# A Generalized Gas-Liquid-Solid Three-Phase Flow Analysis for Airlift Pump Design

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*The object of the present study is to access the performance of an airlift pump under predetermined operating conditions. The gas-liquid-solid three phase flow in an airlift pump is described by a system of differential equations, which derives from the fundamental conservation equations of continuity and momentum. This approach leads to a more general mathematical model which is applicable to a wide range of installations, from small airlift pumps to very large systems, suitable for deep-sea mining. For the frictional pressure drop calculation a new correlation, based on a pseudoliquid model, has been proposed. In addition, parameters such as the drag coefficient of both solid and gas phase, the shape of particles and the compressibility factor, which is very important for deep-sea mining, have been incorporated in the governing equations. The application of the computational algorithm to different geometry and flow conditions of an airlift pump leads to the optimization of the system. The numerical simulation results clearly show a very good agreement with experimental and computational data of other researchers. The analysis methods have been combined in an easily used computer code which is a very useful tool for the optimum design of airlift pump systems.*

## Introduction

The Air-Lift Principle is a well-established method for vertical transport of liquids and solid-liquids mixtures. It is based on the principle of injecting a compressed gas, usually air, into the conveying pipe causing thus, under special conditions, the gradual lifting of the liquid or the mixture. During the early stages, the airlift pump was used for water pumping, later on, for lifting and transporting corrosive and radio-active liquids as well as for pumping crude oil. A special application was to pump liquids with suspended solid particles and more recently to be used in deep-sea mining.

There have been numerous publications suggesting calculation procedures for the design and the satisfactory operation of an airlift pump. Among others, Pickert (1931) and Parsons (1965) developed a procedure while working in excavating metal ore. A systematic review concerning the applications and developments of the airlift pump was presented by Chaziteodorou (1977). Weber and Dedegil (1976), Weber et al. (1978), and Weber (1982) presented a calculation model for an airlift pump to be used for lifting a suspension of solids in a liquid phase. Dedegil (1986) presented the principles of airlift techniques and Bernard and Fitremann (1987) presented a model for a transient vertical three-phase flow which is the prevailing situation when using an airlift pump for lifting of solid particles. The technology of artificial lift methods has been presented by Brown (1977–1984) in a four volumes series concerning fundamental concepts and technical data needed to design the artificial lift installations. Yet all the above studies depend either on experimental data or empirical correlation factors leading to results without general validity.

In the present study, a general calculation method for the three phase flow and a design model for an airlift pump installation are presented. This method can be used in simulation of an airlift system for pumping liquids or solid-liquid mixtures. The differential form of the governing equations derives from the fundamental fluid mechanics equations, i.e., continuity and momentum, and leads to a more general mathematical model which can be extended with the equation of energy to a more complex model, applicable for two- or three-dimensional analysis of a steady or unsteady airlift pump operation. For the prediction of a three-phase flow pattern and the calculation of frictional pressure losses, a pseudoliquid model has been introduced in an analogous way already done in particulate flows by Margaris (1989). In addition to the above, provisions were made in the computational algorithm so as to take into account the influence of more specific factors such as the compressibility factor of the gas phase, the drag coefficient of both gas and solid phase as well as the shape of the solid particles.

## Physical Modeling

The airlift pump consists of two vertical pipes. One for pumping liquid or a mixture of liquid and solid particles and a parallel one for the injection of the gas phase. As can be seen from Fig. 1, the main pipe is divided into three parts. First, there is the suction part $L_S$ extended from the bottom of the pipe to the gas injection point, second, the raising part or injection depth $L_I$ from the injection port to the free surface of the liquid in the feed tank, and last the third one and so-called discharge part $L_D$ from the free surface of the liquid to the pipe outlet. During the operation in the suction part we can have either liquid flow or a hydraulic transport of solids and so, in the parts above the gas injection port, we can have a liquid-gas flow or a hydropneumatic transport of solids, respectively.

In normal pumping installations, high energy cost is caused by the need to increase the total pressure of the liquid in order to overcome the related pressure losses while it is transported to a higher level. These losses are mainly due to gravity and second due to friction and acceleration. In an airlift pump, at first the liquid is in balance within the pipe under an existing pressure difference. The weight reduction of the liquid column caused by the injection of a gas phase forces the fluid to move upwards in order to restore the same static equilibrium. In this manner, with a suitable and continuous feed of gas, we can manage to raise the liquid to the desired level and to discharge it from the pipe. The suspended solids at the inlet of the pipe

Contributed by the Fluids Engineering Division for publication in the JOURNAL OF FLUIDS ENGINEERING. Manuscript received by the Fluids Engineering Division February 2, 1995; revised manuscript received March 10, 1997. Associate Technical Editor: M. W. Reeks.

  

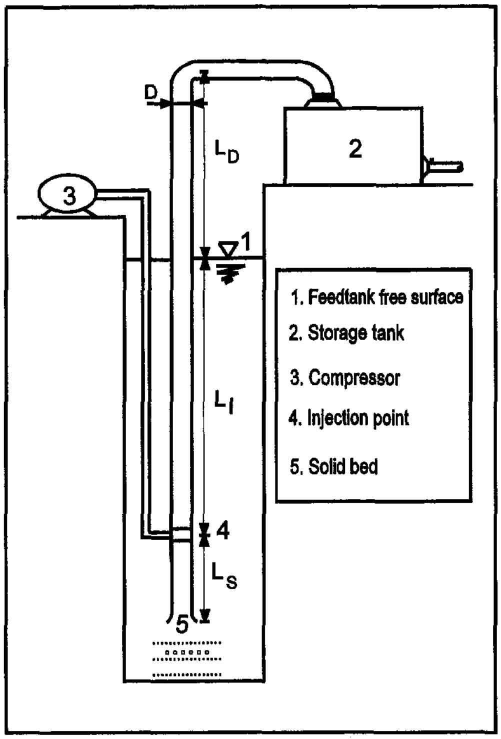


Fig. 1 Airlift pump installation

will be carried over as long as the velocity of the liquid, which tends to fill the pipe, is higher than their settling velocity.

While forming a three-phase model, it was considered that the liquid medium is always the carrier of the solid particles. As a result, the change of the gas phase velocity, will directly influence the liquid phase and indirectly the particle velocity. Because of its expansion, the gas acquires an acceleration higher than that of the liquid phase. The velocity profile thus created, develops unequal shear stresses on each solid particle causing it to revolve and move toward the lower velocity area. In this manner, eventually all the solid particles are captured in the liquid phase. This approach is valid for particles in low volume concentrations and having small dimensions.

In a two-phase flow analysis, the problem of frictional pressure drop prediction can be handled by using correlations based on the homogeneous or separated flow models. This means treating the flow as if the gas-liquid mixture is behaving like a homogeneous fluid with identical velocities of the two phases, or to treat the flow of each phase separately, taking into account the interaction forces between the phases. The fact that in a three-phase flow there are no such correlations, led some researchers to an assumption of a pure liquid on the pipe walls, taking into account only the friction of the liquid phase. In the present study, for the calculation of the frictional pressure drop, a new correlation has been introduced for a vertical gas liquid flow, similar to that proposed by Friedel (1979). In this correlation the influence of the solid particles is taken into account substituting the properties of the liquid with that of an equivalent pseudoliquid, as it is described in the mathematical modeling.

Furthermore, the following assumptions are made for the mathematical formulation of the airlift mechanism. The transport of the solid particles occurs primarily through water. The planes of equal velocity and equal pressure should be normal to the pipe axis. This makes the problem one-dimensional, which is approximately the case in practice. No particular shape of bubbles or solid particles is assumed, due to the generalized relationship used for the calculation of the drag coefficients. Flow pattern prediction for the three-phase flow can be made, applying to the gas-pseudoliquid mixture the well-known criteria of the two-phase flow, but for a first approximation no particular influence of the different flow regimes is taken into account. Finally, an isothermal change of state is assumed for air. This assumption is justified only if the three phases flow very slowly through the pipe. This seems to be the case here since air expands very little in the lower depths during rising and only for long vertical pipelines there is a critical length some meters before the exit of the pipe, where the flow has large velocities, so that a continuous heat exchange with the environment is no longer possible.

## Mathematical Modeling

**Conservation Equations for Separated Three Phase Flow.** A useful starting point for a two-phase flow analysis is to apply the separated flow model and to write conservation equations for mass and momentum for each phase separately taking into account different velocity for each phase and the interaction forces between the phases. However, each pair of conservation equations can be added together to give an overall balance equation for the mixture, and it is the overall balance equation that has been most commonly adopted in pressure drop prediction. Following the same procedure it is possible to write an overall balance equation for a gas-liquid-solid phase flow considering that the velocity as well as the pressure will be constant, in any given pipe cross-section, which makes the problem one-dimensional.

## Nomenclature

$a$ = void fraction
$A$ = area of the pipe
$b, B$ = constants
$C$ = coefficient
$C_D$ = drag coefficient
$c_m$ = mass concentration of particles
$d$ = particle or bubble diameter
$dx$ = step size
$dp$ = pressure loss
$D$ = pipe diameter
$f$ = friction factor
$F$ = force
$g$ = acceleration of gravity
$L$ = pipe length
$m$ = mass flux
$M$ = mass flow rate
$N_C$ = consumed power
$N_G$ = gained power
$p$ = pressure
$R$ = gas constant for air
Re = Reynolds number
$T$ = temperature
$u$ = phase velocity
$v_B$ = bubble volume
$w$ = interaction forces
We = Weber number
$x$ = quality
$Z$ = compressibility factor
$\Delta x$ = calculation step
$\Delta p$ = pressure loss
$\eta$ = pump efficiency
$\mu$ = viscosity
$\nu$ = kinematic viscosity
$\rho$ = density
$\sigma$ = surface tension
$\Phi$ = friction multiplier

### Subscripts

$o$ = reference magnitude
1, $s$ = solid
2, $g$ = gas
3, $l$ = liquid
$D$ = discharge length
FR = friction
$I$ = injection depth
$n$ = nominal
$O$ = one-phase
PS = pseudoliquid
$r$ = relative
$S$ = suction length
Tot = total

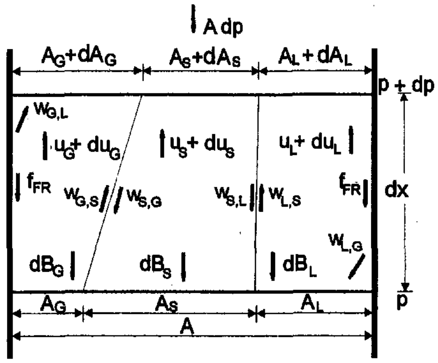


**Fig. 2 Forces acting on an elementary pipe volume**

For a separated flow model, one should consider the balance equations for flow through the vertical pipe element of length $dx$ and cross-sectional area $A$, divided into the subareas with cross section $A_S$, $A_G$, $A_L$, for solid, gas and liquid phase, respectively (Fig. 2). This separated flow concept is an advanced version of the model introduced by Chaziteodorou (1977). The continuity equation for the mixture may be expressed as

$$M_t = \sum_{i=1}^{3} M_i = \sum_{i=1}^{3} \rho_i u_i A_i = \sum_{i=1}^{3} \rho_i u_i a_i A = \text{const.} \quad \text{with} \quad \sum_{i=1}^{3} a_i = 1 \quad (1)$$

where $i = 1$ for solid, $i = 2$ for gas, $i = 3$ for liquid phase and $\rho_i$, $u_i$, and $a_i$ are the density, the velocity, and the void fraction of each phase, respectively.

The momentum equations for the separate phases, resulting from the balance of forces that are exerted on each phase in the control volume, Fig. 2, can be added to give the mixture momentum equation as follows:

$$\sum F = \sum_{i=1}^{3} \left( -\rho_i a_i A dx g - \rho_i a_i A dx \frac{du_i}{dt} - a_i A dp - a_i A dp_{\text{FR},i} + \sum_{j=1}^{3} \delta_{i,j} w_{i,j} \right) = 0 \quad (2)$$

$$\begin{Bmatrix}\text{Gravitational}\\ \text{forces}\end{Bmatrix} \begin{Bmatrix}\text{Accelerational}\\ \text{forces}\end{Bmatrix} \begin{Bmatrix}\text{Pressure}\\ \text{forces}\end{Bmatrix} \begin{Bmatrix}\text{Friction}\\ \text{forces}\end{Bmatrix} \begin{Bmatrix}\text{Interaction}\\ \text{forces}\end{Bmatrix}$$

The above equation may be applied to one, two, or three phase flow, with the last term $w_{i,j}$ indicating the interaction forces between the phases, taking into account that $w_{i,j} = -w_{j,i}$ and that $\delta_{i,j} = 0$ for $i = j$ and $\delta_{i,j} = 1$ for $i \neq j$. These forces will appear in the momentum equation for each phase and will disappear in the momentum equation for the mixture. According to this and recognizing that $du_i/dt = u_i du_i/dx$, Eq. (2) can be rewritten for the mixture pressure drop prediction as

$$-\frac{dp}{dx} = \sum_{i=1}^{3} \left( \rho_i a_i g + \rho_i a_i u_i \frac{du_i}{dx} + \left(\frac{dp}{dx}\right)_{\text{FR},i} \right) \quad (3)$$

with the last term $(dp/dx)_{\text{FR},i}$ indicating the frictional pressure drop which is the most difficult mathematical term to conceptualize.

According to the physical modeling, we assume that the liquid is the main phase and that there is no interaction forces between bubbles and solid particles. So the velocity of both gas and solid phases is expressed in relation to the velocity of the liquid phase and the relative velocity $u_{i,r}$ of the corresponding phase. That is

$$u_i = u_3 + (-1)^i u_{i,r} \quad | i = 1, 2 \quad (4)$$

The settling velocity of the solid particles or the rising velocity of the bubbles are given by the following equations,

$$u_{i,r} = \left( \frac{4}{3} \frac{d_i a_3 g}{C_{D,i}} \left| 1 - \frac{\rho_i}{\rho_3} \right| \right)^{1/2} \quad | i = 1, 2 \quad (5)$$

where for the rising velocity of bubbles we have to consider the change of the bubble diameter due to the expansion of the gas phase.

The calculation of the drag coefficient, $C_{D,i}$, of solid particles or the bubbles is based on a generalized form, similar to that proposed by Margaris and Papanikas (1989).

$$C_{D,i} = \sum_{j=1}^{2} B_{i,j} \text{Re}_i^{b_{i,j}} \quad | i = 1, 2 \quad (6)$$

where the slip velocity between the corresponding two phases, $u_i - u_3$, is used in the Reynolds number

$$\text{Re}_i = \frac{\rho_3 |u_i - u_3| d_i}{\mu_3} \quad (7)$$

and $B_{i,j}$ and $b_{i,j}$ are constants depending upon the shape of the particles or bubbles and the values of the Reynolds number $\text{Re}_i$. (For spherical particles or bubbles see Table 1 (Clift et al., 1978), for other shapes see Margaris and Papanikas (1989).

According to the airlift principle, the gas phase is supplied at the injection point under pressure $p_I$ and it is expanded to the nominal pressure $p_n$ of the storage tank. It can be assumed that its expansion will be done isothermally according to the following equation of state

$$\frac{p}{Z \rho_2} = RT = \text{const.} \quad (8)$$

The compressibility factor $Z$ for air is calculated in the present paper as a function of pressure according to the equation (Schlichting, 1958, Chaziteodorou, 1977),

$$Z(p) = 1 - 5.8198 \cdot 10^{-4} \left(\frac{p_I}{p_n}\right) + 2.809 \cdot 10^{-6} \left(\frac{p_I}{p_n}\right)^2 \quad (9)$$

Since the gas phase is not always air, the compressibility factor $Z$ can be calculated from any appropriate relation (Prausnitz et al., 1986), taking into consideration the composition of the gas and the properties of its components. A separate system

**Table 1 Constants for the calculation of the drag coefficient of spherical particles or bubbles**

| $\text{Re}_i$ | $B_{11}$ | $b_{11}$ | $B_{12}$ | $b_{12}$ | $B_{21}$ | $b_{21}$ | $B_{22}$ | $b_{22}$ |
|---|---|---|---|---|---|---|---|---|
| <0.2 | 24 | −1 | 0 | 0 | 24 | −1 | 0 | 0 |
| 0.2–989 | 26 | 0.8 | 0.4 | 0 | 3.6 | −0.313 | 24 | −1 |
| 989–$10^4$ | | | | | 0.44 | 0 | 0 | 0 |
| $10^4$–$2 \times 10^5$ | 0.4 | 0 | 0 | 0 | | | | |
| >$2 \times 10^5$ | — | — | — | — | 0.1 | 0 | 0 | 0 |

for prediction of transport properties, enabling the calculation of pure substances or mixtures properties, developed by Papanikas et al. (1993a), has been adopted and integrated into the computational system.

**Frictional Pressure Drop Correlation Based on the Pseudoliquid Model.** In order to generalize the applicability of the mathematical model taking into account the influence of solid particles on friction, irrespective of whether the flow inside the pipe is a hydraulic or a hydropneumatic transport of solids, we introduce a pseudoliquid which is formed by liquid and solid phase. The governing equations describing the behavior of this pseudoliquid are the equations for its transport properties. For the density of the pseudoliquid, $\rho_{PS}$, we have

$$\rho_{PS} = \left(\frac{a_1}{a_{PS}}\right)\rho_1 + \left(\frac{a_3}{a_{PS}}\right)\rho_3 = \alpha_1^*\rho_1 + \alpha_3^*\rho_3 \quad \text{with} \quad a_{PS} = a_1 + a_3 \tag{10}$$

where $\alpha_1^*$, $\alpha_3^*$ the void fraction of solids and liquid, with respect to the area occupied by the two phases, and $a_{PS}$ the void fraction of the pseudoliquid, with respect to the total area of the pipe cross-section.

For the dynamic viscosity of the pseudoliquid, $\mu_{PS}$, we have

$$\mu_{PS} = (1 - c_m)\nu_3\rho_{PS} \quad \text{with} \quad c_m = \frac{\rho_1}{\rho_{PS}}\left(\frac{a_1}{a_{PS}}\right) \tag{11}$$

where $\nu_3$ is the kinematic viscosity of the liquid phase and $c_m$ is the mass concentration of solid particles. The equivalent velocity $u_{PS}$ of the pseudoliquid is defined by the equation

$$M_{PS} = M_1 + M_2 = u_{PS}\rho_{PS}a_{PS}A \tag{12}$$

It is obvious that for a flow with no solid particles, the pseudoliquid behavior according to the above model will be the same as that of the liquid phase. Moreover, in most practical cases, the liquid phase appears as a mixture of more than one fluid. For example, when pumping crude oil both water and oil may form the liquid phase. It is suitable in such cases to treat the liquid phases as to be one liquid, the behavior of which may be described by the same relationships used for the pseudoliquid.

With the introduction of the pseudoliquid, any three-phase flow can be treated as a two-phase flow of a gas and a pseudoliquid. According to this the frictional pressure drop correlation, based on the separated flow model, (Hetsroni, 1982), is given by

$$\left(\frac{dp}{dx}\right)_{FR} = \Phi^2_{PSO}\left[\left(\frac{dp}{dx}\right)_{FR}\right]_{PSO} \tag{13}$$

where $\Phi_{PSO}$ is a friction multiplier for the pseudoliquid and $((dp/dx)_{FR})_{PSO}$ is the frictional pressure gradient for a single phase flow at the same total mass velocity and with the physical properties of the pseudoliquid phase. For the calculation of the friction multiplier, the Friedel correlation for a gas-liquid flow (Friedel, 1979) is extended and applied to a gas-pseudoliquid flow substituting the liquid properties with those of the pseudoliquid. According to this correlation the friction multiplier is given by

$$\Phi^2_{PSO} = E + \frac{3.24FH}{\mathrm{Fr}^{0.045} + \mathrm{We}^{0.035}} \tag{14}$$

where

$$E = (1-x)^2 + x^2\frac{\rho_{PS}f_{2O}}{\rho_2 f_{PSO}}, \quad F = x^{0.78}(1-x)^{0.24},$$

$$H = \left(\frac{\rho_{PS}}{\rho_2}\right)^{0.91}\left(\frac{\mu_2}{\mu_{PS}}\right)^{0.19}\left(1 - \frac{\mu_2}{\mu_{PS}}\right)^{0.7} \tag{15}$$

$$\mathrm{Fr} = \frac{m^2}{gD\rho^2_{PST}}, \quad \mathrm{We} = \frac{m^2 D}{\sigma\rho_{PST}},$$

$$\rho_{PST} = \left(\frac{x}{\rho_2} + \frac{1-x}{\rho_{PS}}\right)^{-1} \tag{16}$$

with $m$ the total mass flux of the two phases, $\sigma$ the surface tension, $D$ the pipe diameter and $x$ the quality or the ratio of the gas mass flux to the total mass flux, Fr is the Froude number, We is the Weber number, and $\rho_{PST}$ is the two-phase density of the gas and pseudoliquid mixture. The friction factors, $f_{2O}$ and $f_{PSO}$, for the total mass flux flowing with gas and pseudoliquid properties respectively, are calculated using the Colebrook-White (1939) nonlinear equation, which is a very accurate and valid equation for single phase flow.

For the completeness of the described analysis, it has to be noted that other two-phase flow correlations can be applied also in a similar way and this has been done in the course of this research by using an integrated computational system for a wide range of gas-liquid flows developed by Papanikas et al. (1993b).

## Design Model for an Airlift Pump

A typical airlift pump installation is shown in Fig. 1. Considering the pressure at the free surface of the feed tank to be $p_o$ and in the storage tank to be $p_n$, the pressure balance can be written as

$$p_o + \rho_3 g(L_S + L_I) = p_n + \Delta p_{Tot} \tag{17}$$

where $\Delta p_{Tot}$ are the total pressure losses along the three parts of the pipe, due to the frictional, accelerational and gravitational component of the pressure gradient, $L_S$ is the suction part and $L_I$ is the raising part or injection depth.

In the suction part, pressure losses are calculated as an integral whole according to the equation

$$\Delta p_S = \int_0^{L_S}\frac{dp}{dx}\,dx = \Delta p_{\Delta x}L_S \tag{18}$$

In the other two sections, the pressure losses should be calculated step-by-step because of the expansion of the gas phase, using the following equation:

$$\Delta p_{I,D} = \sum_1^K\left(\frac{\Delta p}{\Delta x}\right)_K\Delta x, \quad K = \frac{L_I + L_D}{\Delta x} \tag{19}$$

where the pressure gradient $dp/dx$ or $\Delta p/\Delta x$ must be calculated using the system of equations described above in the mathematical modeling.

A very important quantity is the pump or lifting efficiency, which is defined as the ratio of the gained power $N_G$ over the consumed power $N_C$. That is

$$\eta = \frac{N_G}{N_C} \tag{20}$$

The power consumed due to the gas compression from pressure $p_o$ to pressure $p_I$ at the injection level is calculated from the integral

$$N_C = \int_{p_o}^{p_I}\frac{M_2}{\rho_2}\,dp = M_2RT\ln\left(\frac{p_I}{p_o}\right) \tag{21}$$

On the other hand, the power gained is defined as the increase of the potential energy of the liquid phase, if the airlift pump is used for pumping liquids, or the solid phase, if it is used for pumping solids. Combining the two cases and taking into account the Archimedes principle for the case of lifting solid particles, the power gained is given by the following general relationship

$$N_G = CM_1 g\left( (L_S + L_l)\left( 1 - \frac{\rho_1}{\rho_3} \right) + L_D \right) + (1 - C)gL_D M_3 \quad (22)$$

with $C = 0$ for pumping liquids and $C = 1$ for pumping solids.

## Numerical Solution of a Gas-Liquid-Solid Vertical Flow

In order to design an airlift pump we have to solve the governing equations for a vertical three-phase flow. Because of the gas-phase expansion, it is necessary to calculate step-by-step the pressure gradient and the related magnitudes, such as the velocities and the void fractions of the phases. To treat this step-by-step calculation procedure, we transform the system of equations to a system of differential equations which can be solved using a suitable numerical method. That is, the momentum equation in differential form is the same with Eq. (3), so we have

$$-\frac{dp}{dx} = \sum_{i=1}^{3} \left( \rho_i a_i g + \rho_i a_i u_i \frac{du_i}{dx} + \left( \frac{dp}{dx} \right)_{\mathrm{FR},i} \right) \quad (23)$$

The continuity equations for the individual phases, ($i$ = 1, 2, 3), in differential form are

$$u_i \rho_i \frac{da_i}{dx} + u_i a_i \frac{d\rho_i}{dx} + \rho_i a_i \frac{du_i}{dx} = 0$$

$$\text{with} \quad \frac{d\rho_1}{dx} = \frac{d\rho_3}{dx} = 0 \quad \text{and} \quad \sum_{i=1}^{3} \frac{da_i}{dx} = 0 \quad (24)$$

The absolute velocity of the solid and gas phase in differential form derives from Eq. (4)

$$\frac{du_i}{dx} = \frac{du_3}{dx} + (-1)^i \frac{du_{i,r}}{dx} \quad (25)$$

with $i$ = 1, 2, while the relative velocity of particles derives from Eq. (5) for $i$ = 1, that is

$$\frac{du_{1,r}}{dx} = \left( \frac{4}{3} \frac{d_1 g}{C_{D,1}} \left( \frac{\rho_1}{\rho_3} - 1 \right) \right)^{1/2} \frac{1}{2a_3^{1/2}} \frac{da_3}{dx} \quad (26)$$

For the rising velocity of bubbles, we have to consider in addition the change of the bubble diameter due to the expansion of the gas-phase. Since the air volume of one bubble $v_B$ is related to the pressure by $p/Z \sim 1/v_B$ and because of $v_B \sim d_2^3$, it follows that $p/Z \sim 1/d_2^3$. Taking also into account the isothermal expansion of gas, the rising velocity of bubbles derives from Eq. (5) for $i$ = 2

$$u_{2,r} = \left( \frac{4}{3} \frac{d_{2,0} a_3 g}{C_{D,2}} \left( \frac{p_0}{Z_0} \right)^{1/3} \right)^{1/2} \left( 1 - \frac{Z_0 \rho_{2,0} p}{p_0 \rho_3 Z} \right)^{1/2} \left( \frac{Z}{p} \right)^{1/6} \quad (27)$$

with $d_{2,0}$ the initial bubble diameter at the injection point where the pressure is $p_0$ and the compressibility factor is $Z_0$. The differential equation for the rising velocity of bubbles follows from Eq. (27)

$$\frac{du_{2,r}}{dx} = C_1 \left( C_2 \left( Z^{-1} \frac{dZ}{dp} - p^{-1} \right) \right) \frac{dp}{dx} + \left( \frac{\rho_3 - \rho_2}{4\rho_3 a_3^2} \right)^{1/2} \frac{da_3}{dx} \quad (28)$$

with the coefficients $C1$ and $C2$ given by the relations

$$C_1 = \left( \frac{4}{3} \frac{d_{2,0} g a_3}{C_{D,2}} \right)^{1/2} \left( \frac{p_0}{p} \frac{Z}{Z_0} \right)^{1/6}$$

$$\text{and} \quad C_2 = \frac{1}{6} \frac{\rho_3 + 2\rho_2}{(2\rho_3 - \rho_2)^{1/2}} \quad (29)$$

For the expansion of the gas phase, differentiated Eq. (8), we get

$$\frac{dZ}{dx} = \left( \frac{dp}{dx} - \frac{p}{\rho_2} \frac{d\rho_2}{dx} \right) \frac{Z}{p} \quad (30)$$

In the framework of the present paper a Runge-Kutta fourth-order method was used in order to solve the above system of differential equations for the gas-liquid-solid flow. When this system is applied to the liquid-solid flow at the lower part of an airlift system, it gives a constant pressure gradient $dp/dx$, which multiplied by the length $L_S$ gives the corresponding pressure drop. Combining this result with those related to the gas phase for the relative velocity and the compressibility factor, we determine the necessary initial conditions for the Runge-Kutta method.

The variables, which must be defined, for the solution of the gas-liquid-solid flow, are the physical properties of the phases (density and viscosity of liquid and solid phase, diameter of solid particles etc.), the geometry of the pipe (diameter, total and injection depth), and the mass flow rate of solids. For these conditions, the computational algorithm calculates the necessary mass flow rate of gas. The calculation starts with the assumption that the pressure at the injection point is equal to the hydrostatic pressure outside the pipe at the same depth. An iterational procedure has been established for the correction of the pressure, taking into account the condition that at the exit of the pipe the pressure must be equal to the atmospheric. The procedure is repeated until the calculated value of pressure differs from the atmospheric one with less than 0.5 percent. This accuracy can be predetermined by the user, depending on the computing time and the final results. A lower limit will increase the computing time without dramatical improvement of the final results, so the difference of 0.5 percent is considered as an effective limit for engineering applications.

## Results and Discussion

In order to validate our analysis, a large number of results were compared against both experimental and computed data by other researchers. But the existing experimental data in the literature do not cover a full range of the operational characteristic curve of an airlift pump, as they are referred every time to different conditions or to airlift pumps for pumping liquids only. Weber (1976, 1982) presented in a table a lot of experimental data obtained at the lignite open-pit mining of the Rheinische Braunkohle AG in the vicinity of Cologne, and for validation purposes he compared calculated versus measured volume flow rate of solids. There is no characteristic curve diagram because the experiments were carried out with the following varying geometry conditions: pipe diameter 300 mm, total depth 50 to 441 m, injection depth 42 to 248 m, air supply 0.22 to 0.713 $m^3/s$, delivered volumetric concentrations 0 to 8.6 percent and maximum out put of solid material 115 t/h. The solid materials were gravel, sand, and lignite with densities 2575, 2610, and

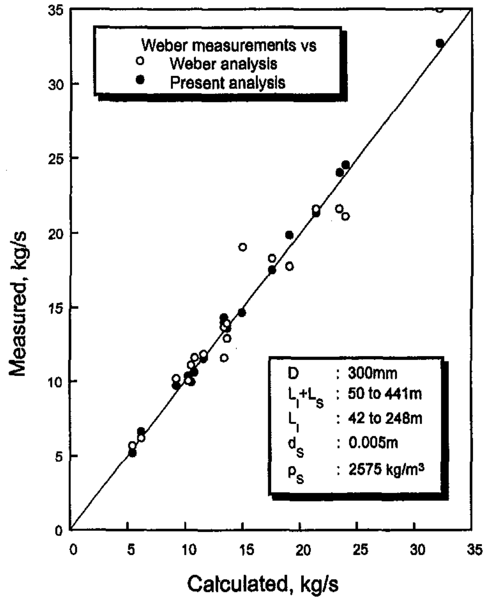


**Fig. 3 Comparison between calculated and measured mass flow rate of solids**

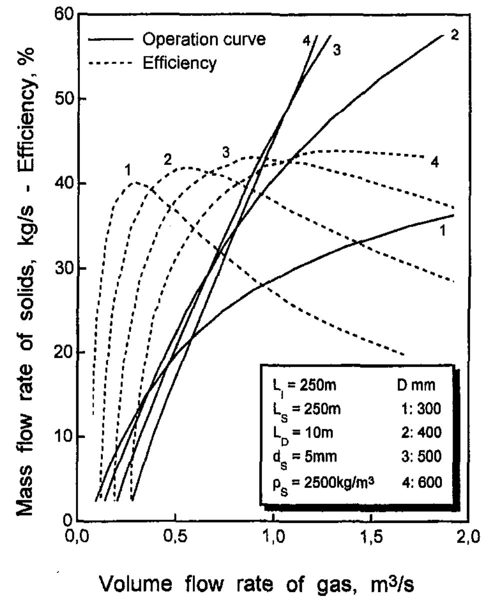


**Fig. 5 Influence of the pipe diameter on the airlift pump operation**

1143 kg/m³ and particle diameter 5, 0.6 and 50 mm, respectively.

The validation of our results is presented in Figs. 3 and 4 through the comparison of our calculated results, according to the above data, versus measured and calculated by Weber, for the mass flow rate of solid and liquid. The agreement of our results with the experimental data, especially for the liquid, is obviously better than that of Weber. The average deviation of our results is 5 percent for the solid flow rate and 0.3 percent for the liquid flow rate, while the corresponding values for Weber's results are 10 percent and 15 percent, respectively. The difference is due to the fact that the analysis of Weber is based on empirical or mechanistic model, while our analysis is based on the fundamental equations of fluid mechanics.

Applying our analysis, we next present the influence of the most important parameters, such as the diameter of the pipe and the injection depth, to the airlift pump operation. In all the diagrams presented here, seawater with density of 1030 kg/m³ is used for the liquid phase, the gas is always air with density 1.2 kg/m³, and three solid materials with densities of 2650, 2500, and 1150 kg/m³ are used. The initial value of the bubble diameter is 3 cm and the volumetric concentration of solids coming into the system at the lower pipe section is 5 percent, which is the proposed value by Weber (1976) for optimum capacity of the pump. The pressure at the free surface of both storage and feed tank were atmospheric and the temperature inside the pipe was considered to be 10°C. In order to be more clear we use for the magnitudes the subscripts $s$, $g$, and $l$ for solid, gas, and liquid instead of 1, 2, 3, respectively, which are used in the mathematical equations.

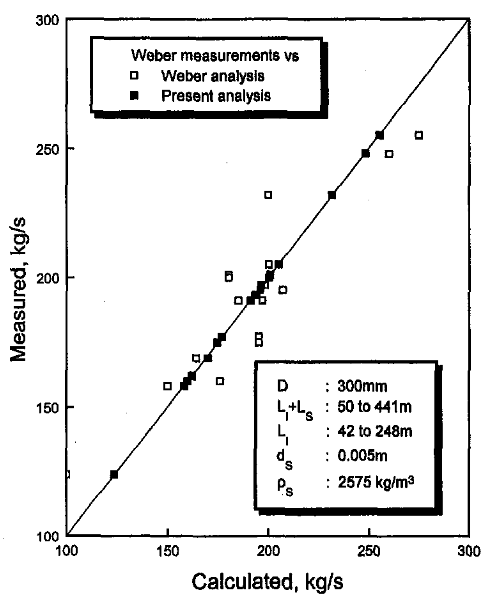


**Fig. 4 Comparison between calculated and measured mass flow rate of liquid**

An operation curve of an airlift pump shows mass flow rate of solids and pump efficiency as a function of gas flow rate delivered by the compressor (Fig. 5). The most significant geometric parameter is the pipe diameter and its magnitude has a great effect in the airlift efficiency. In case of relatively large quantities of the gas phase, resulting in large concentrations, undesirable flow regimes like plug, annular or mist flow is possible to appear. These flow regimes cause a very low efficiency (Chaziteodorou, 1977). This drawback can be corrected by increasing the pipe diameter thus decreasing the gas void fraction. In this manner, the flow can be bubble flow and the pump efficiency will remain at high levels. This gradual increase of the pipe diameter helps to avoid unpleasant situations in airlift pump installations.

In bubble flow, the bubbles undergo random motion in passing through the channel; from time to time, two bubbles collide and may coalesce to form a larger bubble. This process of collision and coalescence ultimately leads to plug flow. Radovcick and Moissis (1962) showed that for void fractions higher than 30 percent, collision and coalescence of bubbles becomes very rapid and bubble flow is very unstable, but at high velocities we may have bubbly flows at higher void fractions. Applying this rule to our analysis for the three-phase flow we assume a bubble flow for void fractions up to 50 percent. Above this limit, the computational algorithm will increase the pipe diameter in order to maintain the void fraction less than 50 percent.

It is obvious that an optimization of the system is important and for the installation depicted in Fig. 6, applying our analysis, the optimum diameter resulted to be 0.44 m. It should also be

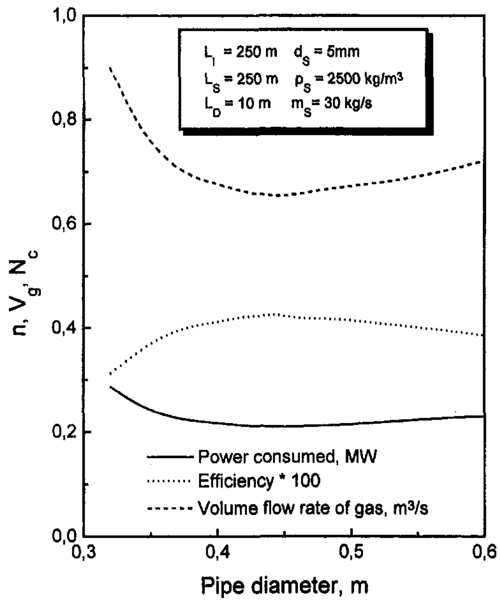


**Fig. 6 Optimization of the pipe diameter of an airlift pump installation**

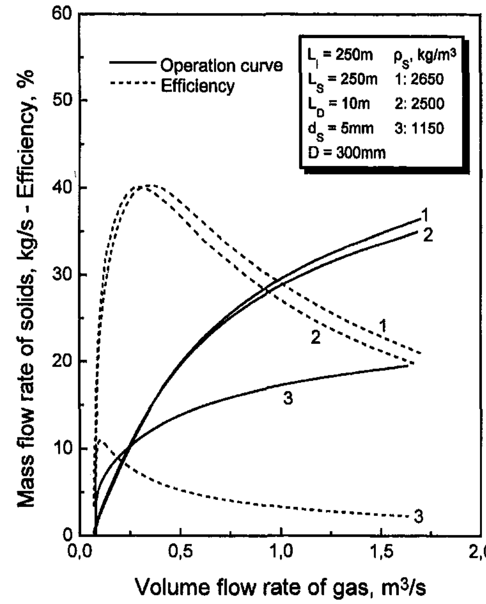


**Fig. 8 Influence of solid density on the airlift pump operation**

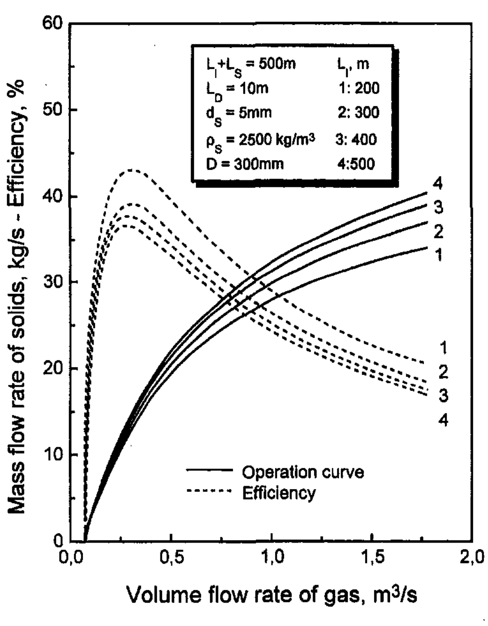


**Fig. 7 Influence of the gas injection depth on the airlift pump operation**

noted that for different mass flow rates of solids different optimum diameters would result. The optimum design of an airlift system should aim to obtain not always the maximum efficiency but a high enough efficiency for a wide range of applications, which means a wide range of mass flow rates of solid particles.

Another important parameter in the airlift design is the injection depth $L_I$ of the gas phase (Fig. 7). A bad choice of gas injection depth can cause a decrease in pump efficiency due to the existence of undesirable flow regimes, such as annular flow, where the liquid flows on the wall of the tube as a film and the gas phase flows in the center. This flow regime can be caused by a large pressure difference between the gas injection port and the pressure of the storage tank. A way to overcome this phenomenon, in large scale installations, is to inject the gas phase to more than one level thus decreasing the pressure differences between the one port to the next.

Generally the increase in the gas injection depth results in an increase of the amount of solids that can be pumped followed by a parallel decrease of the maximum pump efficiency, as related to the power consumed. On the other hand, by increasing the injection depth $L_I$, for pumping a certain quantity of solids, we can also increase the total submerged length, $L_I + L_S$. But as the injection depth increases the suction part decreases, eventually reaching the limit where the gas injection point is at the same level with the solid bed. This phenomenon is of great importance to large scale installations, such as deep-sea mining.

Negative effects on the pump efficiency have the discharge length $L_D$, due to the pressure losses, the diameter of solid particles and the initial diameter of bubbles, due to the increase of their relative velocity. These magnitudes must be as small as possible, depending on the specific demands and design of the installation, in order to achieve higher efficiency of the system. Although the increase of the density of solids results in an increase of their relative velocity it has the opposite effect to that of the diameter of the solid particles. This is due to the fact that in order to pump the same quantity of solids the increase of density means lower solids volume flow rate and eventually lower liquid and gas flow rates. For example, in Fig. 8 we note that for pumping manganese nodules with mass flow rates 15 kg/s and density of 2650 kg/m$^3$ we need 0.4 m$^3$/s of gas while for lignite with density 1150 kg/m$^3$ we need 0.7 m$^3$/s of gas phase.

The purpose of the airlift designer is to combine all the above geometric and functional parameters in order to achieve at first the maximum efficiency while the solids flow rate is large enough, on the other hand, to maintain a satisfactory efficiency over a large range of gas flow rates.

## Conclusions

An applied **A**ir **L**ift **M**odel **A**nalysis for air-water-solid flow has been developed. Based on a system of differential equations, derived from the fundamental equations of continuity and momentum conservation, this model has a very good performance and a more general mathematical form, compared to other models based on a power balance and making a superposition of an air-water flow on a water-solid flow to form the three-phase flow. This model has been combined in an easily used computer code, named ALMA, which is a very useful tool for the optimum design of airlift pump installations.

The optimization of the installation is the more important feature of the code. This means that the code can calculate the optimum value for the pipe diameter, injection depth, and the other parameters in order to minimize the energy consumption. The method can be applied to an air lift pump installation with short or long vertical pipeline system since it is independent of the flow regime in the pipe. From the injection point the flow is bubble and only in the upper part of a long vertical pipeline system we may have transition from bubble flow to annular flow, due to the expansion of the gas phase. For the present, we assume an upper limit of 50 percent for the void fraction of air, in order to avoid the transition, to maintain the flow bubble and to achieve higher efficiency of the system.

The results of the present analysis are in good agreement with the existing data in the literature, but in order to approximate the phenomenon better an extended modeling is in progress now, taking into account the flow regimes that may exist in the pipeline, as well as the influence of the solid particles to the development of the flow regimes. This extended modeling is expected to contribute more to the optimum design of the airlift pump installations.


## Acknowledgments

The present study was supported in part by the Directorate General for Energy of the Commission of the European Communities Union in the framework of the THERMIE project OG/041/90.


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