

Damir ŽARKO, Stjepan STIPETIĆ, Tanja POLJUGAN

University of Zagreb Faculty of Electrical Engineering and Computing

## Simplified model of transformer hysteresis curve

**Abstract.** A simplified model of transformer hysteresis curve utilized for calculation of core losses is presented. It incorporates eddy current losses and hysteresis losses into the same hysteresis curve thus simplifying a set of equations used for solving the magnetic circuit. The parameters of the hysteresis curve model are determined by evolutionary optimization which minimizes the difference between measured and calculated current waveforms from open-circuit test of a three-phase, three-leg transformer rated 400 V, 20.8 kVA. The core losses calculated from the surface area of hysteresis curves calculated for all three legs with supply voltage variation from zero to 10% above rated show a good match with measured losses.

**Streszczenie.** W artykule zaprezentowano uproszczony model krzywej histerezy w transformatorze, służący do obliczania strat w rdzeniu. Model zawiera zarówno straty wiroprądowe jak i straty histerezy w jedną krzywą histerezy przez co upraszcza się układ równań używany w rozwiązywaniu obwodu magnetycznego. Parametry modelowanej krzywej histerezy określone są w procesie optymalizacji ewolucyjnej, która prowadzi do minimalizacji różnicy pomiędzy mierzoną i obliczoną wartością chwilową prądu próby stanu jałowego trójfazowego, trójkolumnowego transformatora o znamionach: 400 V i 20,8 kVA. Straty w rdzeniu obliczane przez całkowanie powierzchni pętli histerezy dla wszystkich trzech kolumn przy zasilaniu napięciem od zera do 10% wartości znamionowej pokazują dobrą zgodność z pomierzonymi stratami. (**Uproszczony model krzywej histerezy transformatora.**)

**Keywords:** transformer, hysteresis curve, optimization, core losses

**Słowa kluczowe:** transformator, krzywa histerezy, optymalizacja, straty w rdzeniu

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### Introduction

Reliable models of transformer core with inclusion of nonlinearities in the form hysteresis curve can be used in the design stage for correct calculation of core losses and open-circuit current [1], [2]. They can be also used to simulate transients in the transformer like inrush current. One of the problems that one might face is variation of the core properties in seemingly symmetrical sections of the transformer core that can emerge due to unequal assembly of laminations at the connection of the transformer leg (vertical portion of the core) and the transformer yokes (top and bottom horizontal portions of the core). In a three-leg, three-phase transformer such asymmetry is inherent for the middle leg, but it can also occur for the outer legs resulting in different open-circuit current waveforms in all three phases as shown in this paper. A simplified approach to core modelling presented in the paper which relies on measured open-circuit voltage and current waveforms can capture these asymmetries in an actual transformer and correctly calculate the total core losses.

### Model of the transformer core

A physically correct lumped parameter model of the transformer magnetic circuit should include magnetomotive forces (MMF) of primary and secondary windings in each leg and MMFs due to induced eddy currents which are proportional to the derivative of the core flux [3]

$$(1) \quad F_{ei} = k_{ei} \frac{d\Phi_i}{dt}, \quad i = 1, 2, 3$$

where  $F_{ei}$  is the component of magnetomotive force due to eddy currents in the  $i^{\text{th}}$  leg,  $k_{ei}$  is the eddy current coefficient and  $\Phi_i$  is the total flux of the  $i^{\text{th}}$  leg. The core fluxes  $\Phi_i$  are at the same time the unknown variables to be solved. The eddy current coefficient  $k_{ei}$  is a function of lamination size and magnetic properties and is defined by an expression given in [3].

The simplified model presented in this paper considers the fact that eddy currents induced in the core laminations will produce power losses which will result in an additional component of current from the power supply that will be in phase with the supply voltage. Similarly, due to hysteresis losses in the core, a component of primary current in phase with the supply voltage will emerge. Since both current

components that originate from core losses are in phase with the voltage, the component due to eddy currents can be substituted with an increased hysteresis component which will result in slightly wider hysteresis loop (dynamic loop). If such wider hysteresis loop can be determined to result in the same waveforms of phase currents as measured in an open-circuit three-legged transformer, then surface area of this hysteresis loop multiplied by the core volume and frequency should match the measured core losses which include both eddy current and hysteresis losses. With this assumption, a set of equations can be formed using notation defined in Fig. 1. Transformer connection is Yy0.

$$(2) \quad \left. \begin{aligned} H_1 l_1 - H_2 l_2 &= N_p i_1 - N_p i_2 \\ H_2 l_2 - H_3 l_3 &= N_p i_2 - N_p i_3 \\ H_1 l_1 - H_3 l_3 &= N_p i_1 - N_p i_3 \\ i_1 + i_2 + i_3 &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} i_1 &= \frac{2H_1 l_1 - H_2 l_2 - H_3 l_3}{3N_p} \\ i_2 &= \frac{-H_1 l_1 + 2H_2 l_2 - H_3 l_3}{3N_p} \\ i_3 &= \frac{-H_1 l_1 - H_2 l_2 + 2H_3 l_3}{3N_p} \end{aligned}$$

In (2),  $H$ ,  $l$  and  $i$  denote the field strength, core section length and current of phases 1, 2 and 3 respectively, while  $N_p$  and  $N_s$  and the number of turns of the primary and secondary windings.

Flux linkages of each phase are obtained by integrating the difference between applied voltage and voltage drop on the winding resistance, i.e.

$$(3) \quad \psi_i(t) = \int_0^t [u_i(t) - i_i(t) R_i] dt, \quad i = 1, 2, 3$$

where  $u_i$ ,  $i_i$  and  $R_i$  are the voltage, current and resistance of the  $i^{\text{th}}$  phase winding respectively. Thus calculated  $\psi$ - $i$  loops are shown in Fig. 2. The asymmetry of  $\psi$ - $i$  loops in outer legs is evident indicating that most probably the quality of overlap of laminations at the connection of leg and yoke is not the same at both legs.

The average flux densities in the core legs are obtained from the expression

$$(4) \quad B_i(t) = \frac{\psi_i(t)}{N_p A_c}$$

where  $A_c$  is the core cross-section area.

Transformer ratings are given in Table 1. Phase winding resistances are measured using low voltage DC power supply with high precision voltmeter and ampermeter connected to each phase separately. The phase resistances thus measured in the laboratory at the ambient temperature of 21.5 °C are:  $R_{1,21.5}=62.033 \text{ m}\Omega$ ,  $R_{2,21.5}=61.329 \text{ m}\Omega$ ,  $R_{3,21.5}=61.801 \text{ m}\Omega$ .

From the known flux densities the field strengths  $H_1$ ,  $H_2$  and  $H_3$  required for calculation of phase currents in (2) are determined using hysteresis model defined in [4] and modified in this particular case by replacing the *sign* function with *tanh* and adding the additional coefficients  $k_1$  and  $k_2$ . The model is defined with

$$(5) \quad H(t) = \frac{B(t)}{a - bB(t) \tanh[k_1 B(t)]} + K_h \left| \frac{dB(t)}{dt} \right|^\alpha \tanh \left[ k_2 \frac{dB(t)}{dt} \right]$$

where  $a$ ,  $b$ ,  $\alpha$ ,  $K_h$ ,  $k_1$  and  $k_2$  are the unknown coefficients determined by Differential evolution (DE) [5] optimization algorithm for each core leg.

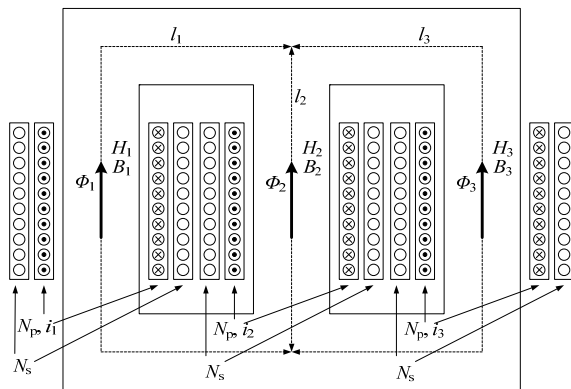


Fig.1. Three-leg transformer core

The Yy0 connection of the transformer does not provide the path for zero sequence current to flow in the transformer winding. As a consequence a third harmonic of flux emerges in the core. However, since core has three legs without independent magnetic paths, the third harmonic of flux is forced to close through the surrounding air, which is a path of high magnetic reluctance. Therefore, in spite of the fact that ampere turns which excite the third harmonic of flux exist, the magnitude of the third harmonic flux density is negligible since it is inversely proportional to magnetic reluctance. This fact allows us to use a hysteresis model which does not include minor hysteresis loops that would otherwise emerge in the case of significant presence of third harmonic flux density like in a five-leg core of three independent cores. In order to verify the absence of harmonics in flux density that lead to minor loops, the time variation of flux density (Fig. 3) is calculated using (3) and (4) at maximum terminal line-to-line voltage of 440 V (maximum saturation) at which it is most likely to expect the highest deviation of flux density waveform.

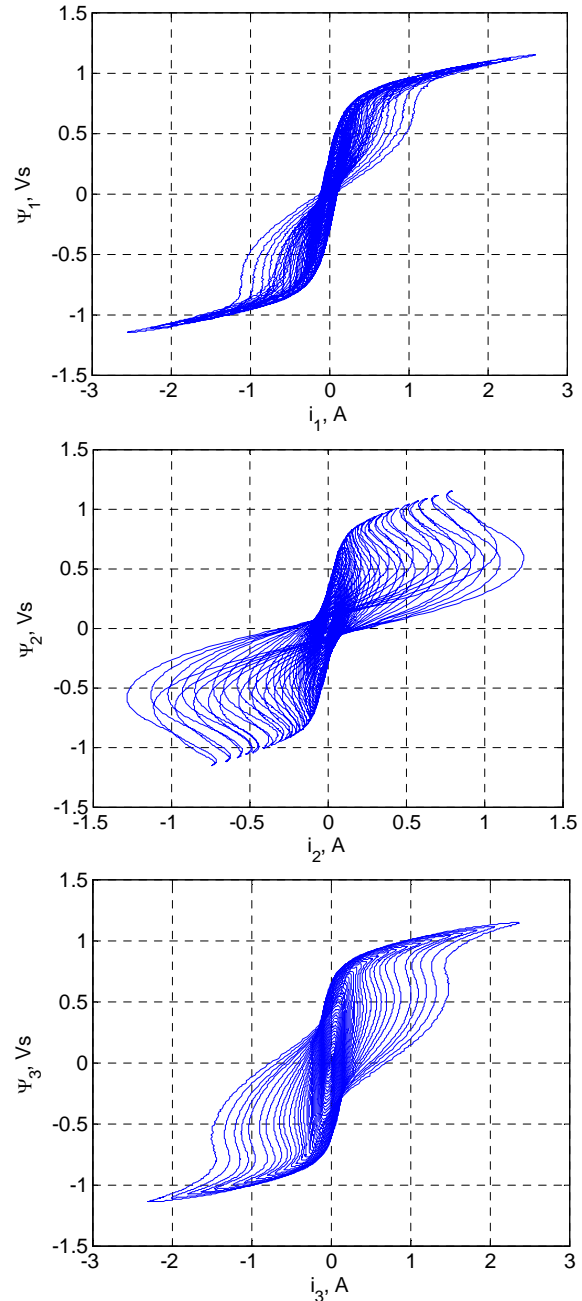


Fig.2.  $\psi$ - $i$  loops of phase windings 1 to 3

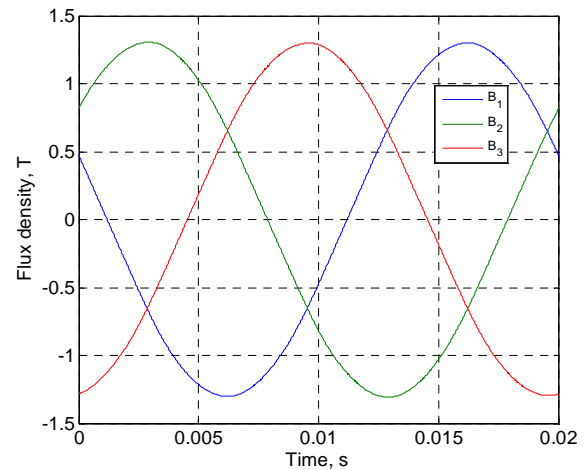


Fig.3. Flux density waveforms in transformer legs at maximum terminal voltage of 440 V

Table 1. Transformer data

Power	20.8 kVA
Primary	400 V, 30 A
Secondary	100 V/100 V, 60A/60 A
Connection	Yy0
Frequency	50 Hz
Primary number of turns, $N_p$	114
Core cross-section area, $A_c$	$77.38 \cdot 10^{-4} \text{ m}^2$
Length of left and right leg, $l_1$ and $l_3$	0.795 m
Length of middle leg, $l_2$	0.355 m
Volume of left and right leg, $V_1$ and $V_3$	$0.005824364 \text{ m}^3$
Volume of middle leg, $V_2$	$0.00274699 \text{ m}^3$

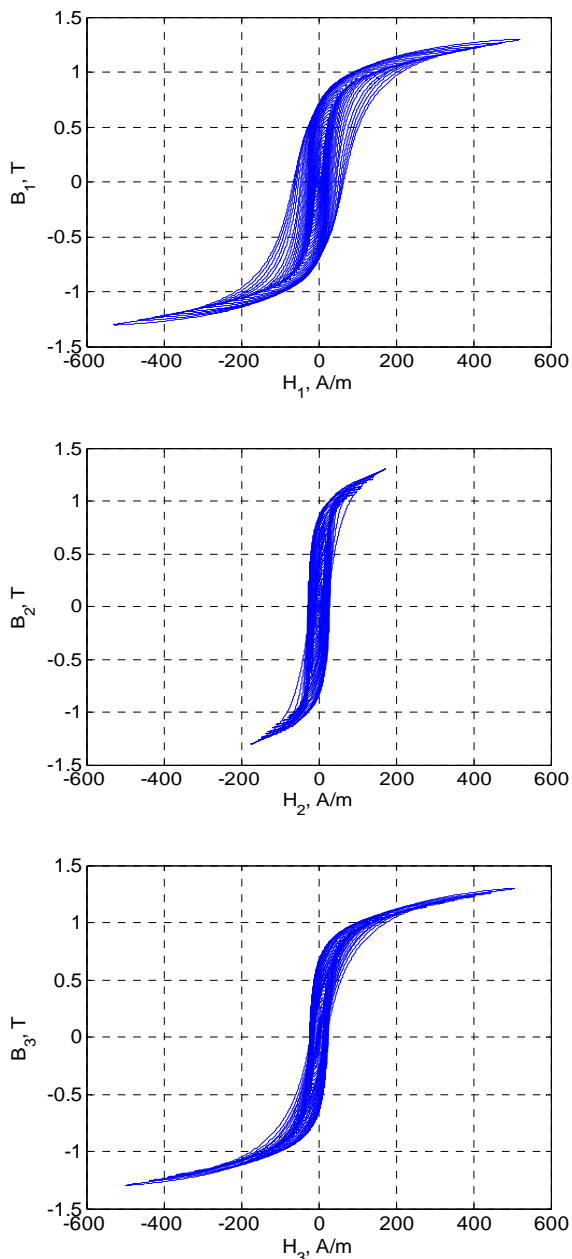


Fig.4. Calculated B-H loops of core legs 1 to 3 with different supply voltages

**Calculation of B-H loops**

The B-H loops for all three legs are determined for each voltage level applied to the transformer. The voltage was varied from 440 V down to 10 V in 10 V steps. The voltage and current waveforms, and average power (active, reactive and apparent) have been measured using NORMA 4000 power analyzer. From each phase current and voltage waveform the time variation of flux density  $B(t)$  is calculated

using (3) and (4). The corresponding field strength  $H(t)$  is determined using (5). The unknown coefficients in (5) are calculated using Differential evolution optimization algorithm. There are a total of 18 coefficients to be determined for the three B-H loops. The list of coefficients with lower and upper limits that define intervals inside which the optimal value of each coefficient is sought is provided in Table 2. The population size is 360, while DE control parameters are set to  $F=0.8$ ,  $CR=0.9$ . The strategy used is DE/best/1/exp.

The cost function  $F_c$  to be minimized is the sum of squared differences of calculated and measured phase current waveforms within one full cycle of 20 ms which can be written as

$$F_c = \sum_{k=1}^N \left[ \left( i_{1k}^{calc} - i_{1k}^{meas} \right)^2 + \left( i_{2k}^{calc} - i_{2k}^{meas} \right)^2 + \left( i_{3k}^{calc} - i_{3k}^{meas} \right)^2 \right] \tag{6}$$

where  $N=684$  is the number of discrete values of current measured and calculated at the time instants

$$t_k = \frac{0.02}{N-1} k, \quad k = 0, 1, \dots, 683 \text{ within the period of 20 ms,}$$

while  $i_{1k}^{calc}$ ,  $i_{2k}^{calc}$ ,  $i_{3k}^{calc}$ ,  $i_{1k}^{meas}$ ,  $i_{2k}^{meas}$ , and  $i_{3k}^{meas}$  are the calculated and measured instantaneous currents of phases 1, 2 and 3 respectively. The results of optimization are B-H loops shown in Fig. 4. For the case of rated voltage of 400 V, Fig. 5 compares measured and calculated currents obtained from (2) and (5) using optimized values of 18 coefficients calculated with DE for that particular voltage. Connecting the tops of the B-H loops yields magnetization curves for all three legs shown in Fig. 6. Those magnetization curves take into account the combined magnetic reluctance of iron and air at the joints of leg and yoke lamination sheets.

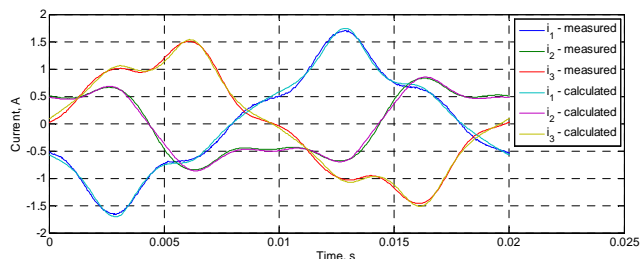
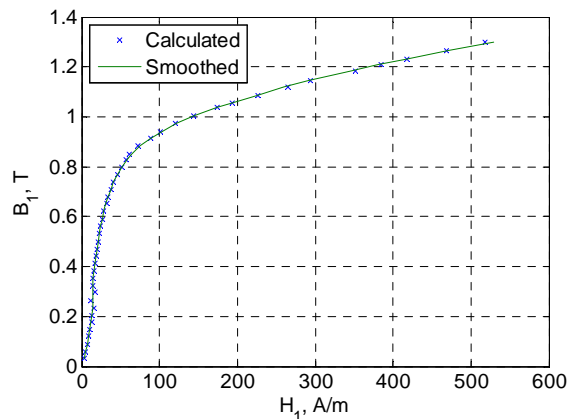


Fig.5. Comparison of measured and calculated open-circuit currents at rated voltage of 400 V



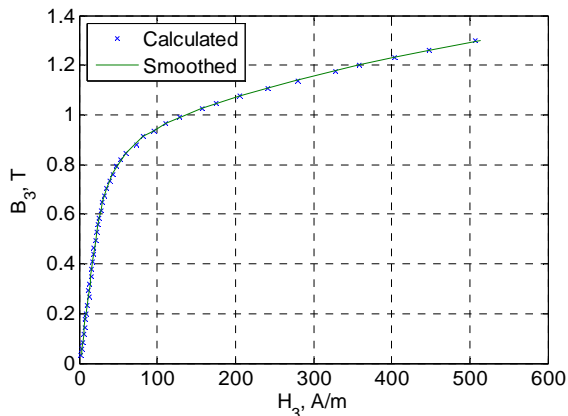
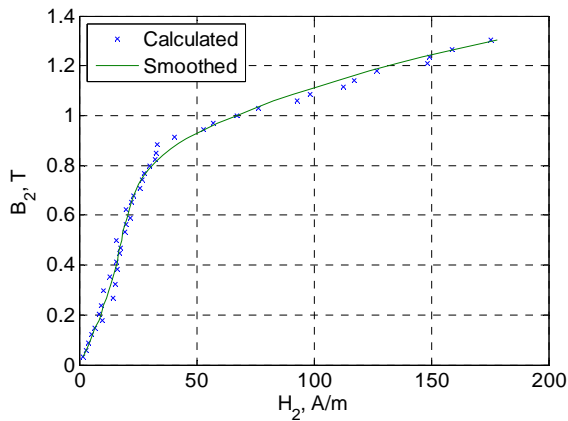


Fig.6. Calculated magnetization curves of core legs 1 to 3

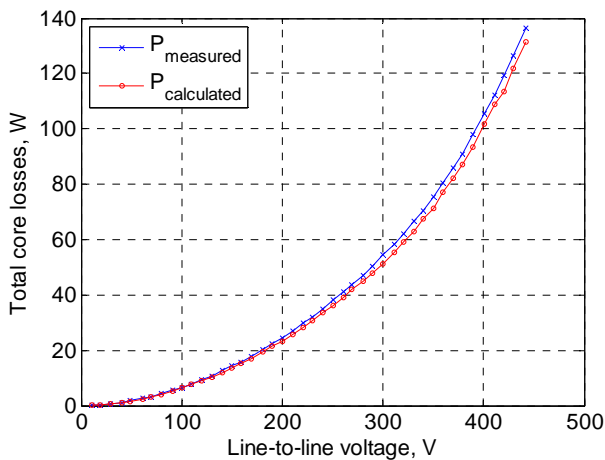


Fig.7. Comparison of measured and calculated total core losses as a function of terminal voltage

Table 2. List of variables to be optimized with their minimum and maximum values

No.	Coefficients	min	max	No.	Coefficients	min	max
1	$a_1$	0	10	10	$b_3$	0	10
2	$b_1$	0	10	11	$K_{h3}$	10	150
3	$K_{h1}$	10	150	12	$a_3$	0	1
4	$\alpha_1$	0	1	13	$k_{11}$	0	500
5	$a_2$	0	10	14	$k_{12}$	0	500
6	$b_2$	0	10	15	$k_{21}$	0	500
7	$K_{h2}$	10	150	16	$k_{22}$	0	500
8	$\alpha_2$	0	1	17	$k_{31}$	0	500
9	$a_3$	0	10	18	$k_{32}$	0	500

### Core losses

Total losses have been measured in no-load operation for each voltage level as previously described. The core losses are obtained by subtracting the copper losses from total losses, i.e.

$$(7) \quad P_c^{meas} = P - I_1^2 R_1 - I_2^2 R_2 - I_3^2 R_3$$

Core losses are determined from B-H loops by calculating their surface area using equation

$$(8) \quad P_c^{calc} = V_1 f \oint H_1 dB_1 + V_2 f \oint H_2 dB_2 + V_3 f \oint H_3 dB_3$$

where  $f$  is the supply frequency (50 Hz), and  $V_1, V_2, V_3$  are the volumes of left, middle and right leg respectively.

Calculated and measured total core losses are compared in Fig. 7. The difference between measured and calculated core losses for voltage variation between 10 V and 440 V is from 3 % to 5.8 %.

### Conclusion

The presented simplified model of transformer hysteresis curve is capable of predicting correctly the open-circuit current waveforms and core losses even in the case of asymmetries in the core assembly. The determination of the model parameters is performed using evolutionary optimization and relies on measured waveforms of no-load phase voltages and currents.

The calculated B-H curves of core legs and calculated specific core losses take into account the deviation from the inherent properties of core laminations due to processing and handling (punching, assembly, pressing) and the presence of small air gaps at the joints of lag and yoke lamination sheets.

The model in its presented form is limited to the case when no minor B-H loops originating from third harmonic of flux density are present which is common for three-leg transformers with or without neutral conductor.

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**Authors:** prof.dr. Damir Žarko, Stjepan Stipetić, dipl.ing., Tanja Poljungan, dipl.ing., University of Zagreb Faculty of Electrical Engineering and Computing, Department of Electric Machines, Drives and Automation, Unska 3, HR-10000 Zagreb, Croatia.  
E-mail: damir.zarko@fer.hr, stjepan.stipetic@fer.hr, tanja.poljungan@fer.hr