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IMPROVED CURRENT STATISTIC MODEL AND ADAPTIVE FILTERING

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ABSTRACT

Current statistical model needs to pre-define the value of maximum accelerations of maneuvering targets. So it may be difficult to meet all maneuvering conditions. In this paper a novel adaptive algorithm for tracking maneuvering targets is proposed. The algorithm is implemented with fuzzy-controlled current statistic model adaptive filtering and unscented transformation. The Monte Carlo simulation results show that this method outperforms the conventional tracking algorithm based on current statistical model.

Keywords: Current statistical model; Fuzzy logic; Unscented transformation

INTRODUCTION

The problem of tracking maneuvering targets has received a great of attention. The key to this problem lies in building the optimal target motion model. Various mathematical models of target motion have been developed over the past three decades, among which interacting multiple model (IMM) and current statistical model (CSM) are representative [1]. The current statistical model is in essence a Singer model modified to have a nonzero mean of the acceleration. Since the target acceleration hasn't always zero mean at any moment, the CSM works better than Singer model in practice. But the performance of CSM depends on three parameters: maneuvering frequency α, positive maximum acceleration $a_{\max}$ and negative maximum acceleration $a_{-\max}$, especially the last two. When people give a large $a_{\max}$ and $a_{-\max}$ a constant value, if the target is in no-maneuvering or a lower acceleration, the system variance will be large and tracking precision is low; If the target is maneuvering in a high acceleration, the system variance will be small, tracking precision is high. Since in CSM $a_{\max}$ and $a_{-\max}$ pre-defined, they can't adaptively adjust in tracking process to suit to different movement of target. For solving this problem, many algorithms have been studied. Jing [2] uses a pair of parallel filters to adapt to different cases of movement by the information fusion technique together with the CSM and BP neural network. But this algorithm's performance relies on the training data of neural network. Ji [3] uses fuzzy method to select the best $a_{\max}$ and $a_{-\max}$ from the discrete acceleration set. The shortcoming is the discrete acceleration set can't consist of all maneuvering conditions. In this paper, we propose a tracking method based on a fuzzy filter to cope with this problem. This method incorporates fuzzy inference in a conventional CSM filter by the use of a set of fuzzy if-then rules. Given the measure error and change-of-error in the last prediction, these rules are used to determine the scale parameter β to adjust $a_{\max}$ and $a_{-\max}$. The tracker has several advantages: quickly adjusting the magnitude of $a_{\max}$ and $a_{-\max}$ in response to changes in target movement; making better decisions by taking into consideration several different or even conflicting situations at the same time.

The paper is organized as follows. Section 2 describes the fuzzy-controlled current statistic model and adaptive filtering (FCSMAF) algorithm. Section 3 gives the simulation result. Concluding remarks are given in Section 4.

FUZZY-CONTROLLED CURRENT STATISTIC MODEL AND ADAPTIVE FILTERING

First we give a brief introduction about CSM. The CSM algorithm assumes that when a target is maneuvering with certain acceleration, its acceleration during the next period is limited within a range around the current acceleration. Hence it is not necessary to take all possible values of maneuvering acceleration into consideration when modeling the target acceleration probability. A modified Rayleigh density function whose mean is the current acceleration is utilized, and the relationship between the mean and variance of Rayleigh density is used to set up an adaptive algorithm for the variance of maneuvering acceleration [4].

Theory analyses show that the system variance σ_w^2 of this model is in direct proportion to acceleration variance σ_a^2

$$\sigma_a^2 = \begin{cases} \frac{4-\pi}{\pi}\left[a_{\max} - \hat{a}(k/k)\right]^2 & \hat{a}(k/k) \geq 0 \\ \frac{4-\pi}{\pi}\left[a_{-\max} - \hat{a}(k/k)\right]^2 & \hat{a}(k/k) < 0 \end{cases} \quad (1)$$

From (1) we can see since $a_{\max}$ and $a_{-\max}$ is a constant, if the target is in no-maneuvering or a lower acceleration, the system variance will be large and tracking precision is low; If the target is maneuvering in a high acceleration, the system variance will be small, tracking precision is high. The main shortcoming of the CSM is that in tracking process CSM can't adaptively adjust $a_{\max}$ and $a_{-\max}$ according to target maneuvering conditions.

In the above CSMAF, since $a_{\max}$ is predefined and affects the system variance, we use a fuzzy system to modify the value of $a_{\max}$ in order to get the most appropriate level in every case.

The fuzzy system is characterized by a set of fuzzy rules. Based on the error and change of error in the last prediction, these rules determine the magnitude of $a_{\max}$ in CSM. Therefore, the fuzzy decision system consists of two input variables and one output variable. Assumed that the dimension of measurement is equal to 2, the input variables $E(k)$ and $\Delta E(k)$ at the kth scan are defined by

$$E^{'}(k) = \frac{\sqrt{E_1^{'2}(k) + E_2^{'2}(k)}}{\sqrt{2}}, \quad \Delta E^{'}(k) = \frac{\sqrt{\Delta E_1^{'2}(k) + \Delta E_2^{'2}(k)}}{\sqrt{2}} \quad (2)$$

where $E_1(k)$, $E_2(k)$ and $\Delta E_1(k)$, $\Delta E_2(k)$ are normalized error and change of error of each component of measurement, respectively. $E_1(k)$ and $\Delta E_1(k)$ are defined by [5].

The fuzzy sets for the input variables $E(k)$ and $\Delta E(k)$ are labeled as the linguistic terms of LP (large positive), MP (medium positive), SP (small positive), and ZE (zero). These membership functions are defined by the trapezoidal function shown in Figure 1. The output variable is scale factor β. The fuzzy sets for β are labeled in the linguistic terms of EP (extremely large positive), VP (very large positive), LP (large positive), MP (medium positive), SP (small positive), and ZE (zero). The specific membership is defined by triangular functions shown in Figure 2. Given the range of $a_{\max}$, e.g. $a_{\max} \in [a_1, a_2]$, the updated $a_{\max}$ is calculated as

$$a_{\max} = a_1 + (a_2 - a_1)\beta \quad (3)$$

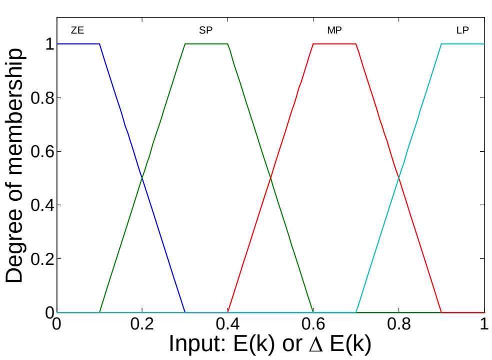

Figure 1 Membership functions for the input variables

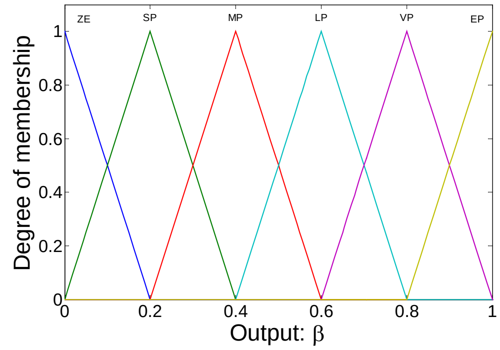

Figure 2 Membership functions for the output variables

With the input and output variables defined above, a fuzzy rule can be expressed in Table 1. The rules were obtained by interviewing some defense experts [5].

		E			
		ZE	SP	MP	LP
Δ *E*	ZE	VP	SP	EP	EP
	SP	LP	LP	VP	VP
	MP	EP	VP	MP	MP
	LP	VP	ZE	MP	EP

Table 1 Fuzzy associations for β

In tracking applications, target dynamics is usually modeled in Cartesian coordinates, while the measurements are directly available in the original sensor coordinates, which maybe give rise to nonlinear problem. Recently Julier and Uhlmann developed a new nonlinear filter (unscented filter) based on unscented transformation (UT) [6]. Some simulation results have shown that the unscented filter leads to more accurate results than the classical extended Kalman filter (EKF) [7]. So in this paper we use the UT to dealing with non-linear problem instead of the linearization.

Without loss of generality one dimension FCSMAF is derived. The discrete state equations is

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}\bar{a}(k) + \mathbf{B}_w w(k) \quad (4)$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & T & \frac{1}{\alpha^2}(-1 + \alpha T + e^{-\alpha T}) \\ 0 & 1 & \frac{1}{\alpha}(1 - e^{-\alpha T}) \\ 0 & 0 & e^{-\alpha T} \end{bmatrix},$$

$$\mathbf{B}_w = \begin{bmatrix} \frac{1}{\alpha^3}(1-\alpha T+\frac{1}{2}\alpha^2T^2-e^{-\alpha T}) \\ \frac{1}{\alpha^2}(-1+\alpha T+e^{-\alpha T}) \\ \frac{1}{\alpha}(1-e^{-\alpha T}) \end{bmatrix}, \quad \mathbf{G}=\alpha\mathbf{B}_w .$$

and $w(k)$ is a zero mean white noise sequence with variance $\sigma_w^2 = 2\alpha\sigma_a^2$. The measurement equation is

$$\mathbf{y}(k) = \mathrm{h}\left(\mathbf{x}(k)\right)+\mathbf{v}(k) \tag{5}$$

where $\mathbf{v}(\mathrm{k}) \sim N\left(0,\mathbf{R}\right)$, $\mathbf{R}$ is measurement noise covariance.

According to Eqs. (4) and (5) the UFCSMAF equations are as follows:

(1) Initialization:

$$\hat{\mathbf{x}}(\text{k-1}) = \mathrm{E}\left[\mathbf{x}(\mathrm{k}-1)\right],\ \mathbf{P}(\text{k-1}) = \mathrm{E}\left[(\mathbf{x}(\text{k-1})-\hat{\mathbf{x}}(\mathrm{k}-1))(\mathbf{x}(\text{k-1})-\hat{\mathbf{x}}(\mathrm{k}-1))^{\mathrm{T}}\right] \tag{6}$$

(2) Calculate sigma points:

$$\begin{cases} \chi_0 = \hat{\mathbf{x}}(k-1) & i=0 \\ \chi_i = \hat{\mathbf{x}}(k-1)+\left(\sqrt{(n_{\mathbf{x}}+\lambda)\mathbf{P}(k-1)}\right)_i & i=1,2,\cdots,n_{\mathbf{x}} \\ \chi_i = \hat{\mathbf{x}}(k-1)-\left(\sqrt{(n_{\mathbf{x}}+\lambda)\mathbf{P}(k-1)}\right)_i & i=n_{\mathbf{x}}+1,n_{\mathbf{x}}+2,\cdots,2n_{\mathbf{x}} \end{cases} \tag{7}$$

The associated weights of sigma points are

$$\begin{cases} w_0 = \lambda/\left(n_{\mathbf{x}}+\lambda\right) \\ w_i = 1/\left\{2\left(n_{\mathbf{x}}+\lambda\right)\right\} & i=1,2,\cdots,2n_{\mathbf{x}} \end{cases} \tag{8}$$

where λ is a scaling parameter and $\left(\sqrt{(n_{\mathbf{x}}+\lambda)\mathbf{P}(k-1)}\right)_i$ is the ith row or column of the matrix square root of $(n_{\mathbf{x}}+\lambda)\mathbf{P}(k-1)$.

(3) Calculate predicted state and its covariance

$$\begin{aligned} &\chi_i(k\mid k-1) = \mathbf{F}\chi_i(k-1)+\mathbf{G}\left[\chi_i(k\mid k-1)\right]_3 \\ &\Rightarrow \chi_i(k\mid k-1) = \mathbf{\Phi}\chi_i(k-1) \end{aligned} \tag{9}$$

where notation $\left[\chi\right]_i$ stand for the ith element of vector χ and

$$\mathbf{\Phi} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\hat{\mathbf{x}}\left(k\mid k-1\right) = \sum_{i=0}^{2n_{\mathbf{x}}} w_i\chi_i(k\mid k-1) \tag{10}$$

$$\mathbf{P}(k\mid k-1) = \sum_{i=0}^{2n_{\mathbf{x}}} w_i\left[\chi_i(k\mid k-1)-\hat{\mathbf{x}}(k\mid k-1)\right]\left[\chi_i(k\mid k-1)-\hat{\mathbf{x}}(k\mid k-1)\right]^{\mathrm{T}}+2\alpha\sigma_a^2\mathbf{B}_w\mathbf{B}_w^{\mathrm{T}} \tag{11}$$

(4) Calculate predicted measurement and its covariance

$$\xi_i(k\mid k-1) = \mathrm{h}\left(\chi_i(k\mid k-1)\right) \tag{12}$$

$$\hat{\mathbf{y}}(k\mid k-1) = \sum_{i=0}^{2n_{\mathbf{x}}} w_i\xi_i(k\mid k-1) \tag{13}$$

$$\mathbf{P}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^{\mathrm{T}}} = \sum_{i=0}^{2n_{\mathbf{x}}} w_i\left(\xi_i(k\mid k-1)-\hat{\mathbf{y}}\left(k\mid k-1\right)\right)\left(\xi_i(k\mid k-1)-\hat{\mathbf{y}}\left(k\mid k-1\right)\right)^{\mathrm{T}} \tag{14}$$

(5) Calculate filter gain

$$\mathbf{P}_{\mathbf{x}_k\mathbf{y}_k} = \sum_{i=0}^{2n_{\mathbf{x}}} w_i\left(\chi_i(k\mid k-1)-\hat{\mathbf{x}}(k\mid k-1)\right)\left(\xi_i(k\mid k-1)-\hat{\mathbf{y}}(k\mid k-1)\right)^{\mathrm{T}} \tag{15}$$

$$\mathbf{K}(k) = \mathbf{P}_{\mathbf{xy}}\mathbf{P}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^{\mathrm{T}}}^{-1} \tag{16}$$

(6) Calculate maximum acceleration

$$\begin{cases} E(k) \\ \Delta E(k) \end{cases} \xrightarrow{\text{Fuzzy system}} a_{\max} \tag{17}$$

(7) Calculate updated state and its covariance

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}(k\mid k-1)+\mathbf{K}(k)\left(\mathbf{y}(k)-\hat{\mathbf{y}}(k\mid k-1)\right) \tag{18}$$

$$\mathbf{P}(k) = \mathbf{P}(k\mid k-1)-\mathbf{K}(k)\mathbf{P}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^{\mathrm{T}}}\mathbf{K}^{\mathrm{T}}(k) \tag{19}$$

SIMULATION AND PERFORMANCE ANALYSIS

We will apply the filter (FCSMAF) described by the above algorithm to a target tracking problem and compare its performance to that of a conventional filter (CSMAF), i.e. filter based on CSM and UKF. Here we use the root mean squares error (RMSE) as the criterion.

The target initial position is (80km, 50km) and initial velocity is (-0.3km/s, 0km/s). In first segment from 0 to 40s, the target moves with a constant velocity (-0.3km/s, 0km/s); in the second segment from 41 to 60s, the target makes a left turn with a centripetal acceleration 30m/s^2; in the third segment from 61 to 100s, the target moves with a constant velocity; in the fourth segment from 101 to 150s, the target makes a right turn with a centripetal acceleration 15m/s^2; in the fifth segment from 151 to 180s, the target moves with a constant velocity; in the sixth segment from 181 to 200s, the target makes a left turn with a centripetal acceleration 35m/s^2; in the seventh segment from 201 to 220s, the target moves with a constant velocity; in the eighth segment from 221 to 236s, the target makes a right turn with a centripetal acceleration 45m/s^2; in the last segment from 237 to 270s, the target moves with a constant velocity.

Here two distributed observers measure the target line-of-sight (LOS) angles. In Cartesian coordinate, the measurement equations are non-linear. The observers fly in a circle of radius 3km and at the speed of 0.3km/s. Their angular measurements standard deviations both are 1mrad. The center of circle of observer 1 locates in (10km, 25km). The center of circle of observer 2 locates in (10km, 45km).

The others parameters are as follows: $T=1s$; $\alpha=0.1$; $a_{\max}=8g\ m^2/s$; $\lambda=0$.

We carry out 100 Monte Carlo runs and give the results of the position and velocity along x-axis in Figure 3-4. Since the results along y-axis similar to that along x-axis, we don't give the results along y-axis.

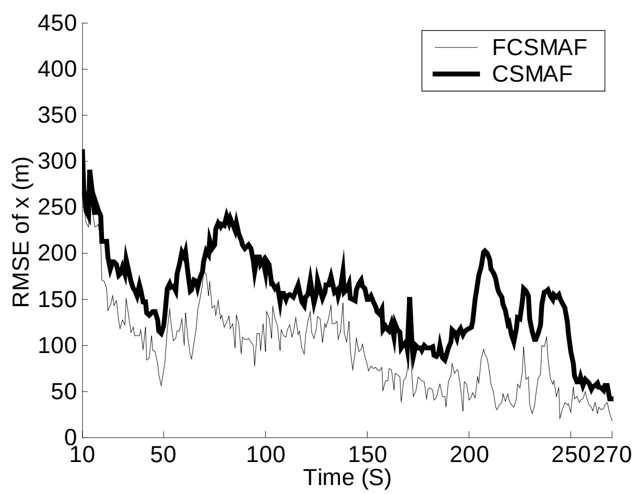

Figure 3 RMSE of position along x-axis

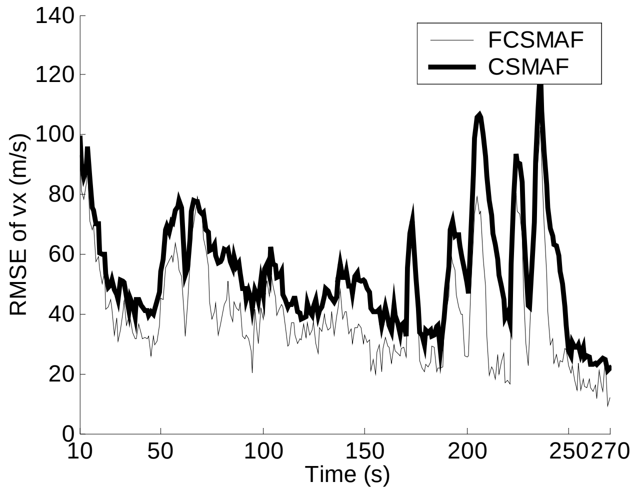

Figure 4 RMSE of velocity along x-axis

From the Figure 3-4, we can see the performance of the FCSMAF always outperforms that of CSUKF in both non-maneuvering and maneuvering motions. These plots also show the convergence rate of FCSMAF faster than that of CSMAF. However, in continuous sharp maneuvering motion (after 180s in these figures), the RMSE of FCSMAF is relatively large. How to resolve it is our issue in next step.

CONCLUSIONS

A FCSMAF algorithm has been presented in this paper for tracking a maneuvering target. The theoretic analysis and computer simulations have confirmed that the presented adaptive algorithm has a robust advantage over a wide range of maneuvers and overcomes the shortcoming of the traditional current statistic model and adaptive filtering algorithm.

Future work will need to address the better model characterizing target maneuvers.

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