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CONSIDERATIONS FOR MINIMUM LAP TIME CALCULATION ON A RACE TRACK

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ABSTRACT

While methods for vehicle modeling are well established for simulation of handling behavior, there is still a lack of driver models, which are important for the realization of closed-loop maneuvers in a virtual environment.

This paper will present preliminary considerations for the development of such a driver model. First, trajectory planning strategies have to be generated and evaluated. To achieve this, a method will be deduced, which calculates the maximum velocity at each point of an arbitrary trajectory, taking into account simplified vehicle characteristics in terms of maximum longitudinal and lateral accelerations and considering the frictional ellipse.

Thus, the minimum necessary time for each trajectory can be calculated, this being a possible parameter to rate the quality of a trajectory for a given course. The feasibility of the method is demonstrated with the Nuerburgring race track.

NOMENCLATURE

r_i	Radius of a curve.
t	Time.
v	Speed.
κ	Curvature
a_{long}	Longitudinal acceleration
a_{lat}	Lateral acceleration
s	Traveled distance
n	Number of sampling points
(x_{li}, y_{li})	Coordinates of left road boarder at i-th sampling point
(x_{ri}, y_{ri})	Coordinates of right road boarder at i-th sampling point
(x_{mi}, y_{mi})	Coordinates of middle of the at i-th sampling point
W_i	Width of road at i-th sampling point
S_i	Sampling point number i
$L(s_i, s_{i+1})$	Distance between sampling points number i and number $i+1$

INTRODUCTION

Since the invention of the automobile in the 18th century, there has been a continuous urge for its improvement. First, open-loop test maneuvers like step-steer, ramp-steer etc. have been defined in order to rate and compare the dynamic behavior of different vehicle configurations objectively. Next, closed-loop maneuvers like double-lane change were defined, which included the driver behavior in testing, thus leading to subjective evaluation of the vehicle behavior. Soon it became obvious that it is not sufficient to rely on hardware testing.

Therefore, mathematical descriptions (i.e. vehicle models) of this dynamic system have been developed, which were able to predict the handling behavior depending on different vehicle setups. These vehicle models became more and more successful after enough computing power became available to perform complex calculations.

Nowadays, vehicle development relies increasingly on computer simulation. Whereas open-loop testing in the virtual environment is well established, there is still a demand for improving closed-loop simulation, which necessarily requires driver modeling.

These controllers, also named 'driver models', were supposed to reproduce the driving behavior of human drivers in a virtual world. The first approach was presented by McRuer (McRuer and Krendel, 1959) with the "quasilinear model", which described the human action as a transfer function (follow-up controller). Further improvement was introduced by Donges (Donges, 1978), who added an anticipatory unit, when he presented the "Two-Level Model of Driving Steering Behavior".

Over the years, more and more sophisticated models were created, also using modern modelling techniques, like neural networks, Fuzzy Models, etc. They emerged due to the large variety of areas they were applied to. For a complete survey of driver models please refer to Juergensohn (Jürgensohn, 1997).

The operation mode of all these driver models is basically the same: they manipulate the control inputs of the vehicle, forcing it to follow a predetermined path. This path is mostly defined a-priori and no interaction with the handling characteristics of the vehicle is considered. Consequently that path will not be optimal.

A new approach to a driver model which overcomes this drawback was recently presented by Prokop (Prokop, 2001), who implemented a short term path planning strategy in addition to the conventional PID controller. He also takes into account the actual position of the car on the road and the vehicle characteristics. Still, this short term path planning unit does not consider the whole geometry of the track.

A further improvement to driver modelling can be achieved generating an optimal path for the entire track. Therefore, it is essential to consider the whole track geometry and the vehicle's handling capabilities. But, a method for the evaluation of the optimal path has not been established yet.

This paper is going to present an approach to evaluate the quality of a given path. The goal is to calculate the minimum necessary time a car needs through a given path, therefore using the lap time as a rating function.

METHODOLOGY

A method to calculate the minimum time through a given path is now going to be described. Some preliminary considerations have to be made in advance.

Travel Time Through a Curve

If the path through a track is a straight line, then the calculation of travel time is trivial. If we are considering a curve, then the radius and the maximal lateral acceleration (a_{lat_max}) influence the result. An example is shown in figure 1, where two different trajectories through a semi-circle are depicted.

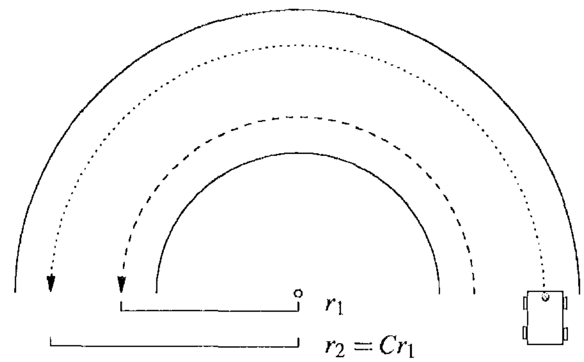

Figure 1. PATH THROUGH A CURVE

The travel time on each trajectory can be calculated with:

$$t = \frac{s}{v} \tag{1}$$

and

$$v = \sqrt{\frac{a_{lat\_max}}{\kappa}} \quad \text{with} \quad \kappa = \frac{1}{r} \quad . \tag{2}$$

The length of each trajectory from figure 1 can be defined as

$$s = \pi r \quad . \tag{3}$$

Thus, two different travel times for the inner and the outer radius r_1 and r_2 are obtained:

$$t_1 = \frac{\pi r_1}{v_1} \quad \text{and} \quad t_2 = \frac{\pi r_2}{v_2} \quad . \tag{4}$$

Using equations 1 and 4 we get:

$$t_1 = \frac{\pi\sqrt{r_1}}{\sqrt{a_{lat\_max}}} \quad \text{and} \quad t_2 = \frac{\pi\sqrt{r_2}}{\sqrt{a_{lat\_max}}} \quad . \tag{5}$$

Choosing $r_2 = Cr_1$, where C is a constant stretch parameter, we find:

$$t_2 = \frac{2\pi\sqrt{C}\sqrt{r_1}}{\sqrt{a_{lat\_max}}} \quad (6)$$

and using t_1 from equation 5 we get:

$$t_2 = \sqrt{C}\cdot t_1 \quad . \quad (7)$$

As we can see from the last equation it is advantageous to choose a tighter path through a semi-circle. This simple result will be extended in the following sections to curves with variable radii [1].

Discretization of the Track

For a numerical treatment a discretization of the track is necessary. Therefore a one dimensional vector $S_i = (x_{mi}, y_{mi})$ is used, which in this case describes the middle of the road. For visualization purposes a left and right border (x_{li}, y_{li} and x_{ri}, y_{ri}) are added. The borders are placed at a constant distance perpendicular to the middle line, as shown in figure 2. Thus, the width of the road is:

$$W_i = \frac{||\overrightarrow{(x_{li}, y_{li})} + \overrightarrow{(x_{ri}, y_{ri})}||}{2} \quad . \quad (8)$$

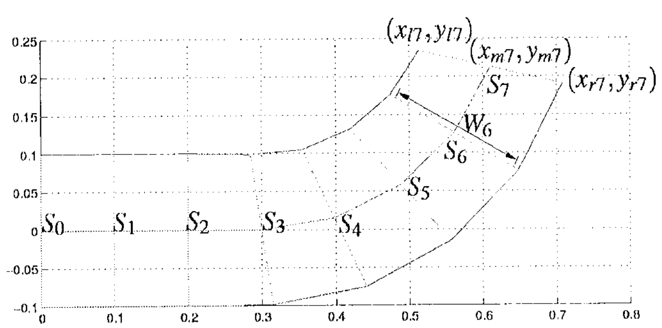

Figure 2. DISCRETISED LEFT AND RIGHT BORDER OF THE TRACK AND MIDDLE OF THE ROAD

[1] It also has to be noted that this result is achieved using a mass point as vehicle model.

Curvature

Using the previously introduced discretization, the curvature of the trajectory (middle of the road) is calculated at each sampling point, as shown in figure 3:

$$\kappa_i = \frac{1}{Radius(S_{i-1}, S_i, S_{i+1})} \quad . \quad (9)$$

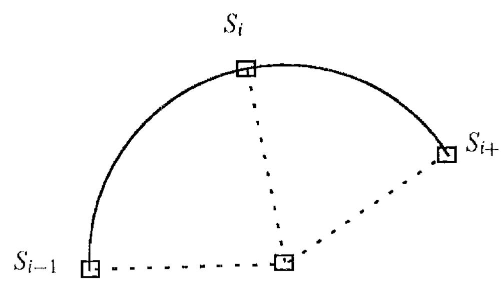

Figure 3. CURVATURE AT SAMPLING POINTS

This yields a vector $\kappa_i \in [0, \infty]$ which defines the path and its curvature.

Instability Zones

In figure 4 a short artificial test track is presented. It is assembled of three straight lines and two curves with different radii.

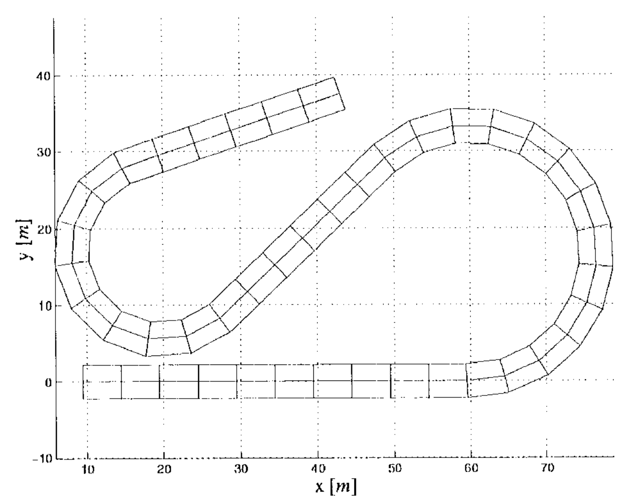

Figure 4. ARTIFICIAL TEST TRACK

Having defined the curvature of the path, the maximum velocity at the limit of adhesion of the car is calculated. Therefore we use again equation 2 (Mitschke, 1990), which gives us a velocity vector at each sampling point:

$$v_i = \sqrt{\frac{a_{lat\_max}}{\kappa_i}} \qquad (10)$$

The limit of adhesion a_{lat_max} will be chosen here to be $10\,\frac{m}{s}$.

The corresponding velocity profile is depicted in figure 5. The hatched areas are regions where the lateral acceleration is greater than $a_{lat\_max} = 10\,\frac{m}{s}$, from now on called **instability zones**. Thus, the possible velocity zone for the car is outside the hatched area.

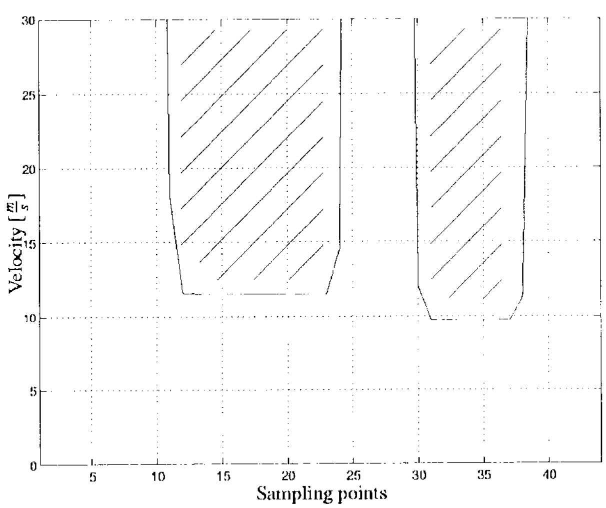

Figure 5. INSTABILITY ZONES

Acceleration and Deceleration

A car in motion can be subjected to acceleration or deceleration. Its velocity history can be calculated using the following equation:

$$v(s) = \sqrt{v_0^2 + 2 \cdot a_{long} \cdot s} \qquad (11)$$

with: v_0 : initial velocity
a_{long} : acceleration of the car (const.)
s : traveled distance

where a_{long} can either be positive (acceleration) or negative (deceleration).

A typical example is shown in figure 6, where we can see how a car accelerates from $v_0 = 0\,\frac{km}{h}$ at $a_{long} = 10\,\frac{m}{s^2}$ and decelerates from $v_0 = 100\,\frac{km}{h}$ at $a_{long} = -10\,\frac{m}{s^2}$.

The graphs resulting from equation 11 will be called **acceleration / deceleration graphs** from now on.

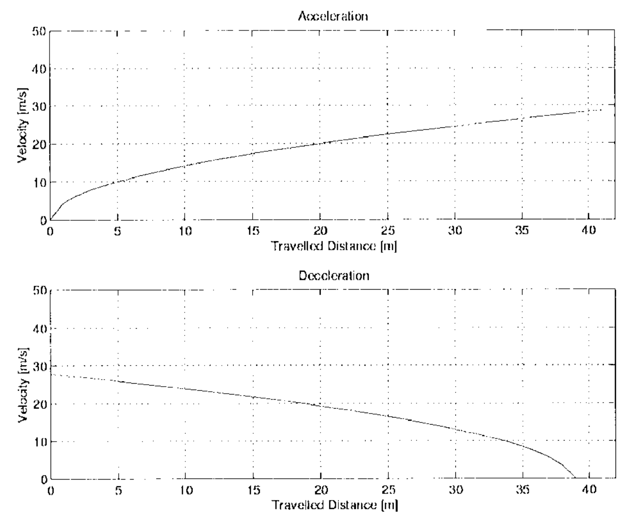

Figure 6. ACCELERATION AND DECELERATION DIAGRAM

To achieve the shortest time for the track[2] from figure 4, we must keep the car going as fast as possible. A possible approach would be the concatenation of acceleration / deceleration graphs (from figure 6) and placing them outside the hatched areas (instability zones) of figure 5.

This works well on straight lines, but cannot be applied to curves. The reason is that it is not possible to apply the full brake force ($a_{long} = -10\,\frac{m}{s^2}$) while cornering, because the wheels would lock up and the car would become unstable. To solve this problem, equation 11 must be enhanced:

$$v(s) = \sqrt{v_0^2 + 2 \cdot a_{long}(s) \cdot s} \qquad (12)$$

where $a_{long}(s)$ is now a function of the distance s. The value of $a_{long}(s)$ depends on the position of the car in the track and varies depending on the actual curvature and velocity:

$$a_{long}(s) \sim f(\kappa_{(s)}, v_{(s)}) \quad . \qquad (13)$$

[2]driving on the middle of the road

This dependency can be expressed mathematically with the frictional ellipse as described in the following section.

Adhesion

The relationship between lateral and longitudinal forces play an important role. Therefore the frictional ellipse has to be considered, which is related to the adhesion between the tire and the ground.

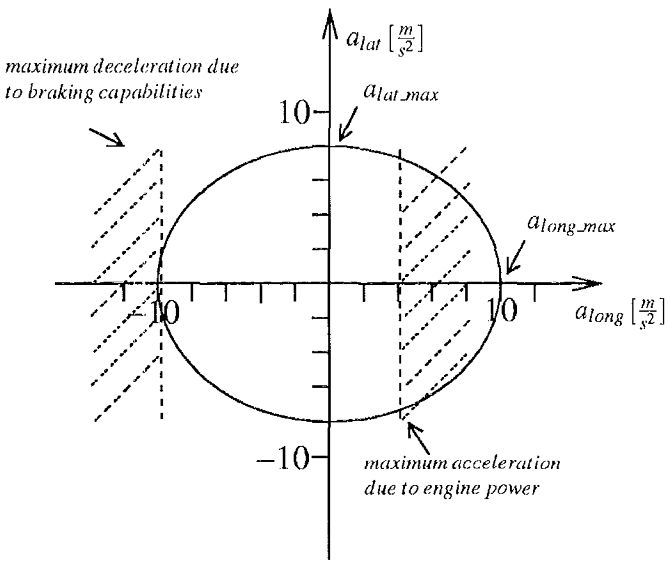

Figure 7. THE FRICTIONAL ELLIPSE DIAGRAM

The diagram shown in figure 7 describes the distribution between the lateral and the longitudinal component of the tire forces.

Introducing the mass of the car as constant, i.e. neglecting the influence of the dynamic load shifts, leads to the acceleration-based frictional ellipse (figure 7). Typical values are $a_{lat\_max} = 8\frac{m}{s^2}$ for maximum lateral acceleration and $a_{long\_max} = 10\frac{m}{s^2}$ for maximum longitudinal acceleration.

However, these values only describe the *possible* accelerations, the *achievable* accelerations are restricted by the characteristics of the tires and the car, as there are engine power and braking capabilities. Thus, the hatched areas of the ellipse are physically not reachable.

The dependency between longitudinal and lateral acceleration is determined by the frictional ellipse from figure 7, yielding:

$$a^2_{long}(s) = \left(1 - \frac{a^2_{lat}(\kappa_{(s)}, v_{(s)})}{a^2_{lat\_max}}\right) \cdot a^2_{long\_max} \qquad (14)$$

with: $a_{long} \in [-10(\frac{m}{s}), 4(\frac{m}{s})]$
$a_{lat} \in [-8(\frac{m}{s}), 8(\frac{m}{s})]$

and:
a_{lat_max} : maximum lateral acceleration
a_{long_max} : maximum longitudinal acceleration
a_{lat} : actual lateral acceleration
a_{long} : resulting longitudinal acceleration

By using equation 14 it is possible to calculate the maximum velocity at each sampling point of the track, taking into account the vehicle characteristics i.e. a_{lat_max} and a_{long_max}:

$$v(s) = \sqrt{v_0^2 + 2 \cdot a_{long\_max} \cdot s \cdot \sqrt{\left(1 - \frac{a^2_{lat}(\kappa_{(s)}, v_{(s)})}{a^2_{lat\_max}}\right)}} \quad . \quad (15)$$

The calculation has to be recursive, since at each sampling point a new deceleration graph has to be calculated, as shown in figure 8.

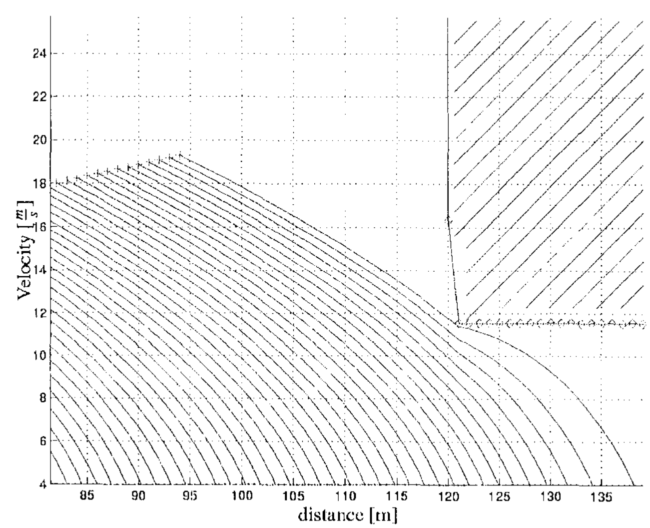

Figure 8. DECELERATION GRAPHS BEFORE CURVE

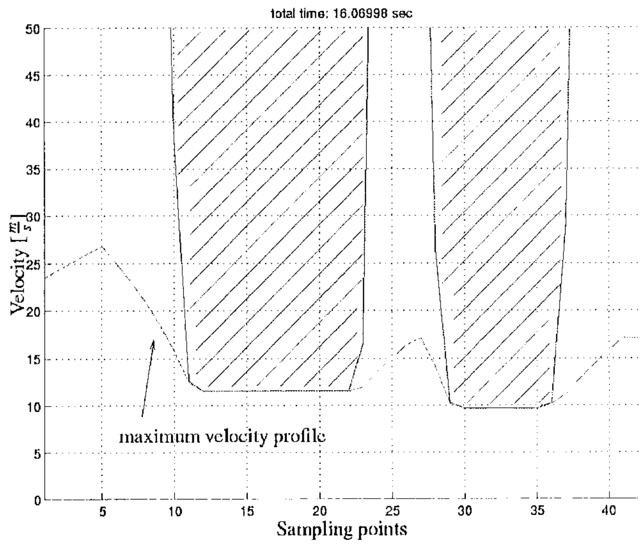

Figure 9. VELOCITY PROFILE

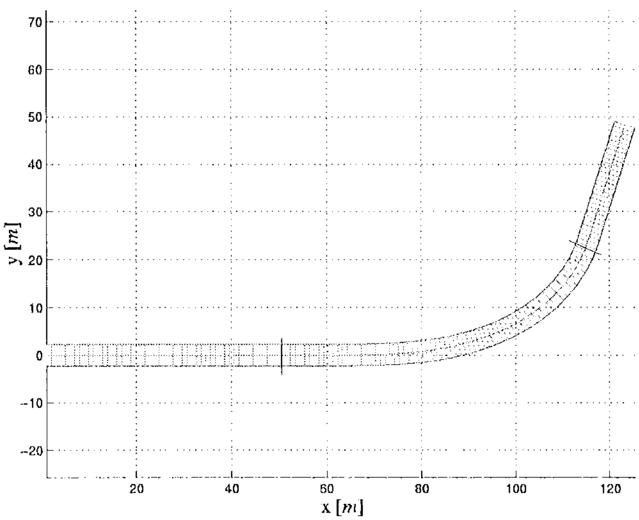

Figure 10. A CLOTHOID

From equation 15 we obtain a vector, from now on called **maximum velocity profile**, describing the fastest possible speed at each (sampling) point of the track (figure 9):

$$v_{(s_i)} = \left[v_{(s_1)}, v_{(s_2)}, \ldots, v_{(s_n)}\right] \tag{16}$$

where n is the total number of sampling points over the test track.

Examples

This methodology is going to be demonstrated in the following example, which is depicted in figure 8. It shows a cutout of figure 5, where the first instability zone is again marked as the hatched area.

An accelerating car (small crosses) approaches the curve from the left. From each position a deceleration graph is calculated, checking if the graph violates the instability zones. As we can see in the figure, the car has to start braking at position $s = 94\ m$ in order not to exceed the maximum speed at the beginning of the curve. We can also observe (in the deceleration diagram) an important reduction of deceleration of the car at position $s = 121\ m$. This is due to the fact that the car would be now traveling near the limit of lateral adhesion and wouldn't be able to brake as strongly as on the straight line (this results from equation 14). From this position on ($s = 94\ m$) the velocity of the car (following crosses) will be on the last possible velocity diagram and then continue under the hatched area.

The example from figure 8 has been created artificially and usually does not occur in reality, since there are no such sudden changes in curvature. Therefore a second example is presented. In this case a smooth transition between a straight line and a curve is achieved via a clothoid, as shown in figure 10.

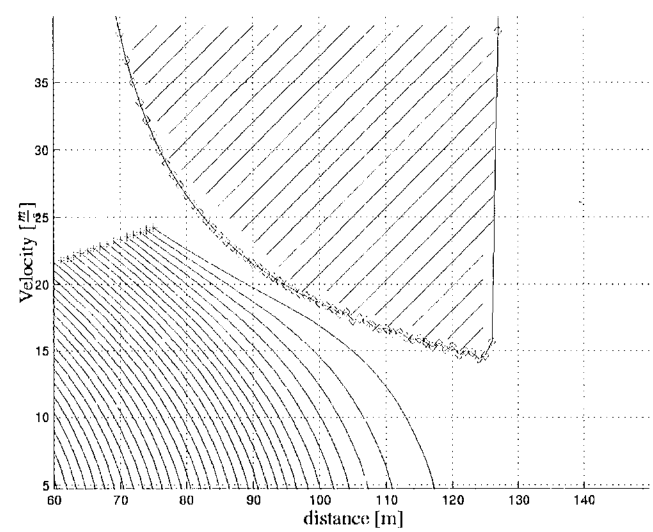

Figure 11. INSTABILITY ZONES AND DECELERATION GRAPHS AT A CLOTHOID

In figure 11 the instability zones and the deceleration graphs are presented. As we can see again, the car approaches the curve accelerating. At each position deceleration graphs are calculated. Due to the fact that the curve has a variable curvature and gets narrower towards the end, the car has to start braking earlier. In figure 12 the complete velocity profile of the car (crosses on the last allowed acceleration / deceleration graph) is shown. The car slows down until its speed reaches $v = 15\ \frac{m}{s}$ at position $s = 126\ m$, then starts accelerating again.

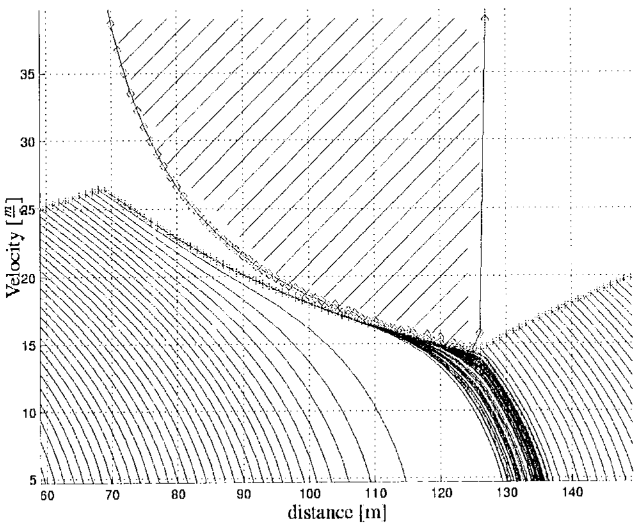

Figure 12. MAXIMUM VELOCITY PROFILE OF THE CAR

Calculation of Necessary Time

Having determined the maximum velocity profile of the car, it is now easy to calculate the total time. First the distance between the sampling points is determined, generating a vector:

$$L(s_i, s_{i+1}) = dist\left[(x_{mi}, y_{mi}),\ (x_{m\,i+1}, y_{m\,i+1})\right] \qquad (17)$$

$$\text{with:} \quad i = 1..n$$

With vectors from equation 16, 17 and adding up over all sampling points using

$$\Delta t = \frac{\Delta s}{\Delta v} \qquad (18)$$

leads to a total time:

$$t_{tot} = \sum_{i=1}^{n} \frac{L(s_i, s_{i+1})}{v(s_i)} \quad . \qquad (19)$$

From a pure mathematical point of view, this calculated t_{tot} is the shortest possible time, which a car would need for the given path, considering the given constraints.

RESULTS

Race Track: Nuerburgring

The presented procedure is now going to be applied on a real racing track. For this purpose one lap of the "Nordschleife" on the Nuerburgring (Germany) is going to be calculated. Figure 13 shows the geometry of the track.

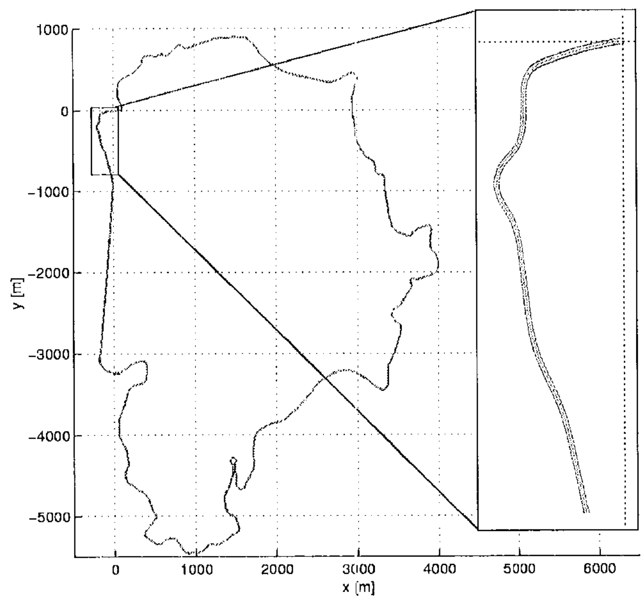

Figure 13. THE NUERBURGRING

The car properties are described by the frictional ellipse from figure 7 and equation 14. The car starts with a initial speed of $v_0 = 10\ [\frac{m}{s}]$ and drives in the middle of the road (no corner-cutting). The track is assumed to be flat.

Due to the length of the course only the magnified section will be shown in detail. In figure 14, the instability zones and the max. velocity profile of the car are presented. We can clearly see how the car slows down while approaching the oncoming corners and how it accelerates afterwards.

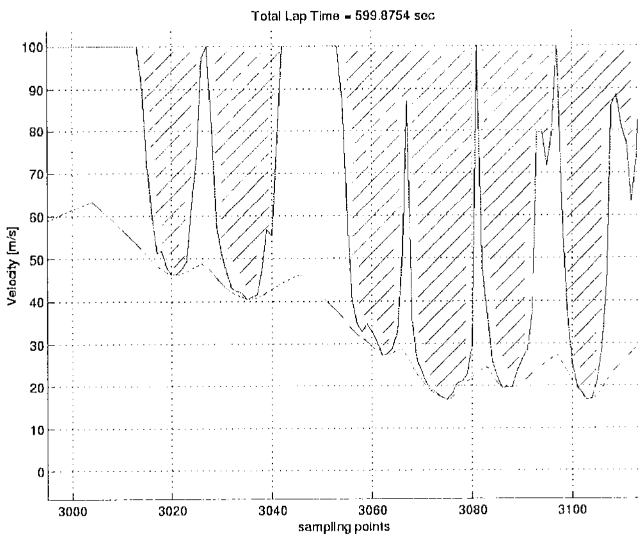

Figure 14. THE VELOCITY PROFILE

Discussion

For 1 lap of the entire race track, the calculation of the necessary time gives a result of 599 *sec* (equivalent to 10 *min*).

This time is comparable to the time professional drivers need for one lap driving a high powered car. Nevertheless, this calculated time is only an approximation of the achievable value, due to the various constraints and simplifications. They influence the lap time in both directions, making it shorter or longer.

Lap time is increased due to:

1. The trajectory, which is chosen to be the middle of the lane; this greatly increases the lap time, since the corners cannot be cut to reduce lateral acceleration and increase speed.
2. Missing aerodynamic considerations; a race car would produce down-force, increasing cornering speeds.

Lap time is reduced due to:

3. Reduction of the vehicle to a mass point; thus the dynamic load transfer, as well as the pitch and roll behavior, are neglected.
4. The neglect of tire properties and slip angles.
5. Missing aerodynamic drag, which plays an important role at high speeds.
6. The travelling of the vehicle model on a flat road, without elevations; at the Nuerburgring the elevation profile of the track influences the run significantly.
7. The assumption that the adhesion between car and ground is $\mu = 1$; on a real track it varies significantly.
8. The instant acceleration capabilities, without a time lag of the engine and power train; it is assumed to have the same acceleration capabilities over the whole speed range.
9. The acceleration forces, which are calculated directly and are applied very precisely; in a real car there must be a mapping to brake and throttle.
10. The fact that between quick left-right turns the car is accelerated; a real driver wouldn't do that.

Future work

Future work will be mainly concerned with the elimination of the shortcomings mentioned in the previous section. The most important changes or enhancements will be:

1. Introduction of a vehicle model to replace the mass-point simplification, in order to eliminate restrictions nr. 3, 4, 5, 6, 7 and 8.
2. Extension of the frictional ellipse to an ellipsoid by inclusion of the elevation profile, thus eliminating restriction nr. 6.
3. Inclusion of corner-cutting strategies (restriction nr. 1).

SUMMARY

In this paper a method to calculate the minimal lap time is presented. It is based on the assumption that a driver always drives close to the physical limit of adhesion. Velocity profiles of this car on a racing track are generated, from which a lap time can be deduced. The results of this calculation show a good approximation of real measured lap times. With a further refinement of the modelling it is expected to increase the accuracy of the results.

REFERENCES

Donges, E. (1978). A two level model of driver steering behavior. In *Human Factors*, volume 20, pages 151–165.

Jürgensohn, T. (1997). *Hybride Fahrermodelle*. PhD thesis, Institut für Fahrzeugtechnik, TU Berlin.

McRuer, D. and Krendel, E. (1959). The human operator as a servo system element part I+II. In *Journal of the Franklin Institute*, volume 267, pages 381–403,511–536.

Mitschke, M. (1990). *Dynamik der Kraftfahrzeuge Band C - Fahrverhalten*. Springer-Verlag.

Prokop, G. (2001). Modeling human vehicle driving by model predictive online optimization. In *Vehicle System Dynamics*, volume 35, pages 19–53.