Pulse Power Enhancement using Mode Locked Arrays of Automatic Level Control Oscillators

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Abstract--A millimeter wave pulse generation system using mode locked arrays of coupled automatic level control oscillators is analyzed. Previous analyses have shown that 90 degrees of coupling phase maximizes the entrainment region size, however this paper shows that pulse power can be significantly enhanced by choosing 0 degrees of coupling phase. A comparison of the entrainment size and phase sensitivity shows that for large arrays peak power enhancement can be utilized without a significant reduction in overall system robustness.

I. Introduction

At millimeter wave frequencies high power sources are difficult to realize with solid state devices. An attractive method is "quasi-optical" power combining utilizing arrays of many small devices. Such arrays have many interesting properties that can exploited for beam steering, beam scanning, and pulsed applications. Mode locked arrays produce periodic pulse trains by transmitting a combined spectrum of equal spaced components, and have been demonstrated experimentally.[1] Unfortunately the oscillator elements must be tuned with great precision because of the exceedingly small entrainment regions. Oscillators with a delayed gain response, such as the automatic level control oscillators considered in this paper, enhance the entrainment region as compared to traditional "instantaneous" gain response oscillators (e.g. Van der Pol).[2] This paper shows how the peak pulse power can be enhanced by choosing a particular coupling phase angle and addresses the principle concerns for the designer: the size of the entrainment region and the sensitivity of the pulse shape with respect to oscillator tunings.

II. Mode Locked Arrays of Oscillators

Mode locking in coupled oscillators is a frequency entrainment condition, similar to the injection locking of two oscillators almost identically tuned, and occurs when the uncoupled oscillator spectra are almost evenly spaced. The locked steady state output frequencies of the array elements are *exactly* evenly spaced, and maintain this spacing even when the elements suffer small detunings. Entrainment occurs between an oscillator fundamental and the beat frequencies generated by nonlinear mixing of the nearby oscillator outputs.[3]

The arrays considered in this paper are "linear," that is, the elements are arranged in a row with nearest neighbor coupling between elements. The mode locked array output is the sum of the individual outputs, and can be expressed as

$$v_{out}(t) = \sum_{n=1}^{N} A_n(t)\cos(\omega_o t + \theta_n(t)) \qquad (1)$$

where $\theta_n(t) = n\omega_b t + \phi_n(t)$. $A_n(t)$ and $\phi_n(t)$ are both periodic functions of time. This signal can be expressed as a periodic envelope modulating a high frequency carrier. The envelope is given by

$$V_e(t) = \left|\sum_{n=1}^{N} A_n(t)\, e^{j\theta_n(t)}\right| \qquad (2)$$

Large mode locked pulses occur when all of the components add coherently. Using phasor diagrams one can show that this occurs at time t when

$$\Delta\Delta\theta_n \equiv \theta_{n+2} - 2\theta_{n+1} + \theta_n = \Delta\Delta\phi_n(t) = 0 \qquad (3)$$

for $n = 1, 2, \cdots, N-2$. If this condition is not met the phasors will not add coherently and the pulse amplitude will be sub-optimum.

If equation (3) is satisfied and, in addition, the amplitudes are uniform across the array, equation (2) can be summed in closed form:

$$V_e(t) = A(t)\left|\sum_{n=1}^{N} e^{jn(\omega_b t + \Delta\phi(t))}\right| = A(t)\frac{\sin\left[\frac{N}{2}(\omega_b t + \Delta\phi(t))\right]}{\sin\left[\frac{1}{2}(\omega_b t + \Delta\phi(t))\right]} \qquad (4)$$

Constant amplitudes and phases result in the classic mode locked pulse train (see figure 1). Time varying amplitudes and phases will change the mode locked waveform, but the change can be advantageous. For instance, if the phases vary rapidly (in the same direction) during the time of coherent power combining the mode locked pulses will be sharper than what is predicted by equation (4). Or, if the amplitudes are large during the coherent combining and small otherwise, the pulses will be enhanced. For practical arrays the time varying phases have little effect on the waveform due to the weak coupling. The amplitude variations, however, can be made quite large in an ALC array due to amplitude resonance effects and can significantly enhance the quality of the mode locked pulses.

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III. Time Domain Characteristics

The differential equations that describe the amplitude, gain control, and phase of each oscillator are (see [2]):

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$$\dot{A}_n = \eta(1-g_n)A_n + \varepsilon\big[A_{n-1}\cos(\tau+\phi_n-\phi_{n-1}+\Phi) + A_{n+1}\cos(\tau+\phi_{n+1}-\phi_n-\Phi)\big]$$

$$\dot{g}_n = -\frac{1}{\eta}g_n + \frac{1}{\eta}A_n \tag{5}$$

$$\dot{\phi}_n = \beta_n - \varepsilon\left[\frac{A_{n-1}}{A_n}\sin(\tau+\phi_n-\phi_{n-1}+\Phi) - \frac{A_{n+1}}{A_n}\sin(\tau+\phi_{n+1}-\phi_n-\Phi)\right], \quad n=1,2,\cdots,N$$

where η is the normalized nonlinearity parameter, ε is the normalized coupling parameter, Φ is the coupling phase, $\tau=\omega_b t$ is the normalized time variable, and the dot denotes differentiation with respect to τ. Approximate solutions are derived using the Poincare-Lindstedt perturbation technique in which the solution is expanded in a power series in ε. To the first order of approximation the amplitudes and phases are

$$A_n(\tau) = 1+\varepsilon\big[\eta\cos(\tau+\Delta\phi_{n-1}+\Phi)+\sin(\tau+\Delta\phi_{n-1}+\Phi) + \eta\cos(\tau+\Delta\phi_n-\Phi)+\sin(\tau+\Delta\phi_n-\Phi)\big] \tag{6}$$

$$\phi_n(\tau) = \phi_{no}+\varepsilon\big[\cos(\tau+\Delta\phi_{n-1}+\Phi)-\cos(\tau+\Delta\phi_n-\Phi)\big]$$

$$n=1,2,\cdots,N$$

where the phase ϕ_{no} is constant and the phase differences are defined as $\Delta\phi_n = \phi_{n+1,o}-\phi_{no}$. For any subscripts of zero or N+1 the phase variable is set to zero. If we further assume the constant phases satisfy equation (3), i.e. $\Delta\Delta\phi_n \equiv \Delta\phi_{n+1}-\Delta\phi_n = 0$ for $n=1,2,\cdots,N-2$, then the amplitudes and phases are

$$A_n(\tau) = 1+2\varepsilon\sqrt{1+\eta^2}\cos(\Phi)\cos(\tau+\Delta\phi-\tan^{-1}(1/\eta))$$
$$\phi_n(\tau) = n\Delta\phi - 2\varepsilon\sin(\Phi)\sin(\tau+\Delta\phi), \quad n=1,2,\cdots,N \tag{7}$$

The amplitude and phase variations of the center elements are uniform across the array. Ignoring the effects of the end elements allows us to use equation (4) for the mode locked waveform, and will be most accurate for large arrays. Because the phase variations are assumed to be uniform they do not affect the mode locked waveform. The peak of the mode locked pulse occurs at times $\tau=\omega_b t+2\pi k$, k=integer. For large η the amplitudes are at their peak value and enhance the peak power of the pulse. Figure 1 shows plots of mode locked waveforms for two cases: amplitudes equal to unity and amplitudes given by equation (7). The peak power in the latter is considerably enhanced. Unfortunately the mode locking entrainment region is smaller for Φ=0 than for $\Phi=\pi/2$. We will see in the next section, however, that for large arrays this difference is small and the phase sensitivity is actually lower for Φ=0.

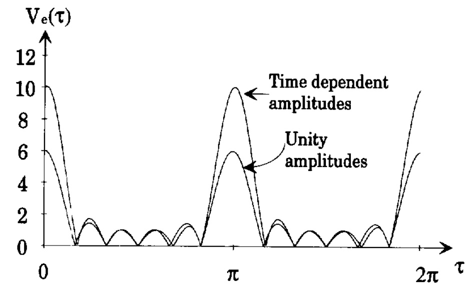

Figure 1 Pulse enhancement using the time varying amplitudes of the ALC oscillators. Waveforms from a six element array are shown for unity amplitudes and for amplitudes given by equation (7). The parameters are ε=0.2, η=2, Φ=0.

IV. Entrainment Region

The mode locking entrainment region for arrays of ALC oscillators has been analyzed in detail elsewhere using approximate methods[2]. It was shown that the size of the mode locked entrainment region is proportional to the nonlinearity parameter and the square of the coupling strength. This former dependence is different than that derived for Van der Pol arrays for which an optimum value exists.[4] Thus, for ALC arrays we can increase the entrainment region beyond what is possible for Van der Pol arrays.

Array tuning becomes complicated for large arrays since the pulse shape is affected by the combined tunings of all of the elements. The phase condition of equation (3) represents the ideal phase distribution, and we will refer to small changes of element tunings away from this optimum as element "detunings." The mode locked system has N-2 degrees of freedom since two of the phase are arbitrary (neglecting frequency spectrum shifts). When tuning the array we will vary only the N-2 central element tunings and leave the end elements fixed. This will maintain an approximately fixed frequency spectrum.

The most important entrainment region characteristics are the size of the region and the sensitivity of the phases to tuning variations. Also, the tuning that gives the desired phase distribution should lie at the center of the locking region. Information about the region of stable entrainment can be obtained from the functional relationship between the free running frequencies and time average oscillator phases.[5] Approximate equations were derived in [2] for arbitrary coupling phase. The results showed that the entrainment region size was maximized for Φ=90 degrees, and this special case was analyzed extensively. From the analysis presented in the previous section Φ=0 degrees is also an attractive choice since the peak power in the mode locked waveform can be enhanced significantly. However, this choice of coupling phase produces a smaller entrainment region. In this section we will compare the entrainment region characteristics for these two values of coupling phase.

For Φ=0 or Φ=90 degrees the functional relation between the central free running frequencies and the central time averaged phases is, in vector form,

$$\overline{\omega_{oc}} = \overline{\overline{B}}^{-1}\overline{\overline{A}}\,\overline{u}+\overline{b} = \overline{\overline{M}}\,\overline{u}+\overline{b} \tag{8}$$

where $\overline{\overline{A}}$, $\overline{\overline{B}}$, and $\overline{\overline{M}}$ are N-2 by N-2 matrices, $\overline{b}$ is an N-2 element vector which is the center of the entrainment region in frequency space, and the vector $\overline{u}$ depends on the time average phases as $(\overline{u})_n = \sin(\phi_{n+2,o} - 2\phi_{n+1,o} + \phi_{no})$.[?] The matrix $\overline{\overline{M}}$ and the vector $\overline{b}$ are functions of coupling phase, but are otherwise constant. The existence region can be determined by allowing the phases to span all of their possible values, which causes the components of the $\overline{u}$ vector to span (-1,1). The transformation from u space to frequency space is linear, so the cubical region in u space maps to a rectangular parallelepiped in frequency space. The volume of the entrainment region is equal to the determinant of $\overline{\overline{M}}$. A figure of merit that measures the size of the entrainment region is the length of the side of a cube in N-2 dimensional space that has the same volume as the entrainment region. This length is given by

$$L = \left[\det\left(\overline{\overline{M}}\right)\right]^{1/_{N-2}} \tag{9}$$

This permits comparison of entrainment regions in different dimensional spaces.

Figure 2 shows the size, L, as a function of the number of array elements for the two desirable values of coupling phase, Φ=0 and Φ=90 degrees. For large arrays the two are comparable, and for Φ=0 the size is almost independent of N. Thus one may choose Φ=0 degrees to enhance the pulse power without significantly reducing the entrainment region size.

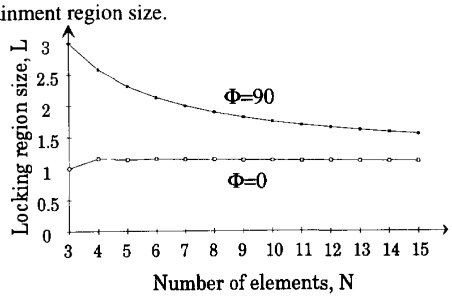

Figure 2 Entrainment region size, L, for two values of coupling phase, as a function of the number of array elements.

The shape and orientation of the entrainment region is related to the eigenvalues and eigenvectors of $\overline{\overline{M}}$. If we inscribe a unit sphere in the cubical region in u space, the sphere maps to an ellipsoid inscribing the parallelepiped in frequency space. The eigenvalues and eigenvectors of the transformation matrix $\overline{\overline{M}}$ are the lengths and directions of the major and minor axes [6], and indicate the approximate size and shape of the entrainment region. A two dimensional example is shown in figure 3.

Stability is determined by forming the matrix

$$\left(\overline{\overline{C}}\right)_{mn} = \frac{\partial\left(\overline{\omega_{oc}}\right)_m}{\partial\left(\overline{\phi_{oc}}\right)_n} \tag{10}$$

where $\left(\overline{\phi_{oc}}\right)_n$ is the n[th] central phase, that is $\left(\overline{\phi_{oc}}\right)_n = \phi_{n+1,o}$.[5] A vector $\overline{\phi_{oc}}$ represents a stable mode locked state when the real parts of the eigenvalues of $\overline{\overline{C}}$ are all positive. One can

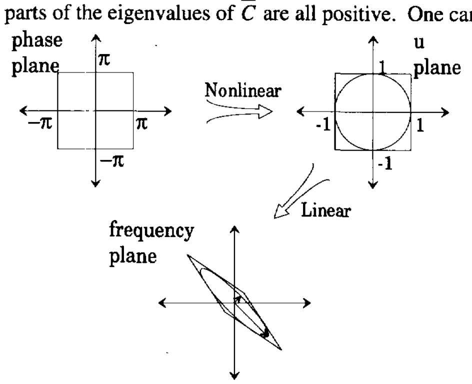

Figure 3 Consecutive nonlinear and linear transformations give the region of stable mode locking in the frequency plane. The circle in the u plane maps to an ellipse in the frequency plane, with major and minor axes given by the eigenvalues and eigenvectors of the linear transformation matrix.

show that for the relation of equation (8) the region of stable mode locking is the same as the region of existence.[2]

The sensitivity of the phases to tuning variations, an important parameter to the system designer, is directly related to the stability matrix $\overline{\overline{C}}$. Detuning the elements in some fashion changes the phases according to

$$\overline{d\omega_{oc}} = \overline{\overline{M}}\,\overline{\overline{B}}\,\overline{d\phi_{oc}} = \overline{\overline{B}}^{-1}\overline{\overline{A}}\,\overline{\overline{B}}\,\overline{d\phi_{oc}} = \overline{\overline{C}}\,\overline{d\phi_{oc}} \tag{11}$$

where $\overline{\overline{C}}$ is evaluated at the center of the entrainment region. We can define the sensitivity of the phase change with respect to frequency change as

$$S^{\phi}_{\omega} = \left[\frac{\overline{d\omega_{oc}}^{T}\,\overline{\overline{C}}^{-T}\,\overline{\overline{C}}^{-1}\,\overline{d\omega_{oc}}}{\overline{d\omega_{oc}}^{T}\,\overline{d\omega_{oc}}}\right]^{1/2} \tag{12}$$

This quantity depends on the direction in which the frequency change is taken. If that direction is one of the eigenvectors λ of $\overline{\overline{C}}$ then the sensitivity is simply

$S^{\Phi}_{\Omega} = \sqrt{\lambda}$.[6] In addition, one can show that the directions of the eigenvalues are extremums for the sensitivity. The maximum sensitivity is an important parameter since any detuning in this direction leads quickly to loss of entrainment.

Figure 4 is a plot of the maximum sensitivity, S_{max}, as a function of the number of array elements for Φ=0 and Φ=90 degrees. For large arrays the maximum sensitivity for Φ=90 degrees is considerable larger than for Φ=0 degrees. Thus, if one can maintain the tuning tolerances required for the smaller entrainment region, Φ=0 degrees results in more robust performance.

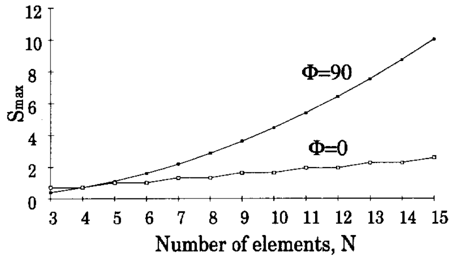

Figure 4 Maximum sensitivity as a function of the number of elements for two values of coupling phase.

V. Conclusions

The preceding results show that one can design a mode locked ALC array with enhanced peak power without sacrificing too much of the entrainment region size. In fact, for large arrays the maximum sensitivity of the phases to tuning variations is significantly smaller for Φ=0 than for Φ=90 degrees. This will lead to more robust mode locked arrays.

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