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FRACTIONAL ORDER MODELS FOR VISCOELASTICITY OF SOFT BIOLOGICAL TISSUES

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ABSTRACT

Dynamic mechanical properties of soft tissues provide information that may be used in medical diagnosis. Developing a better fundamental understanding of the governing constitutive relations could improve diagnostic techniques. The mechanical behavior of soft tissues and tissue mimicking phantoms, such as gels, can be represented by viscoelastic material models. Static loading of viscoelastic materials yields information related to elasticity, creep and stress relaxation. However, a broader measure of rate-dependent properties that affect mechanical wave propagation and wave attenuation in such materials can only be extracted from measured response to dynamic excitation. The well known linear viscoelastic material models of Voigt, Maxwell and Kelvin cannot represent the more complicated frequency dependency of these materials over a broad spectral range. Therefore, fractional calculus methods have been considered to model the viscoelastic behavior of soft tissue-like materials. Fractional order models capture the viscoelastic material behavior using fractional orders of differential equations that may yield a more accurate representation of viscoelastic material behavior. This paper focuses on experimental measurements on the tissue mimicking phantom, CF11. Surface waves on the phantom material are studied experimentally and theoretically. Theoretical calculations using linear and fractional order methods are compared with experimental measurements.

INTRODUCTION

Conventional morphological imaging techniques create contrast based on x-ray attenuation, magnetization relaxation times, or ultrasound reflection. However, a potentially more powerful set of information would be to have a mapping of the viscoelastic properties of the tissue. Stiffness of a medium can be estimated using conventional methods, such as a tensile test, compression test or indentation tests. These methods are usually static or quasi-static and they work well when the material specimen can be isolated with well-defined boundary conditions and is relatively homogenous. However, biological tissue is heterogeneous and anisotropic, with complex boundary conditions *in vivo*. Additionally, static or quasistatic techniques do not provide sufficient information about viscosity, which itself can be a valuable source of contrast. A broader measure of rate-dependent properties that affect mechanical wave propagation and wave attenuation in such materials can only be extracted from measured response to dynamic excitation; for soft tissue this may yield moduli related to propagation and attenuation for the medium.

The mechanical behavior of soft tissues and tissue mimicking phantoms, such as gels, can be represented by viscoelastic material models. Studies have shown that integer order models, such as Voigt, Kelvin and Maxwell, only poorly approximate the rate-dependency of biological tissues and that possibly fractional order viscoelasticity models may be more appropriate (e.g. 1-5,7). Current work in fractional calculus viscoelastic modeling starts with the idea that it is the order of the derivative of the strain that to a first approximation characterizes the material's behavior (assuming a one dimensional stress-strain relation). For example, the order of the derivative is zero for a Hookean solid and one for a Newtonian fluid. Viscoelastic materials occupy the intermediate range with a fractional order "α" between zero and one. Thus, it is possible to build a multi-component fractional equivalent of the "standard" linear solid (or fluid) by replacing one or more springs and dashpots with "Springpots".

This research focuses on experimental measurements on the silicone based tissue mimicking phantom, CF11. Surface (Rayleigh) waves, which are closely related to shear waves in terms of how the medium's material properties affect their phase speed and attenuation, are induced and measured at various frequencies in a manner described in (6).

THEORY

Rayleigh wave propagation on an isotropic homogeneous viscoelastic half-space caused by normal excitation over a region depicted in Fig. 1 is given by (6):

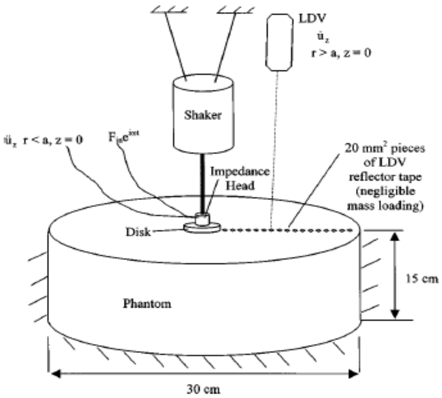

Fig. 1. Schematic of experimental setup

$$\frac{u_z}{P_{in}} = (-1)^{3/4} e^{-\frac{i\pi}{4}} \frac{2ia}{\mu} \frac{J_1(pak_1)\sqrt{p^2-1}}{F_0'(-p)} \ldots \tag{1}$$

$$\times \left\{ 2p^2 e^{-zk_1\sqrt{p^2-\eta^2}} + (\eta^2 - 2p^2) e^{-k_1 z\sqrt{p^2-1}} \right\} K_0(iprk_1)$$

$$F_0 = F_0(\varsigma) = (2\varsigma^2 - \eta^2)^2 - 4\varsigma^2\sqrt{\varsigma^2-\eta^2}\sqrt{\varsigma^2-1}\,. \tag{2}$$

Here, u_z is out-of-plane surface velocity, P_m is force per unit area of the driving disk, a is disk radius, p is a root of the function F_0 that is associated with Rayleigh wave motion, ζ is dummy variable, k_l is the compression wave number, z is zepth from surface, r is radial distance from center of driving disk, η is ratio of compression to shear wave speed, ζ is ratio of compression wave speed to surface wave speed, J_1 is the Bessel function of the first kind (order 1), and K_0 is the modified Bessel function of the second kind (order 0).

Equation (2) couples the dependency of both shear and surface wave speeds to material viscoelastic properties. Assuming a Voigt model, shear wave speed is related to the real (storage) and imaginary (loss) parts of the shear modulus, μ_R and μ_I respectively, and the material density ρ as:

$$c_{shear} = \sqrt{\frac{2}{\rho}\frac{\mu_R^2 + \mu_I^2}{\mu_R + \sqrt{\mu_R^2 + \mu_I^2}}}\,. \tag{3}$$

If surface wave speed and attenuation is measured, material properties that affect shear and surface wave speed can be estimated iteratively using Eqs. (1-3).

Note that both μ_R and μ_I are independent of whether the Voigt model is integer or fractional order. They are equal to μ_1 and $\omega.\mu_2$, shear elasticity and shear viscosity multiplied with rotational frequency, if an integer Voigt model is used. In the case of a fractional Voigt Model the storage modulus and loss modulus are defined as

$$\mu_R = \left[\mu_1 + \mu_1\left(\frac{\mu_2}{\mu_1}\right)^{\alpha} \omega^{\alpha} \cos\left(\alpha\frac{\pi}{2}\right)\right], \tag{4}$$

$$\mu_I = \left[\mu_1\left(\frac{\mu_2}{\mu_1}\right)^{\alpha} \omega^{\alpha} \sin\left(\alpha\frac{\pi}{2}\right)\right]. \tag{5}$$

EXPERIMENT

The surface response of the CF11 phantom (Fig. 1) to an oscillating finite disk is measured using a laser Doppler vibrometer (Polytec). Based on eqs. (1-2), least squares estimates for μ_R and μ_I/ω are shown in Fig. 2. The inadequacy of the integer Voigt model is evident in the widely varying values of the moduli as a function of frequency. (If the integer Voigt model were adequate, then μ_R and μ_I/ω should be independent of frequency.) Alternatively, a fractional Voigt model is fit to the experimental data, by assuming α = 1/2, 1/3, and 1/4 and using eqs. (4-5). A closer match to experiment is achieved.

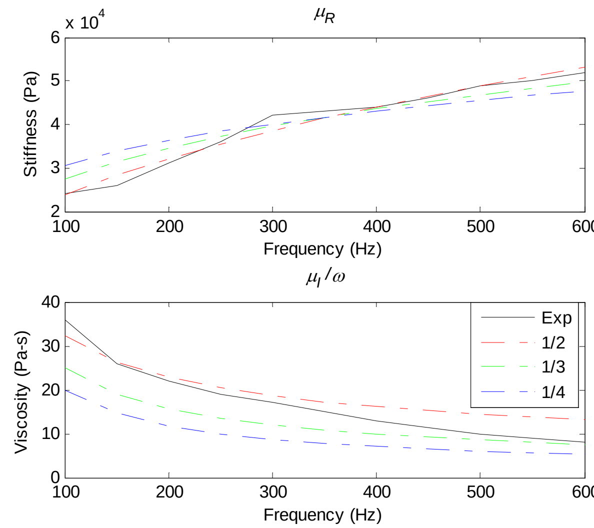

Fig. 2 Fractional material model fit to experimental shear modulus and loss modulus results. Integer Voigt material model suggests constant μ_R and μ_I/ω; fractional models simulate the experimental results better.

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