



ANISOTROPIC BIANCHI TYPE-I COSMOLOGICAL MODEL FOR VISCOUS FLUID IN A MODIFIED BRANS-DICKE COSMOLOGY

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ABSTRACT

We present a new Cosmological solution for a Bianchi type-I Cosmological model filled with viscous fluid in a modified Brans-Dicke theory in which the variable cosmological term is an explicit function of a scalar field. The physical and geometrical properties of this model have been discussed. Finally, this model has been transform to the original form (1961) of Bras-Dicke theory.

Keywords: spatially homogeneous, anisotropic, bianchi type-I, cosmological model, brans-dicke theory.

1. INTRODUCTION

Einstein succeeded in geometrizing gravitation by expressing gravitational potential in terms of metric tensor. Various observations indicate that spatially homogeneous space-time is Bianchi type's model and interpreted as cosmological models. Cosmological models with a cosmological constant are currently serious candidates to describe the dynamics of the universe. The significance of cosmological constant was studied by various cosmologists but no satisfactory results of its meaning have been supported as yet. Zel'dovich [1] has tried to visualize the meaning of this term from theory of elementary particles. Linde [2] argued that the cosmological term arises from spontaneous symmetry breaking and suggested that the term is not a constant but a function of temperature. In cosmology, the term may be understood by incorporation with Mach's principle that suggests the acceptance of Brans-Dicke Lagrangian as realistic one [3]. Endo and Fukui [4] have also studied the variable cosmological term in

Brans-Dicke theory [3] and elementary particle physics. Also Rai, Rai and Singh [5] have studied the variable cosmological term for viscous fluid in modified Brans-Dicke theory by imposing Petrov type -D condition.

Astronomical observation for large scale distribution of galaxies indicates that the distribution of matter can be satisfactorily described by a perfect fluid. Also at the early stage of the universe when galaxies were formed, the matter distribution behaved like a viscous fluid [6]. So, in this paper we consider Bianchi type-I cosmological model for viscous fluid in a modified Brans-Dicke theory in which the variable cosmological term Q in an explicit function of a scalar field ϕ proposed by Bergmann[7] and Wagonar [8], detailed discussed by both Endo and Fukui [4] and Rai, Rai and Singh [5].

The Brans-Dicke field equations with cosmological term Q [5, 9] are:

$$G_{ij} + g_{ij}Q = \frac{8\pi}{\phi} T_{ij} + \frac{\omega}{\phi^2} \left(\phi_i \phi_j - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) + \frac{1}{\phi} (\phi_{;j} - g_{ij} \phi_{;\alpha}^{\alpha}), \quad (1)$$

$$\phi_{;\alpha}^{\alpha} = \frac{8\pi\mu T}{(2\omega+3)}, \quad (2)$$

$$Q = \frac{(2\omega+3)(1-\mu)}{4} \frac{\phi_{;\alpha}^{\alpha}}{\phi} = \frac{8\pi(1-\mu)}{4\phi} T \quad (3)$$

where the constant μ shows how much this theory including $Q(\phi)$ deviates from that of Brans and Dicke and as usual ω is coupling constant and T_{ij} is the energy-momentum tensor for a viscous fluid distribution[9]. Covariant derivative with respect to the metric g_{ij} is denoted by semicolons and partial differentiation with respect to the coordinate x^i is

denoted by comas. Then under the conformal transformation:

$$g_{ij} \rightarrow \bar{g}_{ij} = \phi g_{ij} \quad (4)$$

Equations (1) – (3) becomes (5)

$$\bar{G}_{ij} + \bar{g}_{ij}\bar{Q} = 8\pi\bar{T}_{ij} + \frac{1}{2}(2\omega+3) \left(\Lambda_{,i}\Lambda_{,j} - \frac{1}{2}\bar{g}_{ij}\Lambda_{,k}\Lambda^{,k} \right), \quad (5)$$

$$\bar{\Lambda}_{;\alpha}^{\alpha} = \frac{8\pi\mu\bar{T}}{(2\omega+3)}, \quad \Lambda = \log \phi, \quad (6)$$



$$\bar{Q} = \frac{(2\omega+3)(1-\mu)}{4} \frac{\bar{\Lambda}_{;\alpha}}{\mu} = \frac{8\pi(1-\mu)}{4\phi} \bar{T} \quad (7)$$

where all barred and unbarred quantities are defined in terms of metric \bar{g}_{ij} and g_{ij} respectively.

In this paper, in sect. 2 we considered Bianchi type-I metric and energy- momentum tensor for viscous fluid [5, 9]. In sect.3 we obtained the solution of the field equations for the Bianchi type I metric. In sect.4 we obtained expressions for pressure, density and cosmological term for spatially homogeneous and anisotropic cosmological model. Finally in sect.5 we have transformed this model to the 1961 form of Brans-Dicke theory.

2. FIELD EQUATIONS

The Bianchi type-I metric is considered as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \quad (8)$$

where A, B, C , are functions of $x^4 = t$ only. The energy-momentum tensor T_{ij}

$$\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} \right] + \bar{Q} = 8\pi \left\{ \bar{p} - 2\eta \left(\frac{\dot{A}}{A} \right) - \left(\xi - \frac{2}{3}\eta \right) v_{;a}^a \right\} + \frac{(2\omega+3)}{4} \dot{\Lambda}^2 \quad (11)$$

$$\left[\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} \right] + \bar{Q} = 8\pi \left\{ \bar{p} - 2\eta \left(\frac{\dot{B}}{B} \right) - \left(\xi - \frac{2}{3}\eta \right) v_{;a}^a \right\} + \frac{(2\omega+3)}{4} \dot{\Lambda}^2 \quad (12)$$

$$\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} \right] + \bar{Q} = 8\pi \left\{ \bar{p} - 2\eta \left(\frac{\dot{C}}{C} \right) - \left(\xi - \frac{2}{3}\eta \right) v_{;a}^a \right\} + \frac{(2\omega+3)}{4} \dot{\Lambda}^2 \quad (13)$$

$$\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} \right] + \bar{Q} = -8\pi\bar{\varepsilon} - \frac{(2\omega+3)}{4} \dot{\Lambda}^2 \quad (14)$$

$$\ddot{\Lambda} + \dot{\Lambda} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = -\frac{8\pi\mu}{(2\omega+3)} \left[(3\bar{p} - \bar{\varepsilon}) - 3\xi \frac{d}{dt} \log(ABC) \right] \quad (15)$$

where dot represent differentiation with respect to t. The coefficients of viscosity η and ξ are taken as constants. For complete determinacy of the system one extra condition is needed, which we will impose later.

3. SOLUTION OF FIELD EQUATIONS

The equation (11)-(15) are five equations in six unknowns $A, B, C, \bar{\varepsilon}, \bar{p}$ and Λ .

Let us set $16\pi\eta = L$.

Equation (11)-(12) yields

$$m(\dot{A}\dot{B} - \dot{A}\dot{B})C = e^{Lt}, \quad m = \text{constant} \quad (16)$$

for a viscous fluid distribution is given by [5,9]

$$\bar{T}_i^k = (\bar{\varepsilon} + \bar{p})\bar{v}_i^k + \bar{p}\bar{g}_i^k - \eta(\bar{v}_i^{;k} + \bar{v}_i^k + \bar{v}^k \bar{v}_{;i}^a + \bar{v}_i^a \bar{v}_{;a}^k) - \left(\xi - \frac{2}{3}\eta \right) \bar{v}_{;a}^a (\bar{g}_i^k + \bar{v}_i^k \bar{v}^k) \quad (9)$$

along with

$$\bar{g}_{ij} \bar{v}^i \bar{v}^j = -1 \quad (10)$$

where \bar{p} is the isotropic pressure, $\bar{\varepsilon}$ the density, η and ξ the two coefficient of viscosity and semicolons defined covariant differentiation. \bar{v}^i is the components of the fluid four-velocity satisfying equation (10). We assume that the coordinate to be co-moving i.e. $\bar{v}^1 = \bar{v}^2 = \bar{v}^3 = 0$ and $\bar{v}^4 = 1$. The scalar field Λ is also taken to be a function of t only. The field equations (5) and (6) for the line element (8) turn into

Equation (12)-(13) yields

$$k(\dot{B}C - C\dot{B})A = e^{Lt}, \quad k = \text{constant} \quad (17)$$

Equation (11)-(13) yields

$$\alpha(\dot{A}C - C\dot{A})B = e^{Lt}, \quad \alpha = \text{constant} \quad (18)$$

To avoid the complicity, we assume

$$A = f(t)e^{\mu t}, \quad B = g(t)e^{\nu t}, \quad C = h(t)e^{\rho t}$$



By using above, equation (16) gives two conditions

$$v - \mu = \frac{1}{mfg} - \frac{\dot{g}}{g} + \frac{\dot{f}}{f} = s \text{ (say) (i), } \mu + v + \rho = L \quad \text{(ii)}$$

where m, s are constants.

Similarly equation (17) and (18) give extra conditions except (ii) are respectively

$$\rho - v = \frac{\dot{g}}{g} - \frac{\dot{h}}{h} + \frac{1}{rfgh} = b, \quad \text{(iii)}$$

and

$$\rho - \mu = \frac{1}{\alpha fgh} - \frac{\dot{h}}{h} + \frac{\dot{f}}{f} = c, \quad \text{(iv)}$$

where b, c are constants.

Consider $\int \frac{1}{fgh} dt = \log K(t)$. Then equation (i), (iii) and (iv) yield respectively

$$K^{\frac{1}{m}} f = ge^{st} \text{ (v), } K^{\frac{1}{kr}} g = he^{bt} \text{ (vi) and } K^{\frac{1}{\alpha}} f = he^{ct} \text{ (vii)}$$

Using the above (v), (vi) and (vii), we obtained the relation

$$he^{bt} = K^{\frac{1}{kr}} g = K^{\frac{1}{kr} + \frac{1}{m} - \frac{1}{\alpha}} he^{(c-s)t}$$

Comparing with (vi) we will get

$$\frac{1}{k} + \frac{1}{m} - \frac{1}{\alpha} = 1, \Rightarrow \alpha = \frac{mk}{m+k-mk} \text{ (viii) and } b = c - s \quad \text{(ix)}$$

$$ds^2 = -dt^2 + (\gamma t + l)^{\frac{2k(m\gamma-2)-2m}{3km\gamma}} e^{\frac{32\pi\eta}{3}t} dx^2 + (\gamma t + l)^{\frac{2k(m\gamma+1)-2m}{3km\gamma}} e^{\frac{32\pi\eta}{3}t} dy^2 + (\gamma t + l)^{\frac{2k(m\gamma+1)+4m}{3km\gamma}} e^{\frac{32\pi\eta}{3}t} dz^2 \quad \text{(23)}$$

By the following transformation of coordinate $(\gamma t + l) \rightarrow t$ we get

$$ds^2 = -dt^2 + e^{\frac{32\pi\eta}{3}t} \left\{ t^{\frac{2k(m-2)-2m}{3km}} dx^2 + t^{\frac{2k(m+1)-2m}{3km}} dy^2 + t^{\frac{2k(m+1)+4m}{3km}} dz^2 \right\} \quad \text{(24)}$$

Again from (v) and (vi), we obtained

$$g^2 = K^{\frac{k-m}{mk}} fhe^{(b-s)t} \text{ or } g^3 = K^{\frac{k-m}{mk}} fgh e^{(c-2s)t}, \quad \text{(19)}$$

(by using (ix))

Now, we impose the condition

$$K(t) = (\gamma t + l)^{\frac{1}{\gamma}}, \text{ which gives } fgh = \gamma t + l. \quad \text{(x)}$$

Equation (19) and (x) gives

$$g = (\gamma t + l)^{\frac{k(m\gamma+1)-m}{3km\gamma}} e^{\frac{(c-2s)}{3}t} \quad \text{(20)}$$

Using (20), equation (v) and (vii) give, respectively

$$f = (\gamma t + l)^{\frac{k(m\gamma-2)-m}{3km\gamma}} e^{\frac{(c+s)}{3}t} \quad \text{(21)}$$

and

$$h = (\gamma t + l)^{\frac{k(m\gamma+1)+2m}{3km\gamma}} e^{\frac{(s-2c)}{3}t} \quad \text{(22)}$$

Again, using equations (i), (ii), (iii), (iv) and (ix), we obtained

$$\mu = \frac{L-c-s}{3}, v = \frac{2s+L-c}{3}, \rho = \frac{2c+L-s}{3} \quad \text{(xi)}$$

where $L = 16\pi\eta$

Hence, the required solution is



4. SOME PHYSICAL AND GEOMETRICAL PROPERTIES

The pressure and density in the model (24) are given by

$$8\bar{\rho} = \frac{1024\pi^2\eta^2}{9} + \frac{128\pi\eta}{9t} + \frac{(m^2k^2 - mk - m^2 - k^2)}{9m^2k^2t^2} + 8\pi\xi\left(16\pi\eta + \frac{1}{t}\right) \quad (25)$$

$$- \frac{(mk^2 - mk - m^2 - k^2)}{3m^2k^2t^2} \sec^2 \left\{ \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2(2\omega + 3)}} \log(\beta t) \right\} + \bar{Q}$$

$$8\bar{\varepsilon} = - \frac{(m^2k^2 - mk - m^2 - k^2)}{9m^2k^2t^2} \sec^2 \left\{ \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2(2\omega + 3)}} \log(\beta t) \right\} \quad (26)$$

$$- \frac{256\pi^2\eta^2}{3} - \frac{32\pi\eta}{3t} - \bar{Q}$$

and

$$\bar{Q} = \left(1 - \frac{1}{4\pi\mu}\right) \frac{(m^2k^2 - mk - m^2 - k^2)}{12m^2k^2t^2} \sec^2 \left\{ \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2(2\omega + 3)}} \log(\beta t) \right\} \quad (27)$$

$$- \frac{\eta}{\mu t} \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2}} \tan \left\{ \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2(2\omega + 3)}} \log(\beta t) \right\}$$

$$- \frac{40\pi\eta}{3t} - \frac{70\pi^2\eta^2}{3} - \left(16\pi\eta + \frac{1}{t}\right) \frac{(24\pi\xi - 3\xi)}{4}$$

Also the scalar field Λ is given by

$$\Lambda = \log \sec^2 \left\{ \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2(2\omega + 3)}} \log(\beta t) \right\} \quad (28)$$

The model will be real if the conditions $\bar{\varepsilon} > 0$, $\bar{p} > 0$, $\bar{\varepsilon} > 3\bar{p}$ hold when

$$(m^2k^2 - mk - m^2 - k^2) < 0, \omega < -\frac{3}{2}, \bar{Q} < 0, \mu \geq 1 \quad (29)$$

The average scale factor $R(t)$ is given by

$$R = (ABC)^{\frac{1}{3}}. \quad (30)$$

The Hubble parameter H is given by $H = \dot{R}/R$. Volume expansion θ , deceleration parameter q and shear σ for the metric (1.1) can be written as

$$H = \frac{16\pi\eta t + 1}{3t}, \quad (31)$$

showing that Hubble parameter is decreasing during time evolution and become constant when $t \rightarrow \infty$ and $-4 \leq q \leq -1$.

$$\theta = 3H = 3 \frac{\dot{R}}{R} = 16\pi\eta + \frac{1}{t}, \quad (32)$$

$$\frac{d\theta}{dt} = -\frac{1}{t^2}, \quad (32(a))$$

showing that the rate of volume expansion decreases during time evolution.

The non-zero components of shear tensor σ_j^i are

$$\sigma_1^1 = \frac{1}{3} \left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = -\frac{m + 2k}{3mkt} \quad (33)$$

$$\sigma_2^2 = \frac{1}{3} \left(\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right) = \frac{k - m}{3mkt}$$

$$\sigma_3^3 = \frac{1}{3} \left(\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = \frac{k + 2m}{3mkt}$$

$$\sigma_4^4 = -\frac{2}{3} \left(16\pi\eta + \frac{1}{t} \right)$$

and the shear σ is

$$\sigma^2 = \frac{1}{2} \left\{ (\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2 \right\} \quad (34)$$

$$= \frac{2}{3m^2k^2t^2} (m^2 + km + k^2) + \frac{2}{9} \left(16\pi\eta + \frac{1}{t^2} \right)^2$$

showing that the shear σ decreases during the time evolution.

The deceleration parameter is

$$q = -\frac{\ddot{R}}{RH^2} = -\left\{ 1 + \frac{3}{(16\pi\eta t + 1)^2} \right\} \quad (35)$$

showing that deceleration parameter increases during time evolution and become -1 when $t \rightarrow \infty$ and $-4 \leq q \leq -1$

5. TRANSFORMATIONS OF SOLUTIONS AND DISCUSSIONS

Under the transformation given by [5]:

$$\left. \begin{aligned} \bar{g}_{ij} &\rightarrow g_{ij} = \frac{1}{\phi} \bar{g}_{ij}, \bar{T}_{ij} \rightarrow T_{ij} = \phi \bar{T}_{ij} \\ \bar{T} &\rightarrow T = \phi^2 \bar{T}, \bar{p} \rightarrow p = \phi^2 \bar{p} \\ \bar{\varepsilon} &\rightarrow \varepsilon = \phi^2 \bar{\varepsilon}, \phi \rightarrow \phi = e^\Lambda \\ \bar{Q} &\rightarrow Q = \phi \bar{Q}, \bar{v}^i \rightarrow v^i = \phi^{1/2} \bar{v}^i \end{aligned} \right\} \quad (36)$$

The field equations (5)-(7) are changed into (1)-(3).

$$\phi = \sec^2 \left\{ \sqrt{\frac{m^2k^2 - mk - m^2 - k^2}{3m^2k^2(2\omega + 3)}} \log(\beta t) \right\} \quad (37(a))$$



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$$g_{ij} = \cos^2 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \bar{g}_{ij} \quad 37(b) \quad \text{i.e.}$$

$$g_{11} = \cos^2 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} t^{\frac{2(mk-m-2k)}{3mk}} e^{\frac{32\pi\eta}{3}}$$

$$g_{22} = \cos^2 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} t^{\frac{2(k-m+mk)}{3mk}} e^{\frac{32\pi\eta}{3}}$$

$$g_{33} = \cos^2 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} t^{\frac{2(k+mk+2m)}{3mk}} e^{\frac{32\pi\eta}{3}}$$

$$g_{44} = - \cos^2 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\}$$

$$\begin{aligned} 8\pi p = & \sec^4 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \left[\frac{8\pi\eta}{9t} + \frac{64\pi^2\eta^2}{9} + \frac{m^2 k^2 - mk - m^2 - k^2}{9m^2 k^2} \right. \\ & + \left(16\pi\eta + \frac{1}{t} \right) \left(\frac{3\xi}{4} + 2\pi\xi \right) - \frac{\eta}{\mu t} \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)(2\omega + 3)}{3m^2 k^2}} \tan \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \\ & - \frac{(12\pi\mu + 1)(m^2 k^2 - mk - m^2 - k^2)}{48\mu\pi m^2 k^2 t^4} \sec^6 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \end{aligned} \quad (4.13c)$$

$$\begin{aligned} 8\pi \bar{e} = & - \frac{(m^2 k^2 - mk - m^2 - k^2)(20\pi\mu - 1)}{48\mu\pi m^2 k^2 t^2} \sec^6 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \\ & + \sec^4 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \left[\frac{\eta}{\mu t} \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)(2\omega + 3)}{3m^2 k^2}} \times \right. \\ & \left. \tan \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} - \frac{64\pi^2\mu^2}{3} + \frac{8\pi\eta}{3t} + \left(16\pi\eta + \frac{1}{t} \right) \left(6\pi\xi - \frac{3}{4}\xi \right) \right] \end{aligned} \quad 37(d)$$

$$\begin{aligned} Q = & \left(\frac{4\pi\mu - 1}{16\pi\mu} \right) \frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 t^2} \sec^4 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \\ & - \sec^2 \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \left[\frac{\eta}{\mu t} \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)(2\omega + 3)}{3m^2 k^2}} \right. \\ & \left. \times \tan \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} - \frac{40\pi\eta}{3t} + \frac{640\pi^2\eta^2}{3} - \left(16\pi\eta - \frac{1}{t} \right) \left(6\pi\xi - \frac{3}{4}\xi \right) \right] \end{aligned} \quad 37(e)$$



$$v^1 = v^2 = v^3 = 0, v^4 = \sec \left\{ \sqrt{\frac{(m^2 k^2 - mk - m^2 - k^2)}{3m^2 k^2 (2\omega + 3)}} \log(\beta t) \right\} \quad 37(f)$$

The reality conditions (29) may be imposed on the solutions in 37(a)-37(f). The solution obtained in this paper is new. Both pressure p and density \mathcal{E} are positive and decreases during the time evolution. The cosmological term Q is negative which differ from Rai,Rai and Singh [5] solution. The viscosity prevents the free gravitational field. Equation (32) indicates that the effect of viscosity is to retard the expansion of the model.

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