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Solidification of Metals and Alloys,  
Year 2000, Volume 2, Book No. 44  
Krzepnięcie Metali i Stopów,  
Rok 2000, Rocznik 2, Nr 44  
PAN – Katowice PL ISSN 0208-9386

## NUMERICAL SOLUTION OF 2D STEFAN PROBLEM

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### SUMMARY

The numerical solution of 2D Stefan problem is discussed. The Stefan model describes the solidification of pure metals or eutectic alloys in macro scale. From the numerical view point the solution of this task is very complex, in particular, for 2D or 3D domains. In literature one can find the algorithms basing on the substitution of the Stefan model by the artificial mushy zone one. Such approach is also presented in this paper.

### 1. INTRODUCTION

The concept of the Stefan problem transformation by conventional extension of the solidification point  $T_c$  on interval  $[T^- = T_c - \Delta T, T^+ = T_c + \Delta T]$  was presented in the past by Budak and Solovieva in [1]. The discontinuity of enthalpy at point  $T_c$  is substituted by a continuous function - as in Figure 1. In this way the new sub-domain appears which can be called the artificial mushy zone. A specific heat of casting domain defined as  $dH/dT$  becomes the following function

$$c(T) = \begin{cases} c_L & T > T^+ \\ c_M & T^- \leq T \leq T^+ \\ c_S & T < T^- \end{cases} \quad (1)$$

The course of specific heat  $[J/m^3K]$  is shown in figure 2. So, the numerical algorithm realizing the Stefan problem simulation is exactly the same as the algorithm for the mushy zone model in formulation called the fixed domain approach [2]

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Numerical experiments show that using this formulation introduced in artificial way the value of  $\Delta T$  has the visible influence on the results of numerical computations

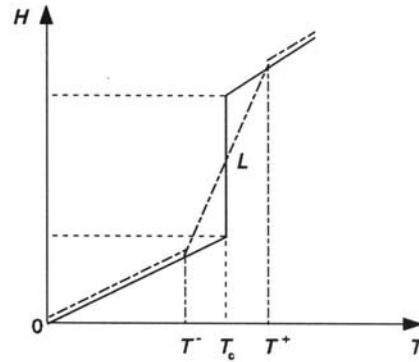


Fig. 1. Modified enthalpy function  
Rys. 1. Zmodyfikowana funkcja entalpii

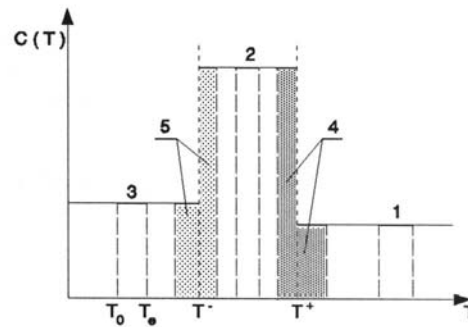


Fig. 2. Specific heat of metal  
Rys. 2. Ciepło właściwe metalu

The essential improvement of the approach discussed was presented by Hsiao in [3]. Let us  $0$  and  $e$  denote two adjoining points resulting from casting domain discretization (for instance, the nodes of differential mesh). The specific heat  $c_{0e}$  is defined as an integral mean:

$$c_{0e} = \frac{1}{T_e - T_0} \int_{T_0}^{T_e} c(T) dT \quad (2)$$

For the assumed course of  $c(T)$  and  $T_e > T_0$  we have to consider the following situations

$$\begin{aligned}
T_0, T_e \rangle T^+ : & \quad c_{0e} = c_L \\
T_0, T_e \in [T^-, T^+] : & \quad c_{0e} = c_M \\
T_0, T_e \langle T^- : & \quad c_{0e} = c_S \\
T_0 \in [T^-, T^+], T_e \rangle T^- : & \quad c_{0e} = \frac{c_L(T_e - T^+) + c_M(T^+ - T_0)}{T_e - T_0} \\
T_0 \langle T^-, T_e \in [T^-, T^+] : & \quad c_{0e} = \frac{c_M(T_e - T^-) + c_S(T^- - T_0)}{T_e - T_0} \\
T_e \rangle T^+, T_0 \langle T^- : & \quad c_{0e} = \frac{c_L(T_e - T^+) + c_M(T^+ - T^-) + c_S(T^- - T_0)}{T_e - T_0}
\end{aligned} \tag{3}$$

The transitions 1, 2, 3, 4 and 5 are shown in Figure 2. The same set of formulas can be written for  $T_e < T_0$ .

Let us  $e = 1, 2, \dots, n$  denotes the indexes of nodes from the neighbourhood of node 0. Hsiao proposed the following definition of specific heat at node 0:

$$c_0 = \frac{1}{n} \sum_{e=1}^n c_{0e} \tag{4}$$

The testing computations presented in [3] and also the numerical experiments done by the authors of this paper show that the results obtained using this approach are quite satisfactory and the value of interval  $\Delta T$  has not essential influence on the final solution.

## 2. MODIFICATION OF HSIAO ALGORITHM

From the numerical view point the Hsiao algorithm is rather complicated (the big number of different formulas, the changing number  $n$  for successive points 0, the relation between  $T_0$  and  $T_e$ . The authors of this paper propose a simpler approach, which gives the same effect. We can notice that

$$\int_{T_0}^{T_e} c(T) dT = \int_0^{T_e} c(T) dT - \int_0^{T_0} c(T) dT = H(T_e) - H(T_0) \tag{5}$$

where  $H$  is the enthalpy function which course is shown in Figure 1. Next

$$c_{0e} = \frac{H(T_e) - H(T_0)}{T_e - T_0} \tag{6}$$

It should be pointed out that the relation  $T_e > T_0$  or  $T_e < T_0$  is here not essential. Application of such approach leads to the simple numerical procedure determining  $c_0$ .

### 3. EXAMPLE OF COMPUTATIONS

The aluminium hot spot of L type produced in typical sand mix has been considered. The thicknesses of walls are equal 3.5 and 4 cm. Two values of  $\Delta T$  has been assumed. In Figure 3 the temporary shapes of solidification front are shown. The different intervals  $\Delta T$ , namely  $\Delta T=2$  and  $\Delta T=3.5$  give practically the same solutions which confirms the effectiveness of approach proposed.

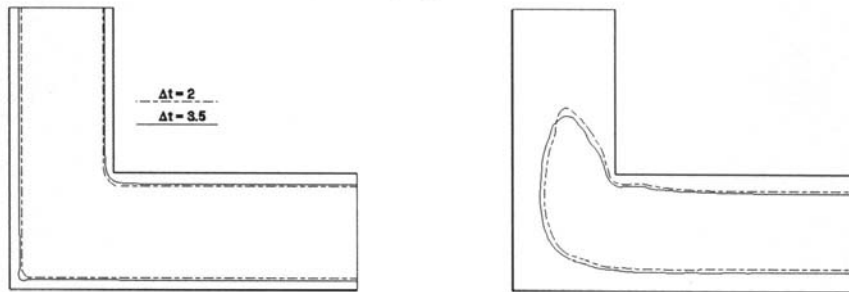


Fig. 3. Solidification front after 13 and 24 seconds  
Rys. 3. Front krzepnięcia po czasie 13 i 24 sekund

### REFERENCES

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### NUMERYCZNE ROZWIZANIE 2D ZADANIA STEFANA

#### STRESZCZENIE

Model Stefana opisuje krzepnięcie czystych metali lub stopów eutektycznych w skali makroskopowej. Z numerycznego punktu widzenia omawiane zadanie jest bardzo skomplikowane, szczególnie dla zadań 2D i 3D. W literaturze można znaleźć opisy algorytmów bazujących na zastąpieniu zadania Stefana zadaniem ze sztucznie wprowadzoną strefą dwufazową. Takie podejście przedstawiono również w tej pracy.

Reviewed by prof. Józef Gawroński