing the data required to produce such a path.

[7 Lambert] discusses path planning using an off-line system, the system works by using bounding box and bounding spheres and surface position and direction information to generate an efficient path than prevents the bounding box of the manipulator from colliding with pre-existent objects. The algorithm forces a pre-generated path that dissects current objects at a tangent to the normal of the object's surface hence ensuring the bounding box does not collide with existent objects. This system could be further developed to analyse the individual joints and their position in respect to the surroundings; this would prevent interference between robot and surroundings. [8 Wu] investigates collision detection in connection with configuration space and develops a model of the free space available for the robot and uses this model to propose a more reliable path (configuration space will be discussed in more depth later in this paper).

As it can be seen a vast amount of information is available regarding path planning and different tasks require different specifications for individual tasks. The use of path planning in automated spray painting requires a highly optimized route so that a controlled and even distribution of paint can be achieved. The pre-computing time could be considerable as long as the application time is minimised, the use of jigs and fixtures could allow a restrictive environment hence reducing the need for complex collision detection. The laser painting system that has been developed will be used on a variety of different models where "*global gouging*" would be a major consideration therefore some form of collision prevention would be required.

2.2 Configuration Space

The placement and orientation of an end effector within a workspace can be represented as a single 6 dimensional point called its configuration. The 6-dimensional space within the workspace where the configurations exist is known as the configuration space.

For on (and off) line path planning it has been shown that the use of configuration space is very efficient. The use of configuration space requires the calculation of a configuration map and the generation of these maps has been found to be very time consuming and complex to produce for a high number of degrees of freedom [9 Branicky]. Research has been carried out in C-space investigating many aspects of robotic path planning. For example [9 Branicky] investigates the ability to rapidly compute obstacles and demonstrates several algorithms that can be used in the conversion from work space to joint space. [10 Lambert] demonstrates the application of safe path planning in C-space taking into account the lack of accuracy that can be attained. In the real world it is demonstrated that with the use of a 4D space(c-space+ uncertainties) it is possible to attain a much more accurate and responsive path which is not necessarily the shortest however; it could be regarded as the most appropriate for the robotic task. [11 Kavraki] investigates the initial construction of the C-space demonstrating that with increased pre-processing it is possible to construct a viable path network which could be searched and analysed to produce the optimum path from a given configuration to the required configuration. The method implemented by Kavraki randomly creates a configuration and analyses its joint angles to provide nodes of acceptance and invalid nodes; this creates a map of nodal positions which can be searched rapidly. Once the map has been implemented a goal position can be specified and a search is done on the map to find the nearest node once the node has been found a search is done to discover the current position of the manipulator in nodal space, with the start node and finish node specified a path can be constructed using a weighted system.

Given the arbitrary nature of the objects being painted it can not be guaranteed that a surface parameterisation can be generated for the shape. Indeed, obtaining "good" parameterisations of meshes is a subject of ongoing research in the 3D graphics community [12 Quinn].

Consequently the work flow envisaged is one where a point cloud is generated by z-depth laminar slicing and then sequenced by subsequent analysis. It is clear from the literature survey and consideration of Figure 4 that a C-space planning approach could potentially over come many of the challenges. The next section describes this approach.

3 Methodology

3.1 Hardware Overview

In the system the output from three lasers (Red, Green, Blue) is combined to form a single tri-colour beam and multi-modal optical fibre is used for the delivery of the illumination to a focusing lens mounted on the end of the SCARA robot arm. This set-up offers the following advantages;

- Compact and robust optical delivery,
- Incremental control of light beam projection,
- Incremental control over each independent colour,
- Integration with robotic system motion.

The cell controller drives and coordinates the motion of the robot's 4-axes, the rotary tilt-table's two axes and the brightness of each of the lasers. Instructions are programmed using the cell controller's native SPEL [13 Epson] language. To enable the creation of programs incorporating millions of points, the system uses SPEL's COM interface to link directly to C++ programs compiled on the PC and then downloaded to the controller. The focal point of the light is approximately 6cm from the lens so effectively the painting algorithm must sweep the surface at a 6cm offset in the most efficient manner.

3.2 Proposed Path Planning Algorithm

The process of generating painting instructions aims to derive a sequence of robot positions from a VRML2 file. The problem initially appears to be relatively under constrained as most of the surface points are visible from large sections of the workspace and combined with the kinematics of a six degree of freedom robot will generate vast numbers of solutions for the configuration of the robot. However; the physical capabilities of the mechanical hardware help to limit the magnitude of the problem (i.e. the tilt table works between -45° and +45°).

Figure 2:- Painted Eggcup at 300dpi

The system aims to produce images of around 300dpi implying a theoretical maximum of 90,000 dots per square inch. Consequently the "pixels" used to form each image should be sorted to reflect the capabilities of the system (i.e. faster axes should always be used in preference of slower ones). The principle constraints on the ordering of the points are:

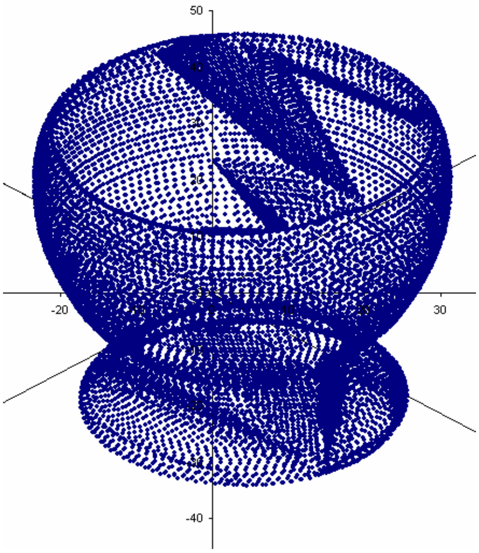

Figure 3:- Point Cloud

1. Orientation: although a point may be visible from many different configurations the maximum amount of energy will be produced when the beam is aligned along the normal axis and at the focus point this will also produce the best image quality. However, when this is not possible the orientation closest to the normal should be selected.
2. Axes Speed: The robot and tilt-table can be configured in numerous ways to deliver a light ray to a point There is a priority to using the robotic axes as these respond quicker and more accurately than the tilt-table and so should be used in preference to the tilt-table.

3. Axes Precision: With all robotic manipulators there is a small variation in the accuracy of the movement throughout the workspace. Generally the greatest precision is obtained in the center of the operational envelope. Consequently, wherever possible the laser delivery system should be positioned near this region of greatest accuracy.

3.3 Overview of Proposed Path Generation Process

The process for generating the robotic path is now described for rendering images on objects with unoccluded surfaces (i.e. genus = 0) where every face is orthogonally viewable. The surface images that drive the process are defined by texture maps that are applied to 3D computer models.

The overall aim of the process can be expressed in terms of inputs and outputs:

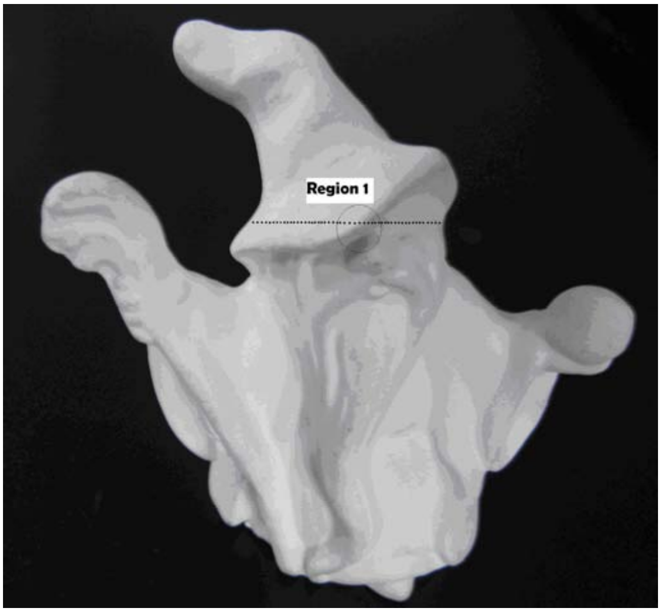

Figure 4:- Gandolf's Slanted Hat

The input to the path planning algorithm is a list of several hundred thousand point records P_i that represent the location on the surface of object O to be painted. Each point record holds coordinates (x_i, y_i, z_i) in model space, the colour (R_i, G_i, B_i) and the surface normal (x_n, y_n, z_n). In a similar fashion to layered manufacturing this list of points is generated by reducing what really is a 3D problem to an easier to solve 2D problem. However, although a brute force approach could be used (i.e. at each "Z" depth work round the perimeter of the model and then increment to the next step) this is not always efficient. For example in the case of Figure 4 (Region 1) where traversing around the surface the orientation of the surface changes dramatically therefore large movements would be required for small moves. Considering the scan line across the hat this is particularly prevalent as it would require the tilt of the table to flip through a large range of angles. This could be done more efficiently if the points were organized so that similar degrees of tilt are done together.

The output of the algorithm will be an organized list of several hundred thousand points and the associated configuration for each of the joints in the robotic system coupled with the beam intensity for each of the lasers. The list will be organized so that points with similar joint angles will be visited together with preference going to the joints on the robot therefore insuring the optimum axes are used.

In other words the objective of the path planning algorithm is to generate a *configuration space* based list where similar joint angles are visited together which is ordered firstly by the robotic arm joints and then by the tilt-table angles.

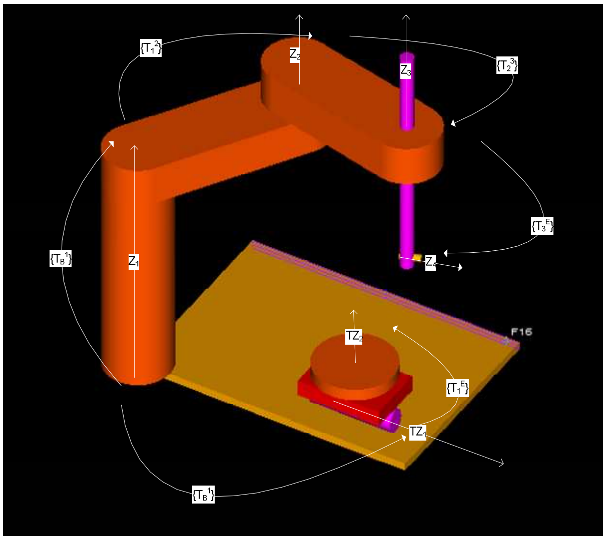

Figure 5:- Expression of Transformations

4 Kinematic Model

The system was modeled using the standard Denavit & Hartenberg (DH) convention to define the schematic shown in Figure 5.
From this homogenous equation transforms for each joint were created

$$RT_{B}^{1}=\begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & dl \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$RT_{1}^{2}=\begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & L_1 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$RT_2^3 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & L_2 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$RT_3^E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -\Lambda \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Using these matrix transforms it is possible to construct the forward kinematics for the robotic arm (i.e. 4 Degrees of Freedom)[*].

$$RT_B^E = RT_B^1 RT_1^2 RT_2^3 RT_3^E \quad (5)$$

$$RT_B^E = \begin{bmatrix} c(\theta_3+\theta_4+\theta_5) & s(\theta_3+\theta_4+\theta_5) & 0 & c(\theta_3+\theta_4)L_2+c(\theta_3)L_1 \\ s(\theta_3+\theta_4+\theta_5) & -c(\theta_3+\theta_4+\theta_5) & 0 & s(\theta_3+\theta_4)L_2+s(\theta_3)L \\ 0 & 0 & -1 & -\Lambda+dl \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Using a similar method it is possible to construct the forward kinematics for the rotational tilt-table as well

$$TT_B^E = TT_B^1 TT_1^2 TT_2^3 TT_3^4 TT_4^5 TT_5^6 TT_6^7 TT_7^E \quad (7)$$

$$TT_B^E = \begin{bmatrix} c(\theta_2) & -s(\theta_2) & 0 & TL_2+TL_1 \\ \frac{1}{2}s(\theta_1+\theta_2)-\frac{1}{2}s(\theta_1-\theta_2) & \frac{1}{2}c(\theta_1-\theta_2)-\frac{1}{2}c(\theta_1+\theta_2) & -s(\theta_1) & -s(\theta_1)TH_2 \\ \frac{1}{2}c(\theta_1-\theta_2)-\frac{1}{2}c(\theta_1+\theta_2) & \frac{1}{2}s(\theta_1+\theta_2)-\frac{1}{2}s(\theta_1-\theta_2) & c(\theta_1) & c(\theta_1)(TH_2-TH_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

As both the kinematic models have been created relative to an origin located in the base of the robot it is possible to register the robotic end effectors' position in work space and it is also possible to convert a position on the model's surface from model space into work space thus allowing us to position the robotic end effector in a suitable position to expose a surface point.

The completed forward kinematic transformation matrix is know as the T-Matrix and can be used to calculate the position and orientation of the end effector.

$$T = \begin{bmatrix} N_X & S_X & A_X & P_X \\ N_Y & S_Y & A_Y & P_Y \\ N_Z & S_Z & A_Z & P_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

Where;

N = normal vector, A = approach vector, S = slide vector and P = is the position vector with respect to the work space.

If we assume that T_R and T_T are the T-matrixes constructed from the kinematic model, then the derivation of the joint angles can be done assuming that:

$$\begin{aligned} T_R &= f(\theta_3, \theta_4, \theta_5, \Lambda) \\ T_T &= f(\theta_1, \theta_2) \\ T_R &= T_T \end{aligned} \quad (10)$$

By equating the elements of T_T and T_R we can calculate the joint angles required to bring robot and tilt table to complimentary positions and orientations. This position and orientation will equate to the position and orientation of one pixel on the surface of the object. The four step procedure is as follows:

Step 1 Using N_x calculate the angle θ_2 from our transformation matrix $T_{T.}$

$$\begin{aligned} N_x &= c(\theta_2) \\ \therefore \theta_2 &= ac(N_x) \end{aligned} \quad (11)$$

Step 2 Obtain the solution for θ_1 by solving the transform for $N_{Y.}$

$$\begin{aligned} N_y &= \frac{1}{2}s(\theta_1+\theta_2) - \frac{1}{2}s(\theta_1-\theta_2) \\ N_X &= c(\theta_2) \end{aligned} \quad (12)$$

Using both equations we can solve simultaneously and this generates two solutions for θ_1 where the sum of both angles is equal to π.

Step 3 With the angles that have been calculated the model can be converted into work space and thus the coordinate reference is changed.

Step 4 With the new model coordinates know in work space the end effector can be assumed to be at an offset distance from the new position and thus simultaneous equation can again be used to solve for the joint angles θ_3 and θ_4.

$$\begin{aligned} newX &= c(\theta_3+\theta_4)L_2 + c(\theta_3)L_1 \\ newY &= s(\theta_3+\theta_4)L_2 + s(\theta_3)L_1 \end{aligned} \quad (13)$$

These again yield two solutions for each joint angle representing the left and the right solution.

The final positional calculation required must workout the required displacement along the *Λ* axis.

$$\begin{aligned} newZ &= -\Delta + dl \\ \Delta &= newZ + dl \end{aligned} \quad (14)$$

The final orientation must also be calculated for $\theta_{5.}$

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[*] Using Notation c=cos, s=sin, ac=arccos, as=arcsin

$$
\begin{aligned}
N_x &= \mathrm{c}(\theta_3+\theta_4+\theta_5)\\
\mathrm{ac}(N_x) &= \theta_3+\theta_4+\theta_5 \qquad (15)\\
\mathrm{ac}(N_x)-\theta_3+\theta_4 &= \theta_5
\end{aligned}
$$

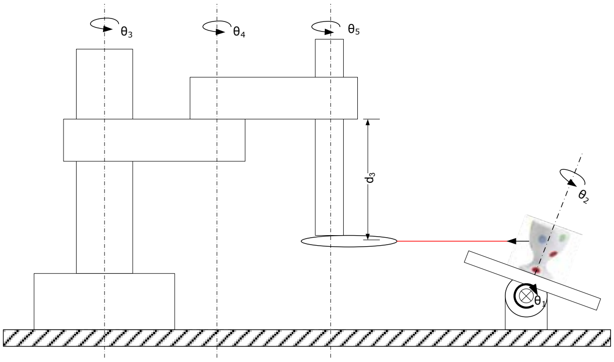

Figure 6:- Joint Angle Representation

5. Experimental Results

Having implemented the algorithm described in the previous sections, it runs on a Pentium 4 PC with 1 GB of memory. To test the effectiveness of the algorithm we have executed a series of tests using two different 3D models.

The first model was a painted eggcup where all points are positioned on a single closed loop at each discrete “Z-height” Figure 7. The second model was a more complex jug shape where horizontal cross-sections will frequently create multiple loops and where the general perimeter is less regular. In order to study the effects of configuration based ordering the joint angles associated with 500 points on the eggcup and 300 points on the jug where calculated. These values were then plotted against the default ordering (based on surface adjacency). The points were re-ordered to minimize the distance traveled by a given joint so different strategies have been examined (e.g. minimize movement of joint 1 or 2 etc).

To quantify the effects of configuration based re-ordering the total angular travel of each path was calculated. Graph 1 shows the large movements that would be required for the tilt-table if the points were not adapted by the path planning algorithm it can however, be seen that as expected regular patterns appear for the symmetrical eggcup.

Graph 2 shows that for less symmetrical components the angles are significantly more erratic with large swings from positive to negative angles; this would mean a greater cost in terms of manufacturing time. Graph 1 and 2 both show large variations in the tilt angle which could be seen to be detrimental to the practical application as the tilt axis moves slower than other axis therefore, minimizing large movements would be beneficial; graph 3 and 4 demonstrate by re-ordering the points with respect to the major joint 3, the required tilt movements are reduced. Graphs 3 and 4 have been normalized so that only the positive solutions are used when calculating movements. At least two solutions exist for every joint (i.e. for the rotary axis theta 2 ±Π) by committing to use the positive solution we reduce the complexity of the problem significantly however, further research will be required to analyse the multiple solutions aspect of this problem and a selection algorithm will be required to use the best solution in each scenario.

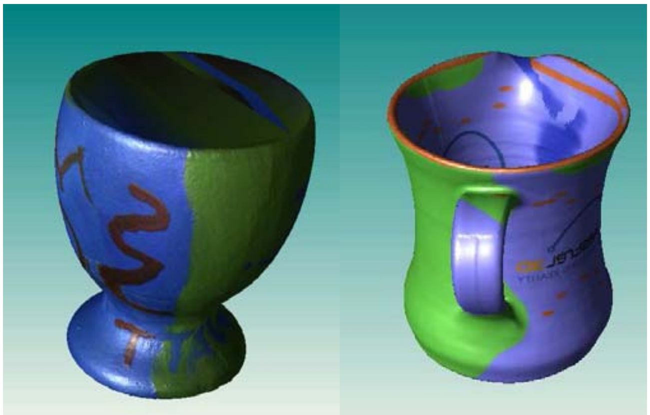

Figure 7:- 3D-Models Used for Experimentation

Graph 5 has re-ordering done in respect to the robots second link angle (theta 4) the robots primary angle (theta 3) can be seen to have an increased range this would impact heavily on the production time as the required travel has increased by approximately 33% from 489 radians to a much larger 736 radians. The variations observed in travel distance can be used as an approximate guide however; further experiments would be required to calculate accurate axis response times.

The simple re-ordering that has been applied to the point data has shown the effects that various re-ordering strategies result in large variations in travel distances. The re-ordering has predominantly been done by identifying primary axis and organizing the points incrementally from smallest to largest this method is suitable for simple problems however, multi dimensional closest point analysis would be required to provide a more efficient algorithm.

6. Conclusion and Future Research

This paper presents a new path planning algorithm which utilized configuration space to build an optimized technique of visiting large numbers of points to enable a rapid painting system to construct images on the surface of 3D objects. The algorithm first calculates the tilt and the rotation of a model so that forward kinematics can be applied to calculate the position in configuration space of the end effector. Once this is done inverse kinematics are applied to calculate the joint angles required for the robotic arm. A matrix is constructed to hold the various joint angles and then re-ordering is applied to calculate the optimum path. This path planning algorithm reduces the setup time required for painting of complex models by grouping similar joint positions together and thus reduces the time required to paint a model.

In future research, more analysis must be done on the solution of the simultaneous equation so that the optimum angle is obtained and not the first angle calculated, this will reduce the range of the results. Further work must also be done on the construction of the sorting algorithm so that more complex and efficient planning can be undertaken this could perhaps be done by manner of a weighted function where joint parameters can be specified such as angular limits and rate of movement.

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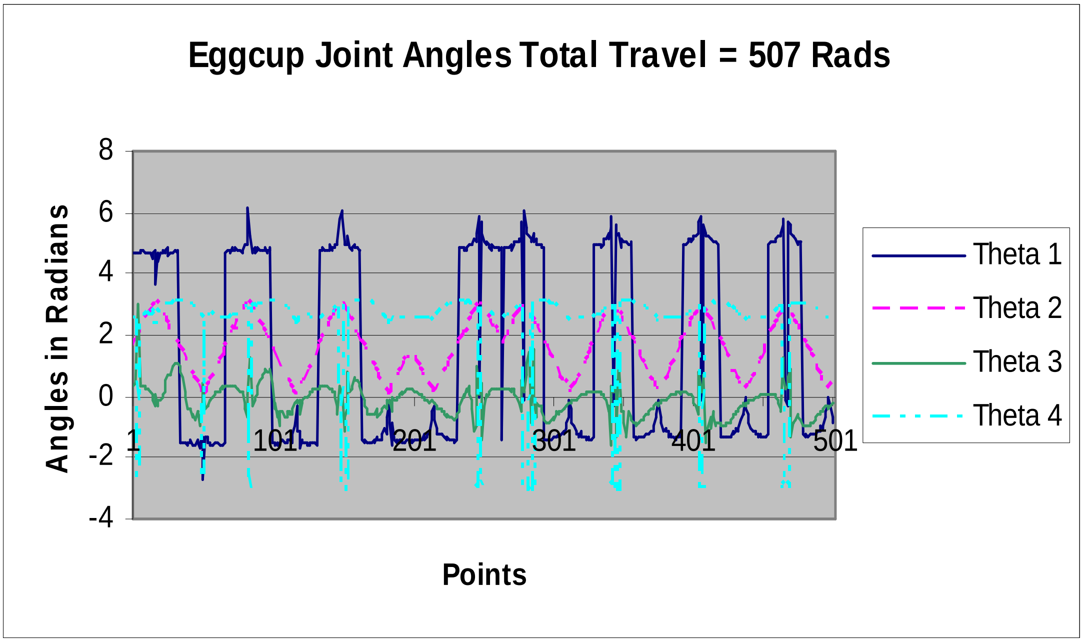
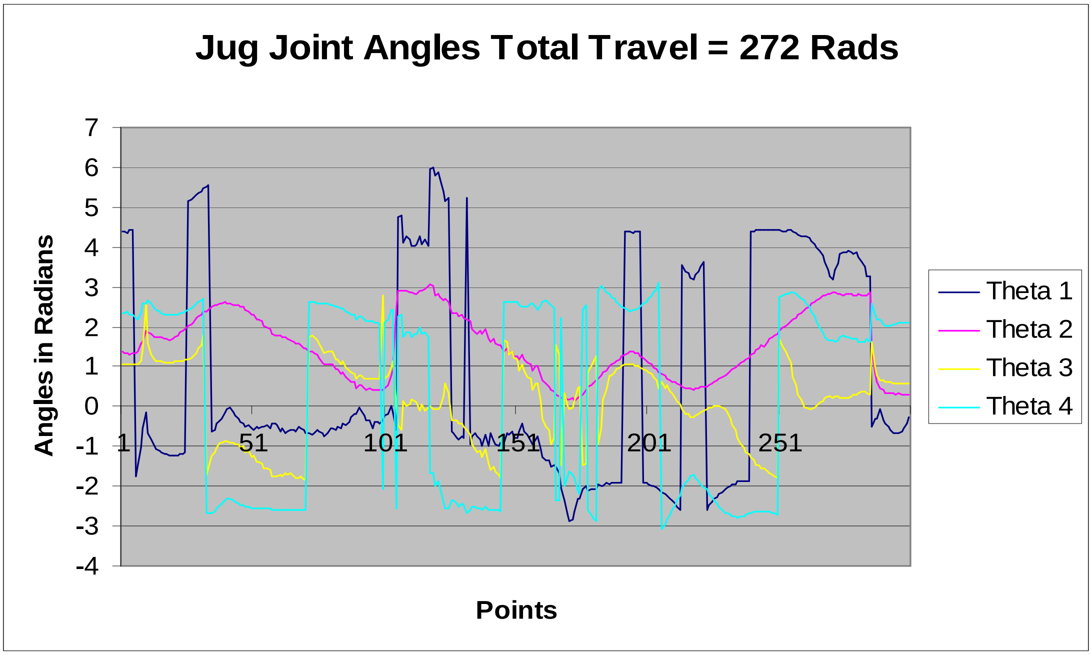
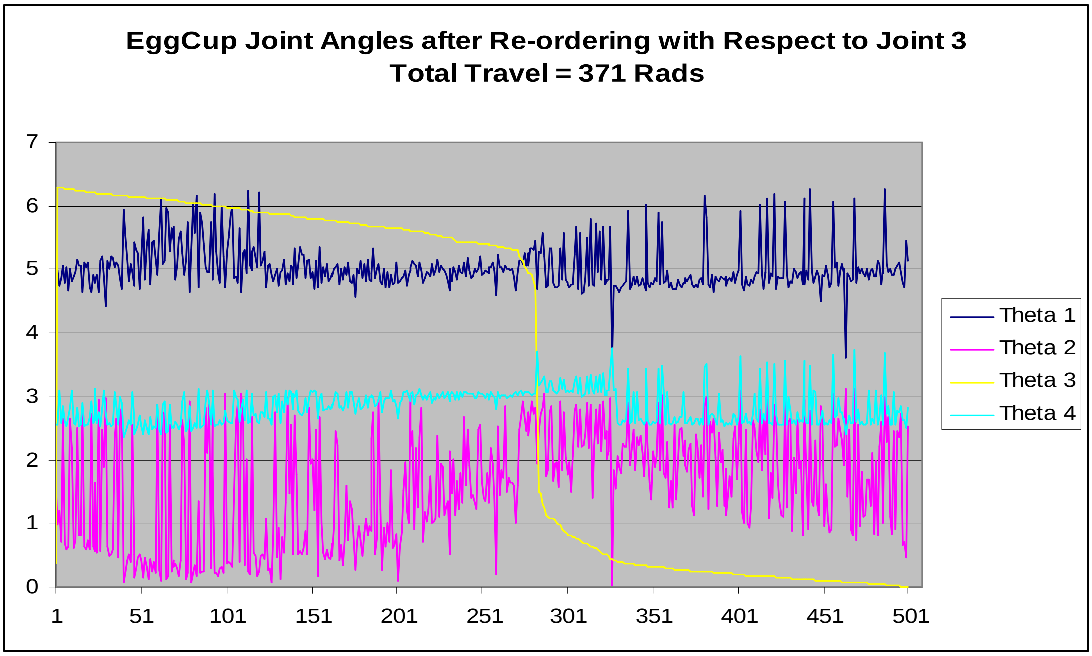


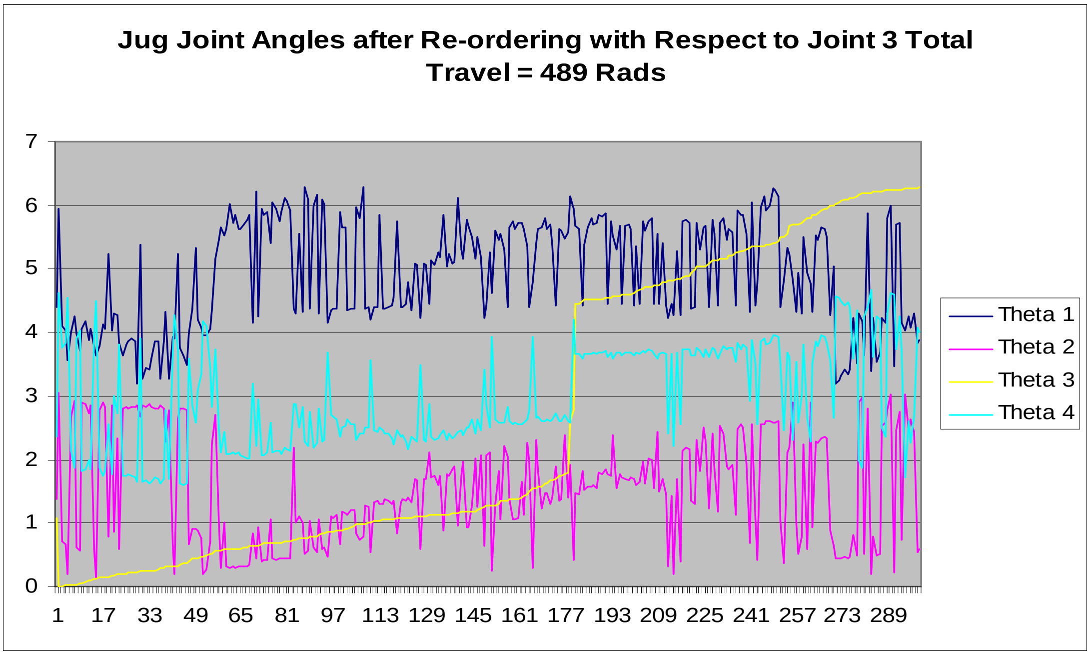
Graph 1:- Eggcup Joint Angles before re-ordering



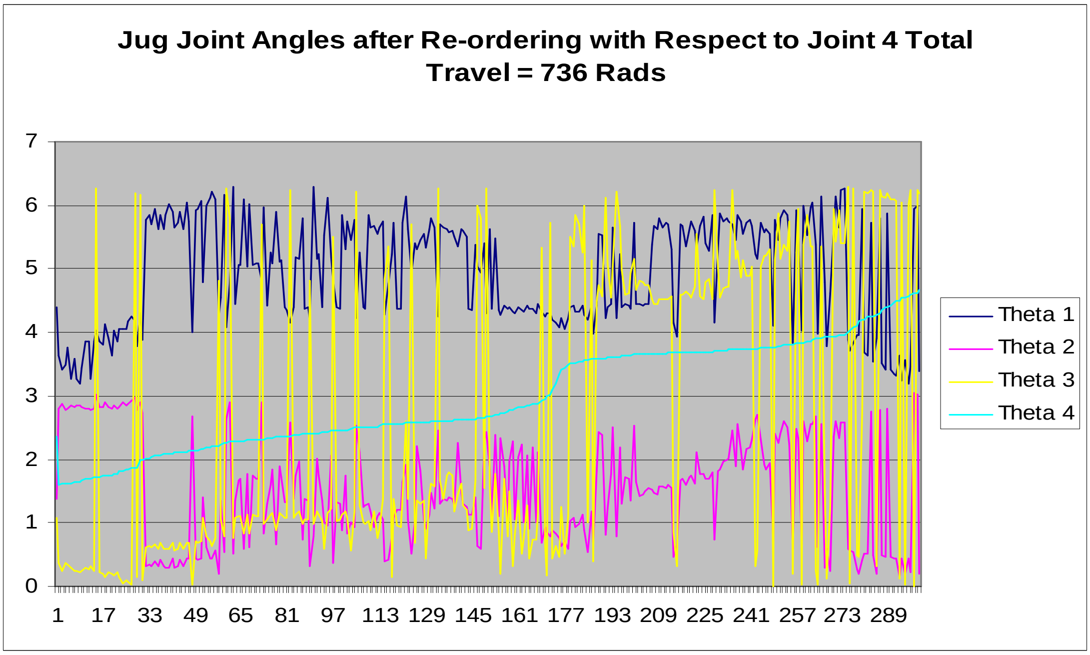
Graph 2:- Jug Joint Angles before re-ordering



Graph 3:- Eggcup Re-Ordered in Respect to Tilt



Graph 4:- Jug Re-Ordered in Respect to joint 3 with normalization applied



Graph 5:- Jug Re-Organized with Respect to theta 4 with normalization applied