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Research Article

Guaranteed Cost Control for Multirate Networked Control Systems with Both Time-Delay and Packet-Dropout

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Compared with traditional networked control systems, the sampling rates of the nodes are not the same in the multirate networked control systems (NCSs). This paper presents a new stabilization method for multirate NCSs. A multirate NCSs with simultaneous considering time-delay and packet-dropout is modeled as a time-varying sampling system with time-delay. The proposed Lyapunov function deceases at each input signal updating point, which is largely ignored in prior works. Sufficient condition for the stochastic mean-square stability of the multirate NCSs is given, and the cost function value is less than a bound. Numerical examples are presented to illustrate the effectiveness of the proposed control scheme.

1. Introduction

Feedback control systems where in the control loops are closed through real-time network are called Networked Control Systems (NCSs) [1]. Compared with the traditional control architecture, NCSs have many advantages such as high reliability, simple installation, and lower cost. Although the NCSs have many advantages, the applying of NCSs makes the system more complicated to analyze. Since the data is transmitted via network, there are two major problems of NCSs. Firstly, the network-induced delay occurs while transferring data between devices and shared medium. Secondly, unreliable network transmission may lead to packet dropout.

For these reasons, it is vital to study NCSs with network-induced delay and packet dropout. Up to now, many good achievements have been investigated to deal with these problems. For the issue of time delay, the stability of NCSs with short random time delay was studied in [1]. The achievement in [1] was expanded for the situation of long time delay in [2]. The system was modeled into switch systems to investigate the stability of networked control systems in [3]. Packet dropout not only exists in time delay progress but also in transmission loss. For the problem of packet dropout, the stabilizing of controller was investigated in [4–18]. The models of NCSs in prior papers were divided into three cases: switch linear systems [6], asynchronous dynamical systems [9], and jump linear systems [13]. It is more complex to deal with the modeling and analysis for NCSs with both delay and packet dropout, compared with separately considering each other. In [15], the method of switched linear systems was applied in modeling for the NCSs of both packet dropout and network-induce delay in NCSs. Sufficient conditions for stochastic stability were discussed in [16]; what is more, packet dropout on both sides of sensor-to-controller and controller-to-sensor was modeled as two Markov chains.

It is important to ensure that the system possesses a strong robust performance. The guaranteed cost control is a good way to deal this problem, which guarantees the system performance affected uncertainty bellow the given performance index bound. The guaranteed cost control was first mooted in [19]. These years it has been applied to networked control system with time delay, and many issues have been developed for this item in [20–25].

Unfortunately, most aforementioned conclusions are under the following assumption: the sampling rates of each node in NCSs are the same. This brings convenience for the

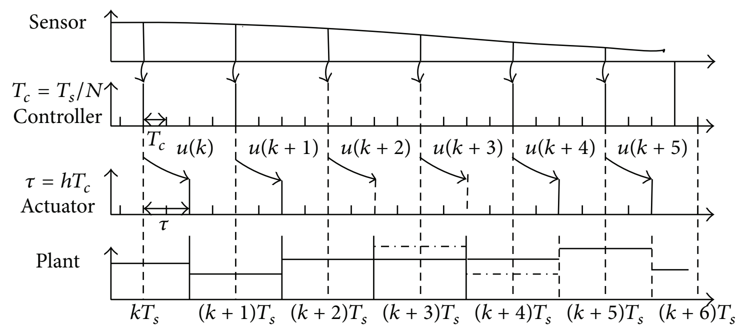

Figure 1: The timing diagrams of multirate NCSs.

theoretical research of NCSs; however, the sampling rate of each node is not identical in practical application. For the multirate network, the rates are not the same, the sampling period of sensor is T_s, and sampling period of the controller is T_c, $T_s \neq T_c$. In recent years, the investigations of multirate control system have made a great progress [26–36]. The NCSs are modeled as switched systems by using multirate method, when sensor, controller, and actuator are all event-driven, in [26], and the stability was analyzed. The exponential stability of multirate NCSs was analyzed including three cases of perfect transmission, delayed transmission, and time-varying transmission, in [28]. In [32], the condition of stabilizing controller of multirate NCSs was discussed by the way of using a V-K iteration algorithm. Controllability and observability of networked control systems with short time delay are analyzed in [35, 36].

It is nice to see that the network control systems theory has been widely applied to practical area. Two online schemes based on the data-driven fault-tolerant control (FTC) systems on the benchmark Tennessee Eastman process are presented in [37]. A subspace-aided data-driven approach for batch processes is proposed in [38]. A comparison between the basic data-driven methods for process monitoring and fault diagnosis (PM-FD) is provided in [39].

Published literature shows that many questions about guaranteed cost control for multirate NCSs with both time delay and packet dropout should be investigated. The main contributions of this paper are as follows. (1) A multirate NCSs with simultaneous consideration time delay and packet dropout are modeled as a time-varying sampling system with time delay. (2) The Lyapunov function deceases at each input signal updating point, which is largely ignored in prior works. Compared with traditional NCSs methods, it can yield less conservative. (3) State feedback controller of multirate NCSs, which render the multirate networked control systems stochastic mean square stable, is proposed. And the cost function value is less than a bound.

This paper is organized into four sections including the Introduction. Problem formulations and main assumptions were presented in Section 2. In Section 3, the guaranteed cost control of NCSs was discussed. The controller is proved to render the system stochastic mean square stable. An illustrative example is provided in Section 4.

Notations. The superscript "T" stands for the transpose of a matrix. R^n and $R^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $\|\cdot\|$ stands for the Euclidean norm. I and 0 stand for identified matrices and zero matrices with appropriate dimensions, respectively. The notation $X > 0\,(X \geq 0)$ means that the matrix X is positive definite (X is semipositive definite). I is the identity matrix of appropriate dimensions. $\left[\begin{smallmatrix} X & Z \\ * & Y \end{smallmatrix}\right]$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Problem Formulation

It is assumed that the controlled process is a linear time-invariant system, which can be expressed as

$$\dot{x}(t) = A^c x(t) + B^c u(t) + E^c v(t), \\ z(t) = Cx(t) + Hv(t), \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $v(t) \in R^q$, and A^c, B^c, E^c, C, H are matrices of appropriate sizes and $v(t)$ is white noise with zero mean. The sampling period of the sensor is noted as T_s, and the sampling periods of controller and actuator are the same, noted as T_c. $T_c = T_s/N$ and N are a positive integer not less than 2. That is to say, the controller and actuator have a higher sampling frequency than the sensor. In order to facilitate the discussion, the delay of the system is considered to be a constant short delay, in this paper. The information transmission sequence of multirate NCSs is in Figure 1.

In convenience of investigation, we make the following rational assumptions.

(A1) The sensor, the controller, and the actuator are all time-driven, sensor-to-controller delay is denoted by τ_{sc}, and the delay of controller-to-actuator is denoted by τ_{ca}. The time delay in the system is $\tau = \tau_{sc} + \tau_{ca} = hT_c, h \leq N$, and h is a positive integer.

(A2) The number of successive packet dropouts is upper bounded, and the bound is denoted a known constant d.

(A3) The system adopts the Zero Order Hold (ZOH) strategy.

In the multirate networked control systems, the inputs of sampling interval are different from sampling interval to interval. The model of multirate NCSs is described as follows.

Case $S_i^{(0)}$. There is no packet dropout which occurs in the current sampling interval. Such as in the interval $[(k+5)T_s, (k+6)T_s]$,

$$\begin{aligned} x[(k+1)T_s] &= Ax(kT_s) \\ &\quad + \left(\int_{kT_s}^{kT_s+hT_c} e^{A^c[(k+1)T_s-\eta]} B^c d\eta\right) \\ &\quad \times u[(k-1)T_s] \\ &\quad + \left(\int_{kT_s+hT_c}^{(k+1)T_s} e^{A^c[(k+1)T_s-\eta]} B^c d\eta\right) u(kT_s) \\ &\quad + E^c v(kT_s) \\ &= Ax(kT_s) + \left(\int_0^{(N-h)T_c} e^{A^c\eta_1} B^c d\eta_1\right) u(kT_s) \\ &\quad + \left(\int_{(N-h)T_c}^{NT_c} e^{A^c\eta_1} B^c d\eta_1\right) u[(k-1)T_s] \\ &\quad + Ev(kT_s) \\ &= Ax(kT_s) + (B_1 + B_2 + \cdots + B_{N-h}) u(kT_s) \\ &\quad + (B_{N-h+1} + B_{N-h+2} + \cdots + B_N) \\ &\quad \times u[(k-1)T_s] + Ev(kT_s), \end{aligned} \tag{2}$$

where

$$\begin{aligned} A &= e^{A^cT_s}, \qquad B_1 = \int_0^{T_c} e^{A^c\eta_1} B^c d\eta_1, \\ B_k &= D^{k-1}B_1, \quad D = \int_0^{T_c} e^{A^c\eta_1} d\eta_1, \ 1 \le k \le N, \\ E &= \int_0^{T_s} e^{A^c\eta_1} E^c d\eta_1. \end{aligned} \tag{3}$$

Case $S_i^{(1)}$. There are i successive packet dropouts in the current sampling interval. Such as in interval $[(k+2)T_s, (k+3)T_s]$ to interval $[(k+3)T_s, (k+4)T_s]$. The inputs of actuator in this period are the latest effective inputs,

$$\begin{aligned} x[(k+1)T_s] &= Ax(kT_s) \\ &\quad + \left(\int_{kT_s}^{kT_s+T_s} e^{A^c[(k+1)T_s-\eta]} B^c d\eta\right) \\ &\quad \times u[(k-i)T_s] + Ev(kT_s) \\ &= Ax(kT_s) + (B_1 + B_2 + \cdots + B_N) \\ &\quad \times u[(k-i)T_s] + Ev(kT_s). \end{aligned} \tag{4a}$$

Case $S_i^{(2)}$. There is no packet dropout within the current sampling interval, but the last i times sampling intervals packet dropouts. Such as in the interval $[(k+3)T_s, (k+4)T_s]$

$$\begin{aligned} x[(k+1)T_s] &= Ax(kT_s) \\ &\quad + \left(\int_0^{(N-h)T_c} e^{A^c\eta_1} B^c d\eta_1\right) u(kT_s) \\ &\quad + \left(\int_{(N-h)T_c}^{NT_c} e^{A^c\eta_1} B^c d\eta_1\right) u[(k-1)T_s] \\ &\quad + Ev(kT_s) \\ &= Ax(kT_s) + (B_1 + B_2 + \cdots + B_{N-h}) u(kT_s) \\ &\quad + (B_{N-h+1} + B_{N-h+2} + \cdots + B_N) \\ &\quad \times u[(k-i-1)T_s] + Ev(kT_s). \end{aligned} \tag{4b}$$

Definition 1. Let $\varsigma = (i_1, i_2, \ldots, i_k\}$ denote the point of each input signal arrived,

$$\xi_k = \{i_{k+1} - i_k\} \in \{1, 2, 3, \ldots, d\}, \quad d = \max(i_{k+1} - i_k). \tag{5}$$

Definition 2. We assume the packet dropout progress on the basis of a discrete-time Markova chain process, the mode of transition probabilities:

$$p_{\xi_k} = \Pr(i_{k+1} = i + \xi_k \mid i_k = i) > 0, \quad i \in \varsigma, \tag{6}$$

where $\sum_{\xi_k=1}^{d} p_{\xi_k} = 1$.

The controlled process can be rewritten as

$$\begin{aligned} x(i_{k+1}) &= A^{\xi_k} x(i_k) + \Gamma_0(\tau_k) u(i_k) \\ &\quad + \left[\Gamma_1(\tau_k) + \delta_{\xi_k}\left(\sum_0^{\xi_k} A^i\right)\Gamma_2(\tau_k)\right] u(i_k - \xi_{k-1}) \\ &\quad + Ev(i_k), \end{aligned} \tag{7}$$

where $\delta_{\xi_k} = 1$ if $\xi_k > 1$, otherwise $\delta_{\xi_k} = 0$. $A^{\xi_k} = \underbrace{A \cdot A \cdot A \cdots A}_{\xi_k}$, $\Gamma_0(\tau_k) = \sum_{i=1}^{N-h} B_i$,

$$\Gamma_1(\tau_k) = \sum_{i=N-h+1}^{N} B_i, \qquad \Gamma_2(\tau_k) = \sum_{i=1}^{N} B_i. \tag{8}$$

The state-based control scheme for the system (7) is described by

$$u(k) = Kx(k), \tag{9}$$

where $K \in R^{m\times n}$ is the controller gain. Substituting (9) into (7) results in the following system:

$$x(i_{k+1}) = \left(A^{\xi_k} + \Gamma_0 K\right) x(i_k) + \left(\Gamma_1(\tau_k) + \delta_{\xi_k}\left(\sum_0^{\xi_k} A^i\right)\Gamma_2\right) K x(i_k - \xi_{k-1}) + E v(i_k), \tag{10}$$

where τ_k is omitted here.

3. Main Results and Proofs

Firstly, we introduce the following lemmas and definitions, which will be cited for the proofs in this section.

Definition 3. The closed-loop networked control systems (10) are stochastic mean-square stable if when $v(i_k) = 0$, $x_0 = x(0)$ such that

$$E\left(\sum_0^{\infty}\|x(i_k)\|^2\right) < \infty. \tag{11}$$

Definition 4. Our object is to design a controller such that the closed loop networked system with both packet dropout and time delay is stochastic mean-square stable and satisfies H_∞ performance constraint γ. That is to say (10) satisfies the following three conditions simultaneously.

(N1) The closed loop networked control systems (10) are stochastic mean-square stable.

(N2) For system (10), the defined cost function:

$$J = E\left\{\sum_{k=0}^{\infty} x^T(i_k) S x(i_k) + u^T(i_k) R u(i_k)\right\} \tag{12}$$

satisfies $J \le J^*$. J^* is a constant, where $S > 0$, $R > 0$.

(N3) Under the zero-initial condition, for all nonzero $v(i_k)$, the controlled output z_{i_k} satisfies

$$\sum_{k=0}^{\infty} E\left(z_{i_k}^T z_{i_k}\right) - \gamma^2 v_{i_k}^T v_{i_k} < 0. \tag{13}$$

Lemma 5 (Schur complement). *For given a matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22}\end{bmatrix}$, *where* S_{11}, S_{12} *are square matrices, the following conditions are equivalent:*

$$\begin{aligned} &(1)\ S < 0; \\ &(2)\ S_{11} < 0, \quad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0; \\ &(3)\ S_{22} < 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0. \end{aligned} \tag{14}$$

Theorem 6. *For the system* (10)*, if positive definite matrixes exist* $P, Q > 0$*, such that*

$$\begin{aligned} &(1)\ 2G^T(P+Q)G - P + S + K^T R K < 0, \\ &(2)\ 2H_{\xi_k}^T(P+Q)H_{\xi_k} - Q < 0, \end{aligned} \tag{15}$$

where $G = A^{\xi_k} + \Gamma_0 K$, $H_{\xi_k} = [\Gamma_1 + \delta_{\xi_k}(\sum_{i=0}^{\xi_k} A^i)\Gamma_2]K$*. Then the system* (10) *with the controller* (9) *is stochastic mean-square stable and the cost function value is less than a bound. The corresponding cost function satisfies* $J < x^T(i_0)Px(i_0) + x^T(i_{-1})Qx(i_{-1})$.

Proof. Define a Lyapunov function as

$$\begin{aligned} V_1(i_k) &= x^T(i_k) P x(i_k), \\ V_2(i_k) &= \sum_{s=\xi_{k-1}}^{i_k-1} x^T(i_k - s) Q x(i_k - s), \\ V(i_k) &= \sum_{j=1}^{2} V_j(i_k), \end{aligned} \tag{16}$$

where $P > 0, Q > 0$

$$\Delta V(i_k) = E[V(i_{k+1}) - V(i_k)] = \sum_{j=1}^{2} E\left(\Delta V_j(i_k)\right). \tag{17}$$

Along the solution of (9), $E(\Delta V_1(i_k))$, $E(\Delta V_2(i_k))$ takes the form of

$$\begin{aligned} E(\Delta V_1(i_k)) = {} & \sum_{\xi_k=1}^{d} p_{\xi_{i_k}}\left[Gx(i_k) + H_{\xi_k i} x(i_k - \xi_{k-1})\right]^T P \\ & \times \left[Gx(i_k) + H_{\xi_k} x(i_k - \xi_{k-1})\right] \\ & - x^T(i_k) P x(i_k), \\ E(\Delta V_2(i_k)) \le {} & \sum_{\xi_k=1}^{d} p_{\xi_k}\left[Gx(i_k) + H_{\xi_k} x(i_k - \xi_{k-1})\right]^T Q \\ & \times \left[Gx(i_k) + H_{\xi_k} x(i_k - \xi_{k-1})\right] \\ & - \sum_{\xi_k=1}^{d} p_{\xi_k} x^T(i_k) Q x(i_k). \end{aligned} \tag{18}$$

By (17) we can obtain

$$\begin{aligned} \Delta V(i_k) \le {} & \sum_{\xi_k=1}^{d} p_{\xi_k}\left[Gx(i_k) + H_{\xi_{i_k}} x(i_k - \xi_{k-1})\right]^T (P+Q) \\ & \times \left[Gx(i_k) + H_{\xi_k} x(i_k - \xi_{k-1})\right] \\ & - x^T(i_k) P x(i_k) \\ & - \sum_{\xi_k=1}^{d} p_{\xi_k} x^T(i_k - \xi_{k-1}) Q x(i_k - \xi_{k-1}). \end{aligned} \tag{19}$$

It is easy to obtain

$$\begin{aligned}
&\sum_{\xi_k=1}^{d} p_{\xi_k}\left[Gx(i_k)+H_{\xi_k}x\left(i_k-\xi_{k-1}\right)\right]^T(P+Q)\\
&\quad\times\left[Gx\left(i_k\right)+H_{\xi_k}x\left(i_k-\xi_{k-1}\right)\right]\\
&=\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k\right)G^T(P+Q)H_{\xi_k}x\left(i_k-\xi_{k-1}\right)\right]\\
&\quad+\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k-\xi_{k-1}\right)H_{\xi_k}^T(P+Q)Gx\left(i_k\right)\right]\\
&\quad+\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k\right)G^T(P+Q)Gx\left(i_k\right)\right]\\
&\quad+\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k-\xi_{i_{k-1}}\right)H_{\xi_k}^T(P+Q)H_{\xi_k}x\left(i_k-\xi_{i_{k-1}}\right)\right].
\end{aligned}\tag{20}$$

By Lemma 5, $[Gx(i_k), H_{\xi_k}x(i_k-\xi_{k-1})]^T\left[\begin{smallmatrix}-P-Q & P+Q\\ P+Q & -P-Q\end{smallmatrix}\right][Gx(i_k), H_{\xi_k}x(i_k-\xi_{k-1})]^T \le 0$.

Therefore,

$$\begin{aligned}
\Delta V\left(i_k\right)\le{}&2\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k\right)G^T(P+Q)Gx\left(i_k\right)\right]\\
&+2\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k-\xi_{i_{k-1}}\right)H_{\xi_k}^T\right.\\
&\qquad\left.\times(P+Q)H_{\xi_k}x\left(i_k-\xi_{i_{k-1}}\right)\right].
\end{aligned}\tag{21}$$

Corresponding to (20),

$$\begin{aligned}
\Delta V\left(i_k\right)\le{}&\sum_{\xi_k=1}^{d} p_{\xi_k}\left[x^T\left(i_k\right)\left[2G^T(P+Q)G-P\right]x\left(i_k\right)\right]\\
&\sum_{\xi_k=1}^{d}\left[p_{\xi_k}x^T\left(i_k-\xi_{k-1}\right)\left[2H_{\xi_k}^T(P+Q)H_{\xi_k}-Q\right]\right.\\
&\qquad\left.\times x\left(i_k-\xi_{k-1}\right)\right].
\end{aligned}\tag{22}$$

Denote $\Phi_i = 2G^T(P+Q)G-P+S+K^TRK$.

It is apparently $\Delta V(i_k)\le 0$, when (1) $\Phi_i<0$, (2) $2H_{\xi_k}^T(P+Q)H_{\xi_k}-Q<0$.

Therefore, $E(V(i_k)) \le -\min\{\lambda_{\min}(-\Phi_i)\}\|x(k)\|^2$. Denote $\mu=\min\{\lambda_{\min}(-\Phi_i)\}$

$$\Xi\left(\sum_{k=0}^{\infty}\left\|x\left(i_k\right)\right\|^2\right)\le\frac{1}{\mu}\Xi\left(V\left(x(0),\xi_0\right)\right)<\infty.\tag{23}$$

Then, the networked control systems (10) are stochastic mean-square stable.

Due to (22),

$$\begin{aligned}
&\sum_{k=0}^{\infty}E\left(V\left(i_{k+1}\right)-V\left(i_k\right)\right)\le -J\\
&\quad\Longrightarrow J\le V\left(i_0\right)-V\left(i_\infty\right)<x^T\left(i_0\right)Px\left(i_0\right)\\
&\qquad+x^T\left(i_{-1}\right)Qx\left(i_{-1}\right).
\end{aligned}\tag{24}$$

□

Theorem 7. *For given matrices R, S if there exist matrix M and positive definite matrices $X=P^{-1}$, $Y=Q^{-1}$ such that*

$$(1)\ \begin{bmatrix}\Xi_1 & \Xi_2\\ * & \Xi_3\end{bmatrix}<0;\qquad (2)\ \begin{bmatrix}-\frac{1}{2}Y & * & *\\ \Gamma_1M+\bar{H}M & -X & 0\\ \Gamma_1M+\bar{H}M & 0 & -Y\end{bmatrix}<0.\tag{25}$$

Then $K=MX^{-1}$ is a guaranteed cost controller gain for the system (10) with disturbance $v_{i_k}=0$ and the corresponding closed loop cost function satisfies

$$J<\lambda_{\max}\left(U^TPU\right)+\lambda_{\max}\left(U^TQU\right),\tag{26}$$

where

$$\begin{aligned}
&\Xi_1=\begin{bmatrix}-X & * & *\\ A^{\xi_k}X+\Gamma_0M & -\frac{1}{2}X & 0\\ A^{\xi_k}X+\Gamma_0M & 0 & -\frac{1}{2}Y\end{bmatrix},\\
&\Xi_2=\begin{bmatrix}M^T & 0\\ 0 & 0\\ 0 & 0\end{bmatrix},\qquad \Xi_3=\begin{bmatrix}-R^{-1} & 0\\ * & -S^{-1}\end{bmatrix},\\
&\bar{H}=\delta_{\xi_k}\left(\sum_{i=0}^{\xi_k}A^i\right)\Gamma_2.
\end{aligned}\tag{27}$$

Proof. By Lemma 5, (15)⇔(28),

$$(1)\ \begin{bmatrix}\Xi_1' & \Xi_2'\\ * & \Xi_3\end{bmatrix}<0;\qquad (2)\ \begin{bmatrix}-\frac{1}{2}Q & H_{\xi_{i_k}}^T & H_{\xi_{i_k}}^T\\ * & -P^{-1} & 0\\ * & * & -Q^{-1}\end{bmatrix}<0,\tag{28}$$

where

$$\Xi_1'=\begin{bmatrix}-P & G^T & G^T\\ * & -\frac{1}{2}P^{-1} & 0\\ * & * & -\frac{1}{2}Q^{-1}\end{bmatrix},\qquad \Xi_2'=\begin{bmatrix}K^T & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}.\tag{29}$$

Pre- and postmultiplying (28)(1) by $\mathrm{diag}(P^{-1}, I, I, I, I, I, I)$, Pre- and postmultiplying (28)(2) by $\mathrm{diag}(Q^{-1}, I, I)$. Define $X=P^{-1}$, $Y=Q^{-1}$; we obtain (25).

The initial state of system is unknown; we suppose the initial state of the system (10) is arbitrary and belongs to the set $S = \{x(-i_k) \in R^n : x(-i_k) = UV(i_k), V^T(i_k)V(i_k) < 1, k = 0, 1\}$, where U is a given matrix. Then, the cost bound $J < x^T(i_0)Px(i_0) + x^T(i_{-1})Qx(i_{-1})$ leads to (26). □

Remark 8. Denote the upper bound of the cost function J as J^* and $J^* = \theta_1 + \theta_2$ depends on matrices X, Y. The optimal guaranteed cost control gain of NCSs (10) can be solved by existing LMI that

$$\begin{aligned} &\text{Minimize} \quad \theta_1 + \theta_2 \\ &\text{s.t} \qquad (1) \begin{bmatrix} -\theta_1 I & U^T \\ U & -X \end{bmatrix} \leq 0 \\ &\qquad\qquad (2) \begin{bmatrix} -\theta_2 I & U^T \\ U & -Y \end{bmatrix} \leq 0. \end{aligned} \tag{30}$$

Theorem 9. *Take given scalar $\gamma > 0$, and matrices R, S, if there exist matrix M and positive definite matrices $X = P^{-1}$, $Y = Q^{-1}$ such that*

$$\begin{aligned} &(1) \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_4 \\ * & \Xi_3 & 0 \\ * & * & \Xi_5 \end{bmatrix} < 0, \\ &(2) \begin{bmatrix} -\frac{1}{2}Y & * & * \\ \Gamma_1 M + \bar{H}M & -X & 0 \\ \Gamma_1 M + \bar{H}M & 0 & -Y \end{bmatrix} < 0, \end{aligned} \tag{31}$$

where $\Xi_4 = \begin{bmatrix} XC^T H & (CX)^T \\ 0 & 0 \end{bmatrix}$, $\Xi_5 = \operatorname{diag}(H^T H - \gamma^2 I, -I)$, *and* $\bar{H} = \delta_{\xi_k}(\sum_{i=0}^{\xi_k} A^i)\Gamma_2$.

Then $K = MX^{-1}$ is a guaranteed cost controller gain for the system (10) with H_∞ performance constraint (13) is achieved for all nonzero $v(k)$ and the cost function value is less than a bound:

$$J < \lambda_{\max}\left(U^T P U\right) + \lambda_{\max}\left(U^T Q U\right). \tag{32}$$

Proof. Consider

$$\begin{aligned} \sum_{k=0}^{\infty} E\left(z_k^T z_k\right) - \gamma^2 v_k^T v_k = \sum_{k=0}^{\infty} E\left(z_k^T z_k\right) - \gamma^2 v_k^T v_k \\ + \Delta V(k) - V_\infty + V_0, \end{aligned} \tag{33}$$

$V_0 = 0$, $V_\infty > 0$ can be obtained from the zero initial conditions. Therefore,

$$\begin{aligned} &\sum_{k=0}^{\infty} E\left(z_{i_k}^T z_{i_k}\right) - \gamma^2 v_{i_k}^T v_{i_k} \\ &\quad \leq \sum_{k=0}^{\infty} E\left(z_{i_k}^T z_{i_k}\right) - \gamma^2 v_{i_k}^T v_{i_k} + \Delta V(i_k) \end{aligned}$$

$$\begin{aligned} &z_{i_k}^T z_{i_k} - \gamma^2 v_{i_k}^T v_{i_k} + \Delta V(i_k) \\ &\leq \sum_{\xi_k=1}^{d} p_{\xi_k} \left[x^T(i_k) \left[2G^T (P+Q) G - P + S \right.\right. \\ &\qquad \left.\left. + K^T R K + C^T C \right] x(i_k) \right] \\ &\quad - x^T(i_k) \left[S + K^T R K \right] x(i_k) \\ &\quad + p_{\xi_k} \sum_{\xi_k=1}^{d} \left[x^T(i_k) C^T H v^T(i_k) + v^T(i_k) H^T C x^T(i_k) \right. \\ &\qquad \left. + v^T(i_k) H^T H v^T(i_k) \right] \\ &\quad + p_{\xi_k} \sum_{\xi_k=1}^{d} \left[x^T(i_k - \xi_{k-1}) \left[3H_{\xi_k}^T (P+Q) H_{\xi_k} - Q \right] \right. \\ &\qquad \left. \times x(i_k - \xi_{k-1}) \right]. \end{aligned} \tag{34}$$

By the Schur complement (34) is equivalent to

$$(1) \begin{bmatrix} \Xi_1' & \Xi_2' & \Xi_4 \\ * & \Xi_3 & 0 \\ * & * & \Xi_5 \end{bmatrix} < 0; \qquad (2) \begin{bmatrix} -\frac{1}{2}Q & H_{\xi_{i_k}}^T & H_{\xi_{i_k}}^T \\ * & -P^{-1} & 0 \\ * & * & -Q^{-1} \end{bmatrix} < 0. \tag{35}$$

Therefore, we can derive $z_{i_k}^T z_{i_k} - \gamma^2 v_{i_k}^T v_{i_k} + \Delta V(i_k) < 0$. Similar to Theorem 6, the networked system (10) is stochastic mean-square stable. Pre- and postmultiplying (35)(1) by $\operatorname{diag}(P^{-1}, I, I, I, I, I, I, I)$, pre- and postmultiplying (35) (2) by $\operatorname{diag}(Q^{-1}, I, I)$, and define $X = P^{-1}$, $Y = Q^{-1}$ we obtain (31).

Similar to Theorem 7, $J < \lambda_{\max}(U^T P U) + \lambda_{\max}(U^T Q U)$. □

Remark 10. The optimal guaranteed cost control gain ($J^* = \theta_1 + \theta_2$) in Theorem 9 can be solved by existing LMI that

$$\begin{aligned} &\text{Minimize} \quad \theta_1 + \theta_2 \\ &\text{s.t} \qquad (1) \begin{bmatrix} -\theta_1 I & U^T \\ U & -X \end{bmatrix} \leq 0 \\ &\qquad\qquad (2) \begin{bmatrix} -\theta_2 I & U^T \\ U & -Y \end{bmatrix} \leq 0. \end{aligned} \tag{36}$$

4. Numerical Examples

Example 1. Consider the following system:

$$\dot{x} = \begin{bmatrix} -0.3 & -1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.01 \\ -0.01 \end{bmatrix} u(t). \tag{37}$$

The sampling period of controller is $T_c = 0.01$ s, and the sampling period of sensor is $T_s = 0.15$ s. And the packet loss upper bounds $d = 2$, and the time delay $\tau = 0.03$ s. Choose $U = \begin{bmatrix} 2.366 & 1.6282 \\ 1.3025 & -1.624 \end{bmatrix}$, $S = \begin{bmatrix} 1 & 0 \\ 0 & 1.1 \end{bmatrix}$, $R = 10$.

By solving the LMIs given in Theorem 7, we can obtain

$$X = \begin{bmatrix} 0.1978 & 0.0595 \\ 0.0595 & 0.1236 \end{bmatrix}, \qquad M = 1.0e^{-10} \begin{bmatrix} 0.6131 & -0.9766 \end{bmatrix}. \tag{38}$$

Therefore, $K = MX^{-1} = 1.0e^{-008} [0.0640 \quad -0.1098]$.

The state responses of closed loop system are shown in Figure 2, where the initial condition is $x(0) = (1, -1)^T$. Solving the LMIs given in Remark 8, $J^* = 51.9179$. The simulation results show that the proposed method is effective.

Example 2. Consider the following system:

$$\dot{x} = \begin{bmatrix} -0.3 & -1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.01 \\ -0.01 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix} v(t),$$
$$z(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + 0.091 v(t),$$
$$v(t) = \begin{cases} 0.1 \sin(t), & 90 \le t \le 105, \\ 0, & \text{otherwise}. \end{cases} \tag{39}$$

The other parameters are the same with Example 1. Solving the LMIs given in Theorem 9,

$$X = \begin{bmatrix} 3.9836 & 0.8675 \\ 0.8675 & 3.9836 \end{bmatrix},$$
$$M = 1.0e^{-009} \begin{bmatrix} 0.5063 & -0.4462 \end{bmatrix}. \tag{40}$$

And we can obtain $K = MX^{-1} = 1.0e^{-009} [0.1353 \quad -0.0375]$. Solving the LMIs given in Remark 10, $J^* = 63.3589$. The state responses of closed loop system are shown in Figure 3, where the initial condition is $x(0) = (1, -1)^T$.

Example 3. To illustrate the proposed method's effectiveness, which is obtained in this issue, we consider the following system in [7, 14]:

$$\dot{x} = \begin{bmatrix} -1.84 & -0.33 \\ 7.18 & -1.14 \end{bmatrix} x(t) + \begin{bmatrix} 2.43 \\ -0.42 \end{bmatrix} u(t) + \begin{bmatrix} 1.86 \\ -0.76 \end{bmatrix} v(t),$$
$$z(t) = \begin{bmatrix} 0.57 & 0.78 \end{bmatrix} x(t) - 0.56 v(t). \tag{41}$$

We can see, [7, 14] supposed that the sampling period of controller is $T_c = 0.01$ s, the sampling period of sensor is $T_s = 0.1$ s. And the packet loss upper bounds $d = 1$, and the time delay $\tau = 0.01$s. Solving the LMIs given in Theorem 9, $K = [0.040, -0.051]$. We can obtain the H_∞ bounds from different methods in Table 1. Figure 4 illustrates the merits of the proposed multirate control system both with time delay and packet dropout.

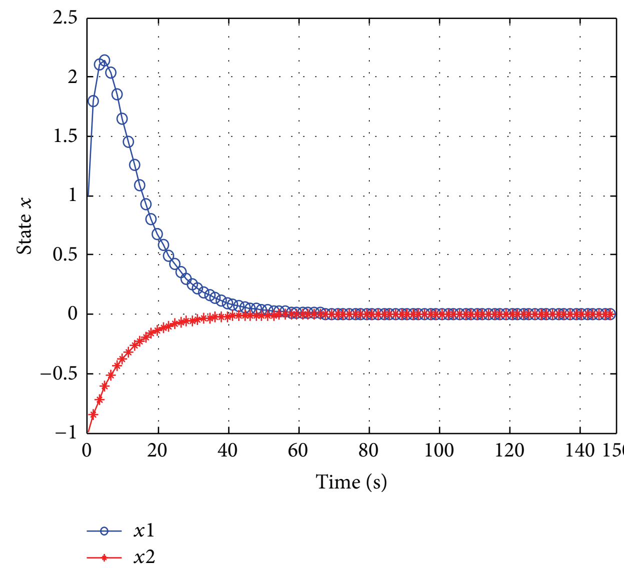

Figure 2: The state response of multirate NCSs.

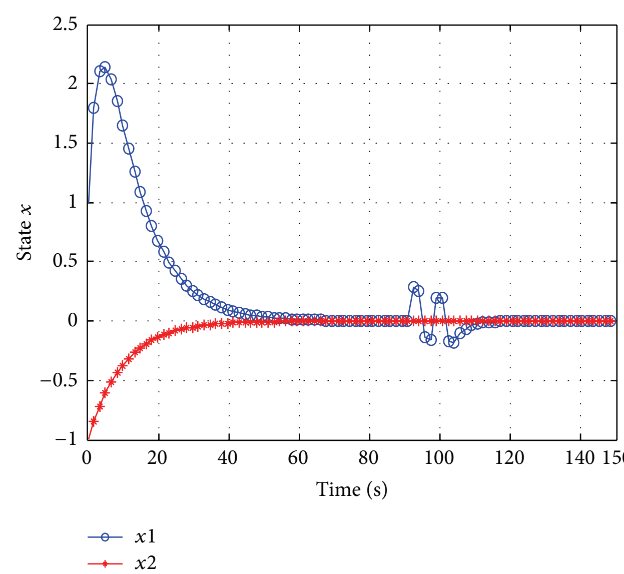

Figure 3: The state response of multirate NCSs with noise.

5. Conclusions

For multirate NCSs, we mean that the sampling periods of the nodes in the system are not the same. In this paper, the guaranteed cost control for multirate networked control system with both time delay and packet dropout of multi-rates networked control is discussed. A multirate NCS with simultaneous consideration time delay and packet dropout is modeled as a time-varying sampling system with time delay, in which the newest control inputs are adopted and the Lyapunov function deceasing at each input signal updating point. Numerical examples are given to demonstrate the effectiveness of the proposed method.

The proposed problems in this paper for the nonlinear networked control systems [40, 41] have not fully been investigated. The method of fuzzy control [42, 43] will be adopted in the future work.

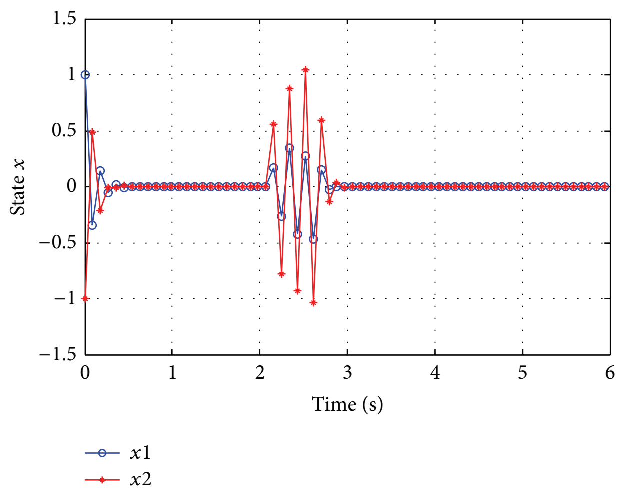

FIGURE 4: The state response of closed loop systems.

TABLE 1: H_∞ bounds.

Method	Theorem 6 [14]	Corollary 1 [7]	Theorem 9
γ	2.4966	4.4967	3.8705

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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